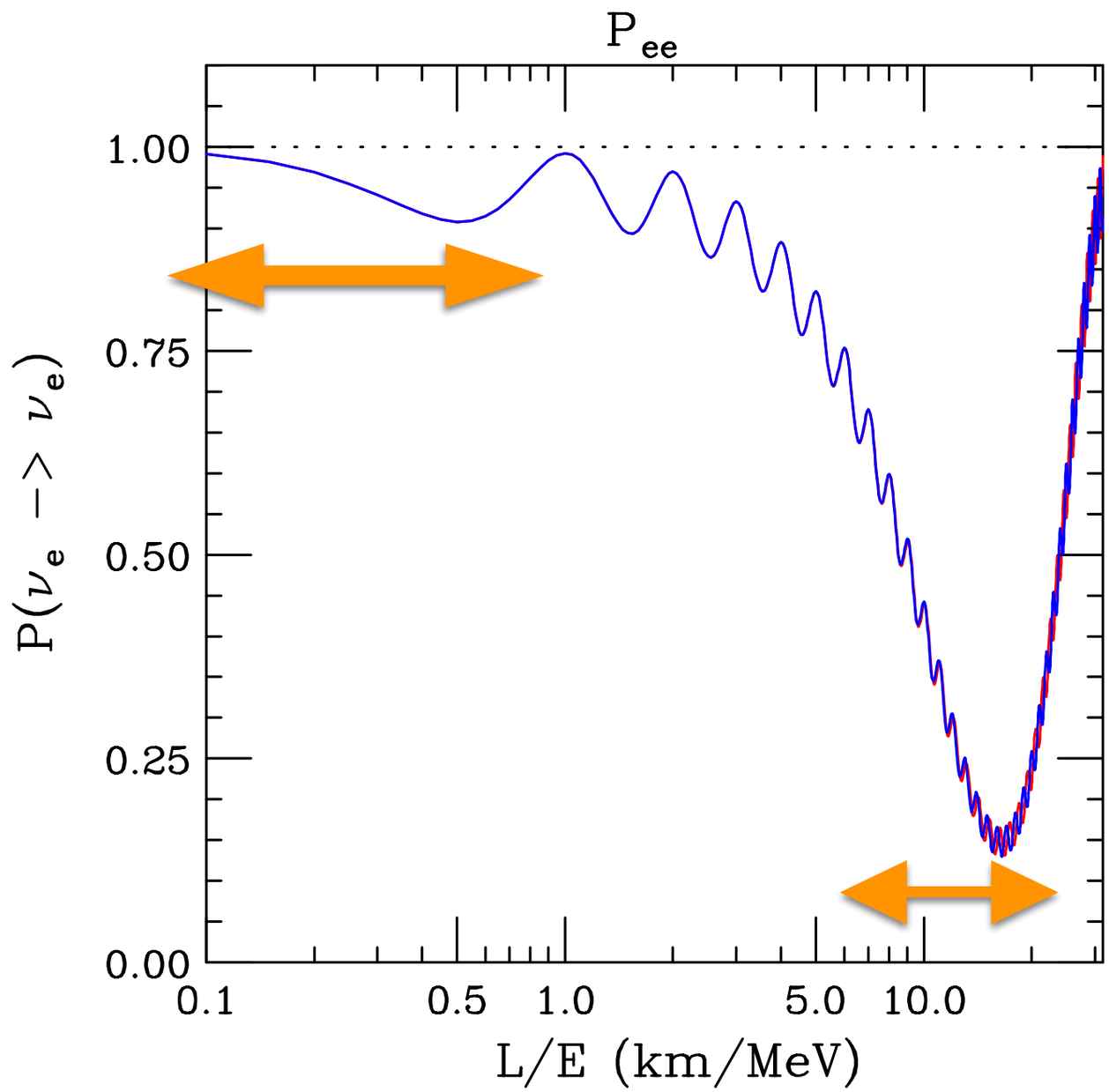
What is δm^2_{ee} ?

Stephen Parke
Fermilab

Daya Bay
RENO
D-Chooz



JUNO
RENO 50

Amplitude Modulation & Phase Advancement (NO) / Retardation (IO)

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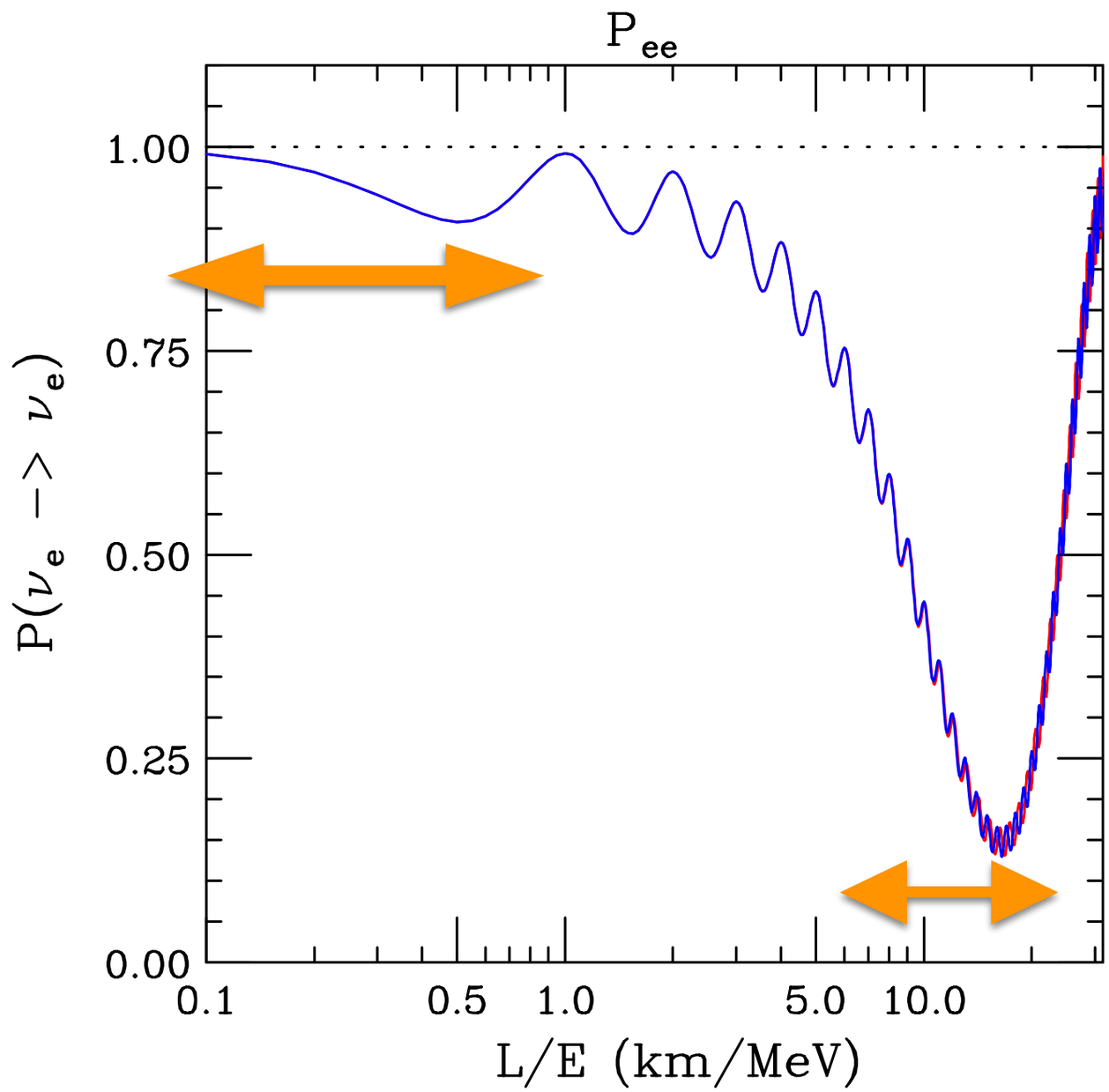
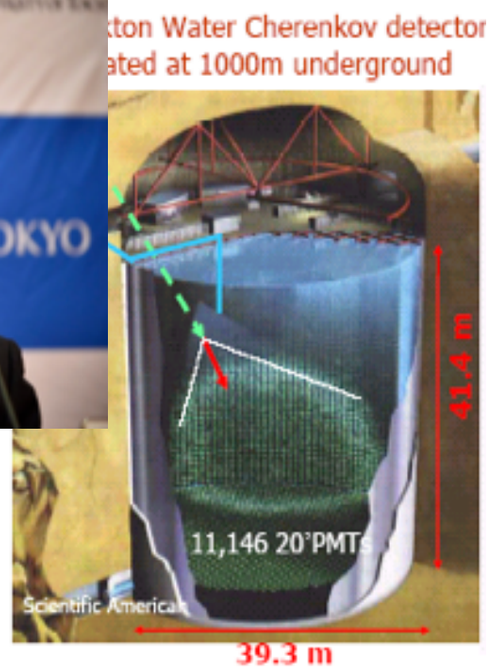


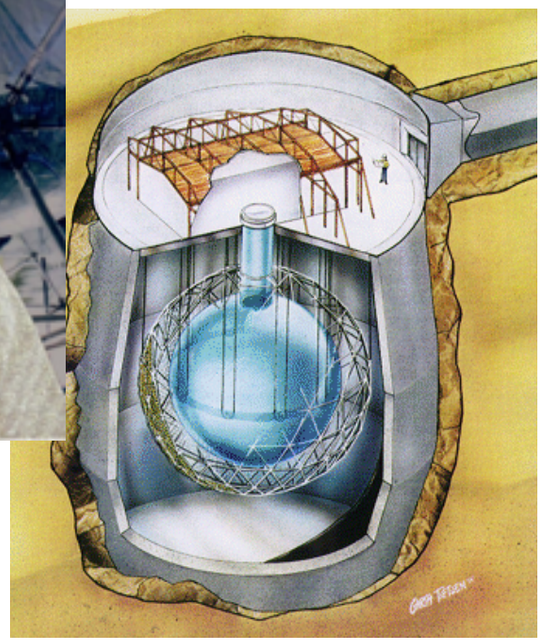
JUNO
RENO 50

Amplitude Modulation & Phase Advancement (NO) / Retardation (IO)

NOBEL 2015



Takaaki Kajita
SuperKamiokaNDE



Art MacDonald
SNO

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

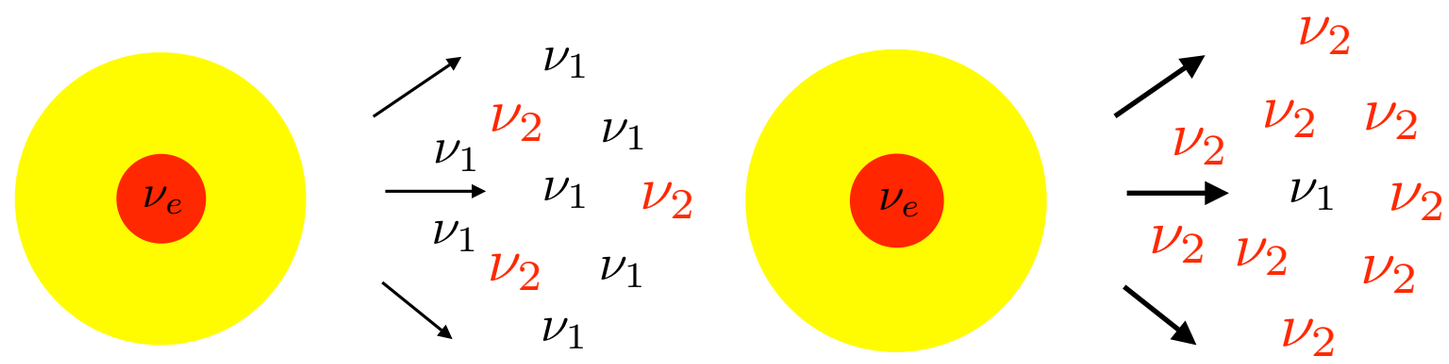
"for the discovery of neutrino flavor transformations, which shows that neutrinos have mass"

SNO circa 2001

CC: $\nu_e + D \to p + p + e^-$

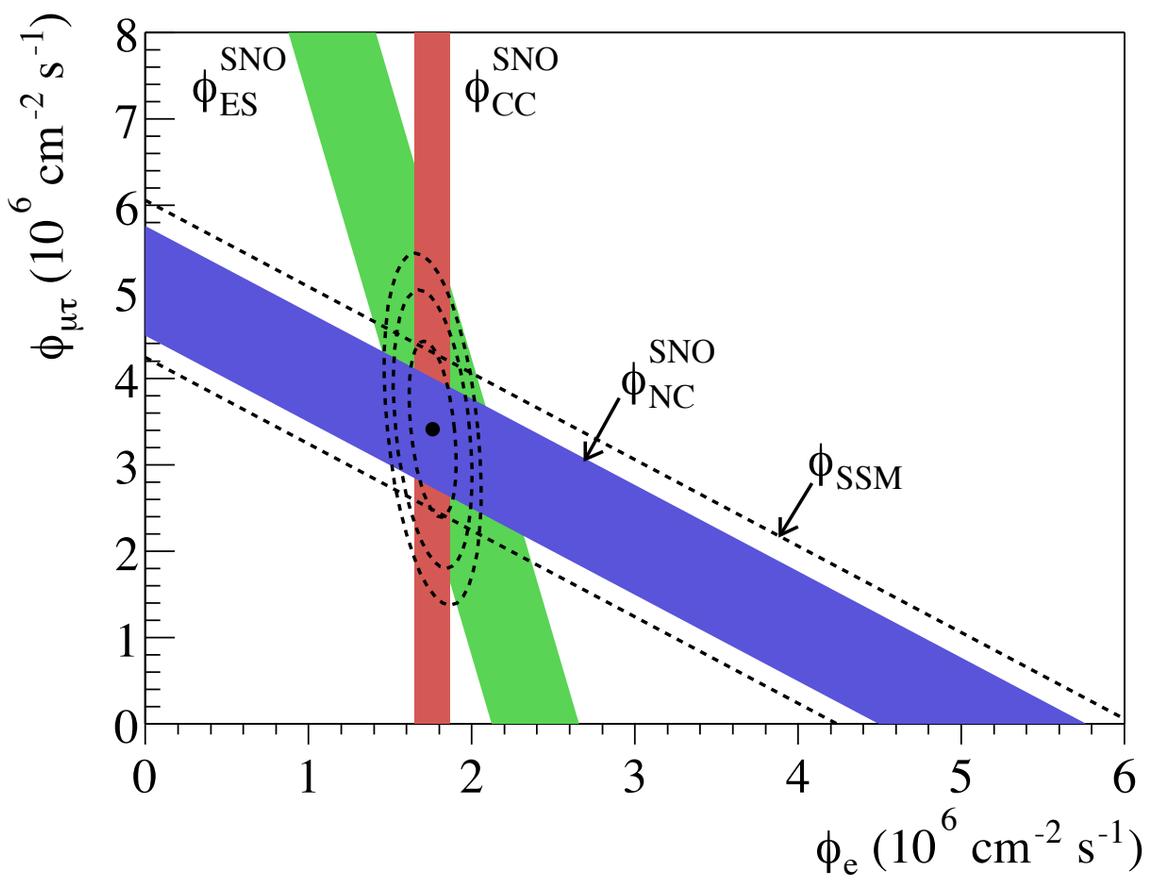
NC: $\nu_x + D \to p + n + \nu_x$

ES: $\nu_e + e^- \to \nu_e + e^-$
and $\nu_{\mu/\tau} + e^- \to \nu_{\mu/\tau} + e^-$



70% ν_1

$> 90\%$ ν_2



- ^8B neutrinos exit the Sun as nearly pure ν_2 ($> 90\%$), so that $\frac{CC}{NC} \approx \sin^2\theta_{12}$

why use $\tan^2\theta$?

- Mass Ordering of ν_1 & ν_2:

$m_2 > m_1$

(using defn $|U_{e2}|^2 < |U_{e1}|^2$)

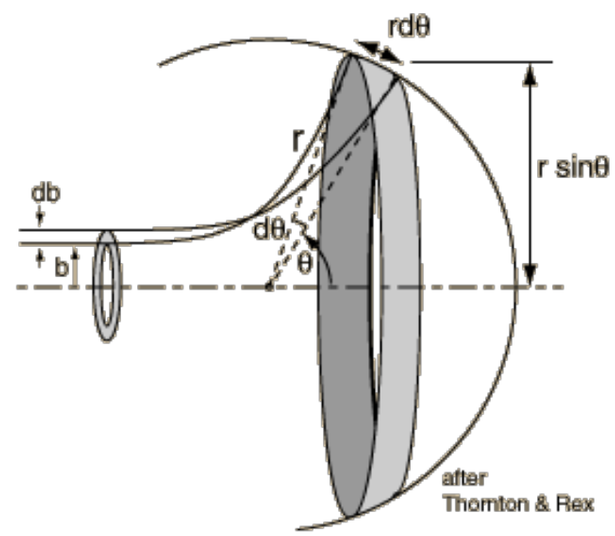
Beacom and SP: hep-ph/0106128

Ernest Rutherford, master of simplicity

By Ashutosh Jogalekar | August 30, 2013 | 6

Ernest Rutherford, emperor of the atomic domain (Image: Wikipedia Commons)

"theorists play games with their symbols while we discover truths about the universe". And yet



$$\sigma = \pi Z^2 \left(\frac{ke^2}{KE}\right)^2 \left(\frac{1+\cos\theta}{1-\cos\theta}\right)$$

he had an eye for theoretical talent that allowed him to nurture Niels Bohr, as dyed-in-the-wool a theoretician and philosopher as you could find.

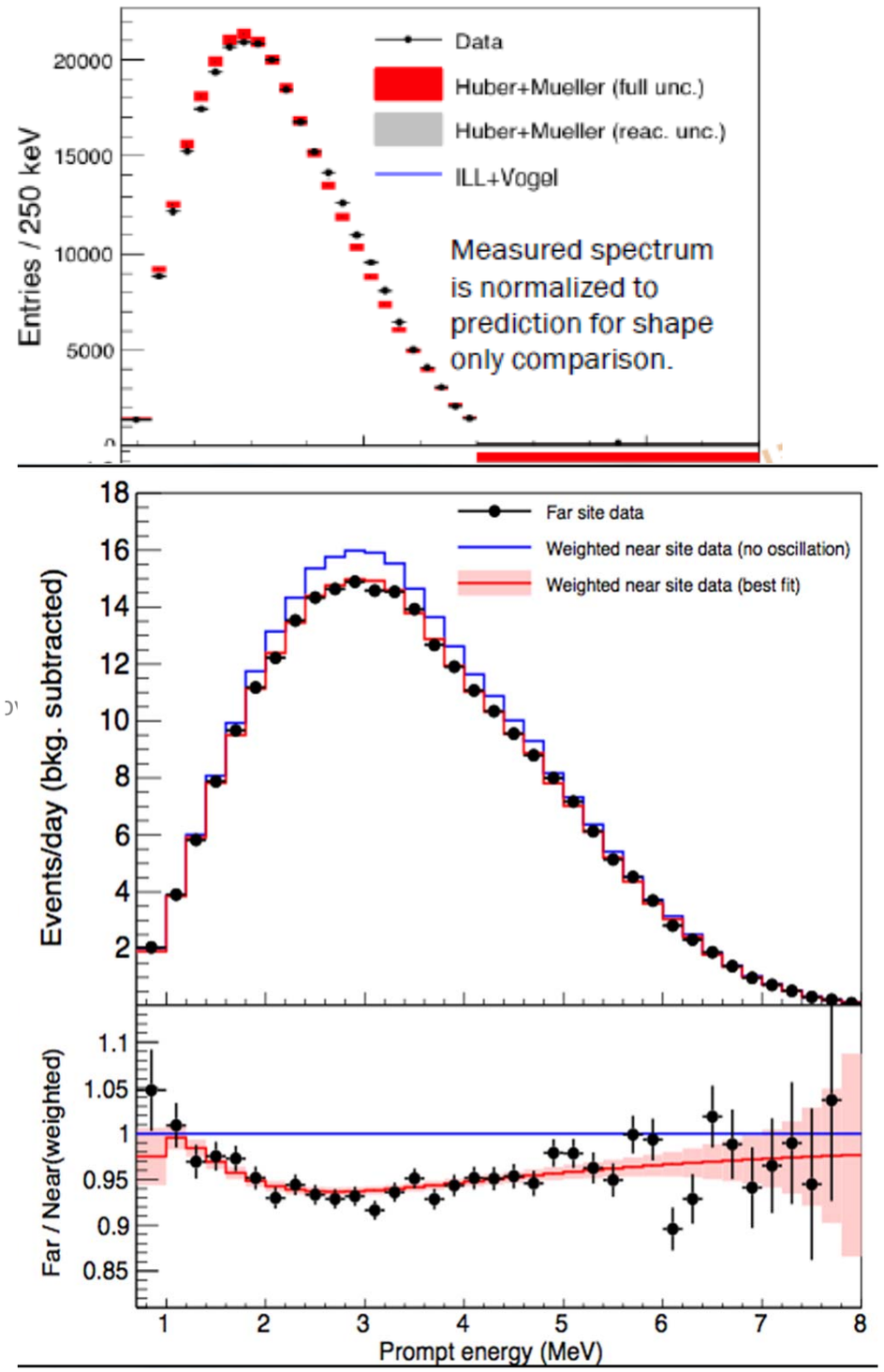
Reactor θ_{13} Experiments

Daya Bay

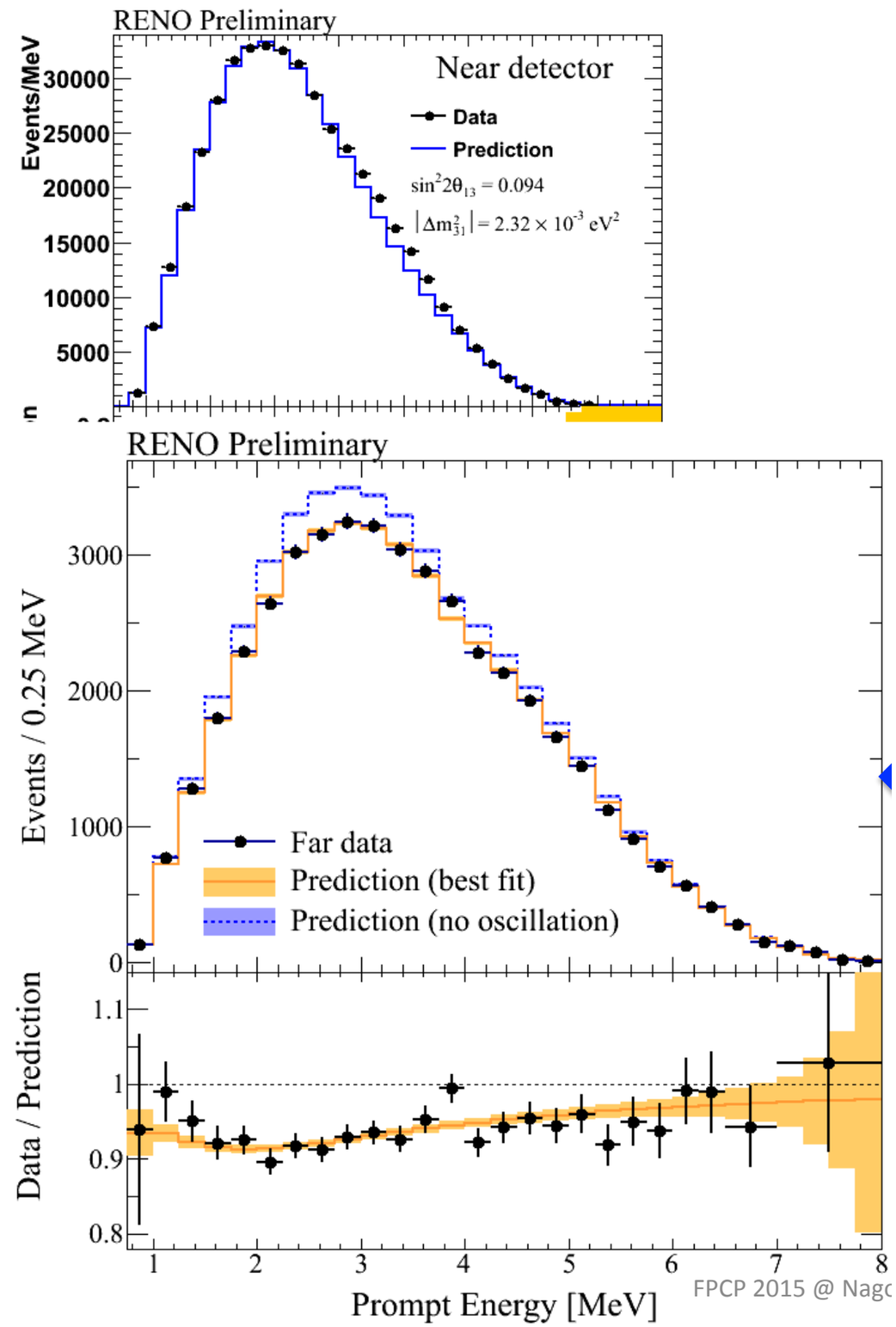
Double Chooz

RENO

Daya Bay



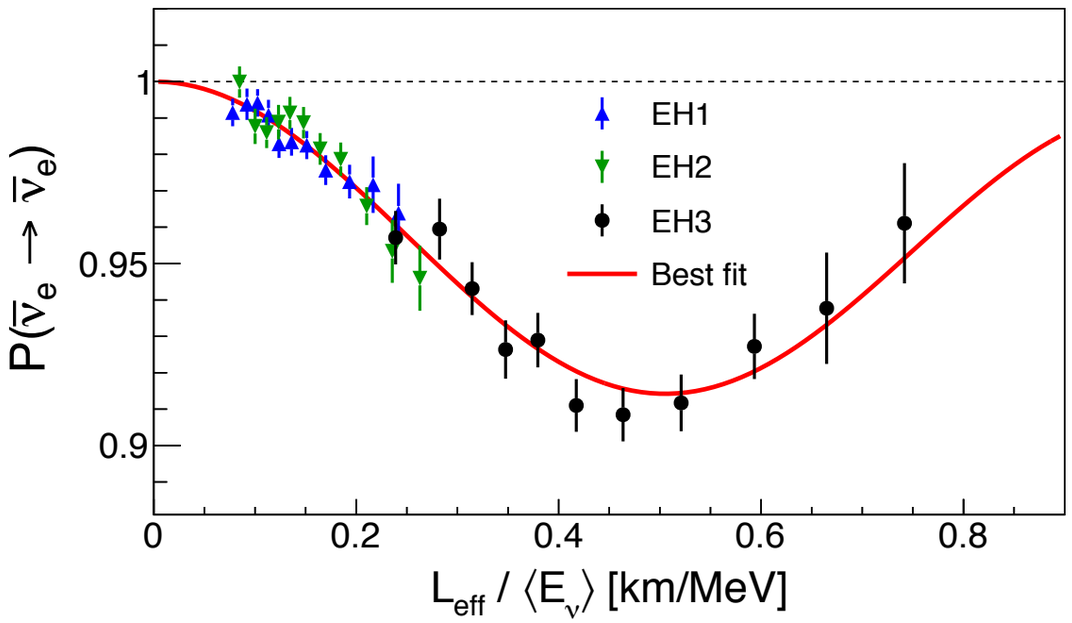
RENO



FPCP 2015 @ Nago

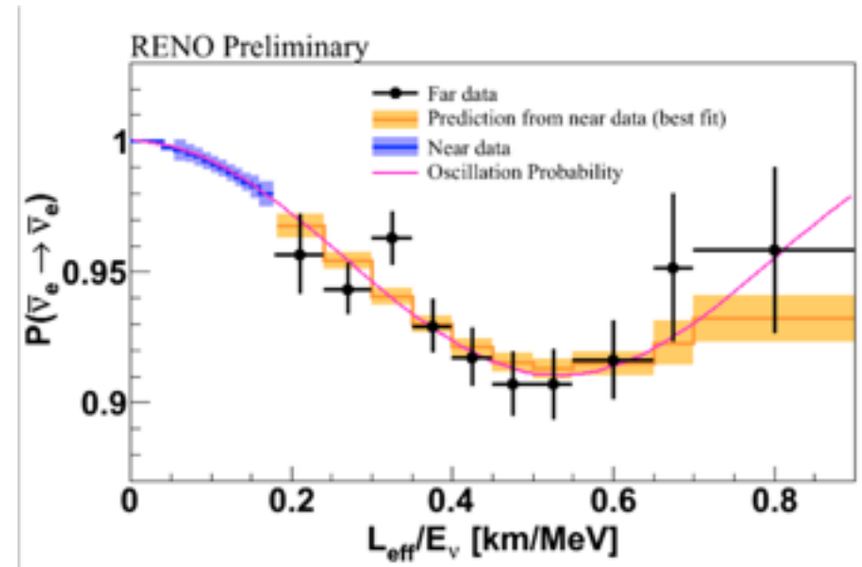
What is δm^2_{ee} ?

from Daya Bay: arXiv:1505.03456



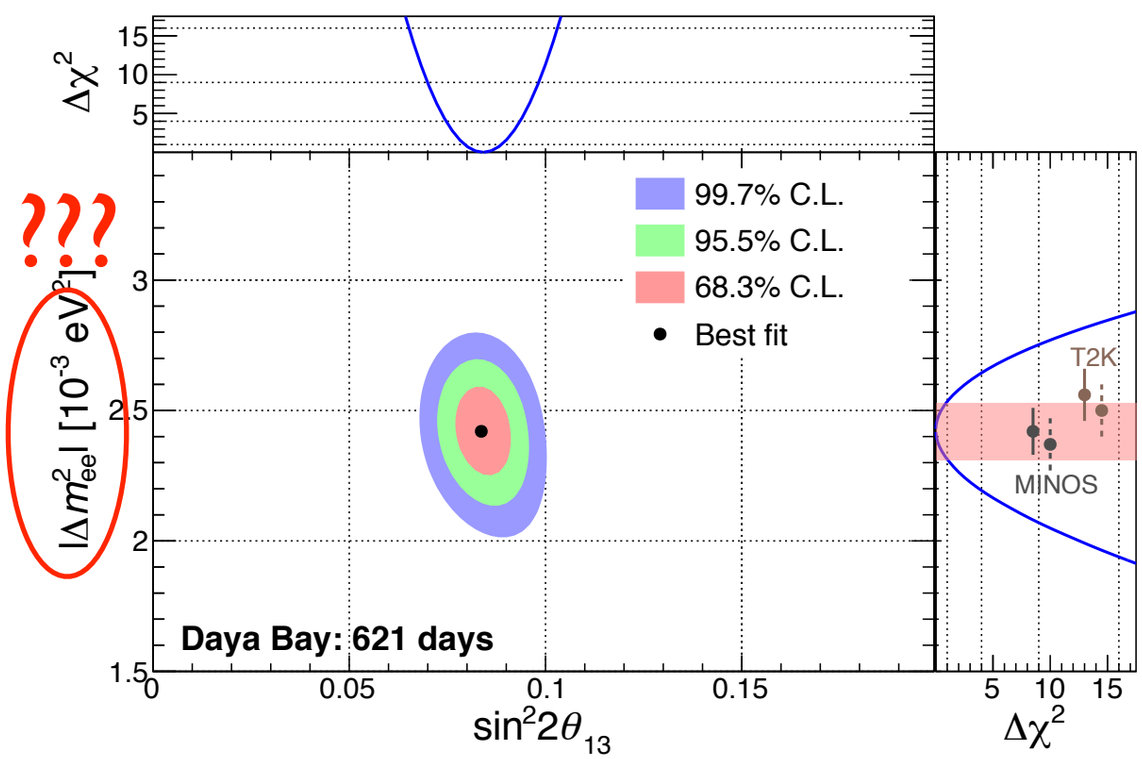
RENO

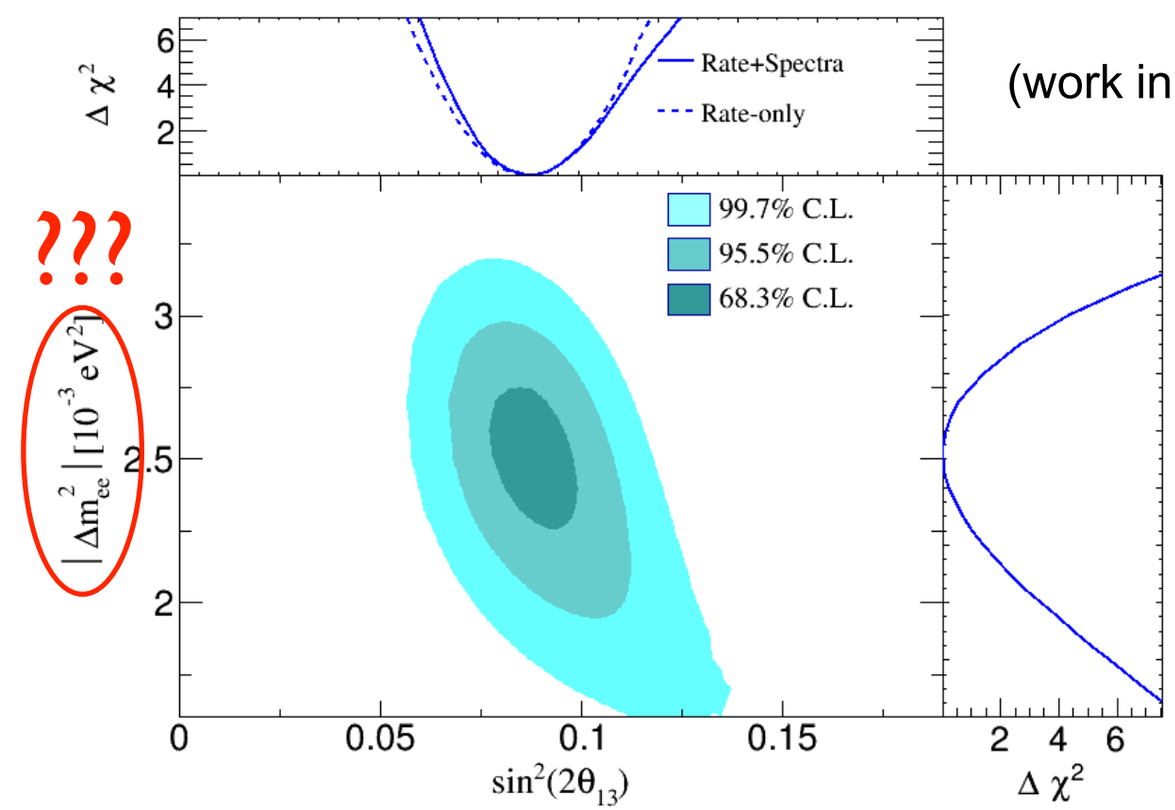
NeuTel 2015



possible in Double Chooz also!

NDM 2015





(work in prog

- **Daya Bay**

$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$ 4 % measurement !

Analysis of the relative antineutrino rates and energy spectra between detectors gave $\sin^2 2\theta_{13} = 0.084 \pm 0.005$ and $|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$ in the three-neutrino framework.

- **RENO**

$\Delta m_{ee}^2 = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$

10 % measurement !

- **JUNO & RENO 50**

expect ~0.5 % measurement !!!

NO: $|\Delta m^2_{31}| = (1+3\%)|\Delta m^2_{32}|$ IO: $|\Delta m^2_{31}| = (1-3\%)|\Delta m^2_{32}|$

Δm^2_{ee} is, obviously, some combination of Δm^2_{31} and Δm^2_{32} !

But What Combination ?

Desirable Features:

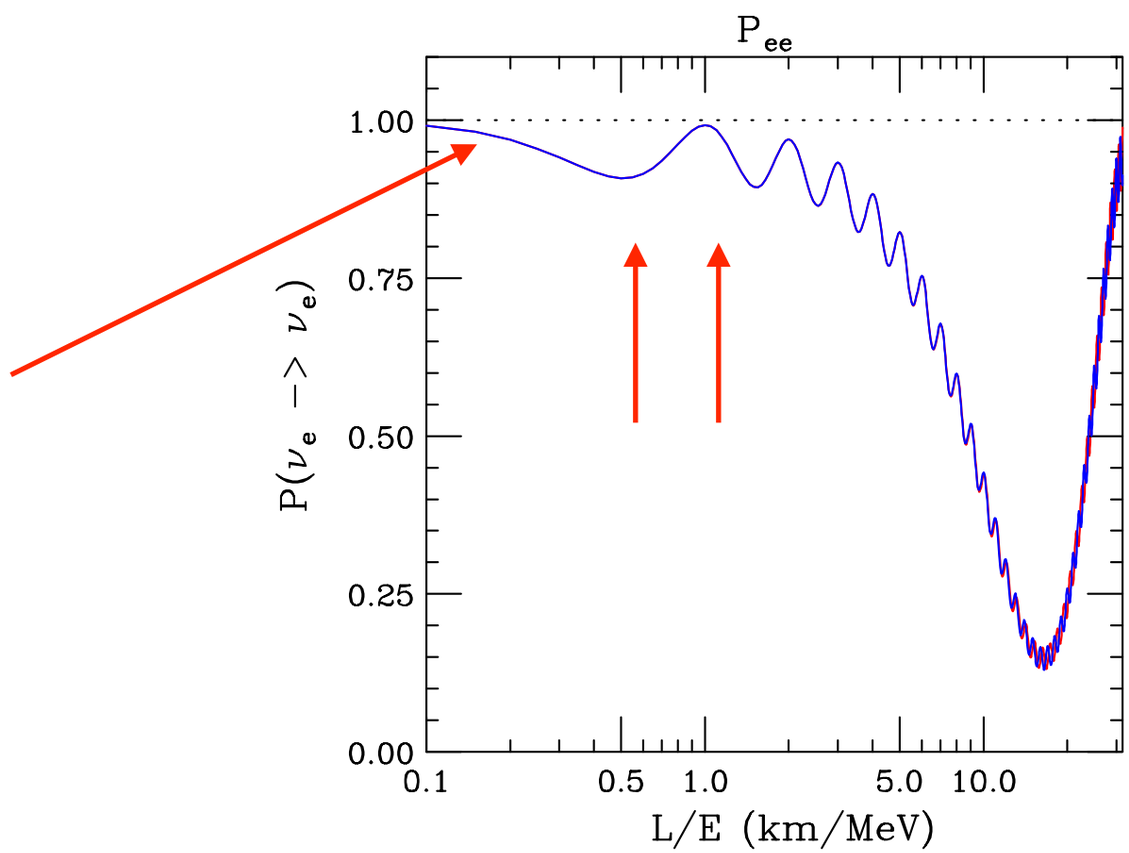
- L/E independent
- Simple relationship to Δm^2_{31} and Δm^2_{32}
- Useful for experiments at any L

In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

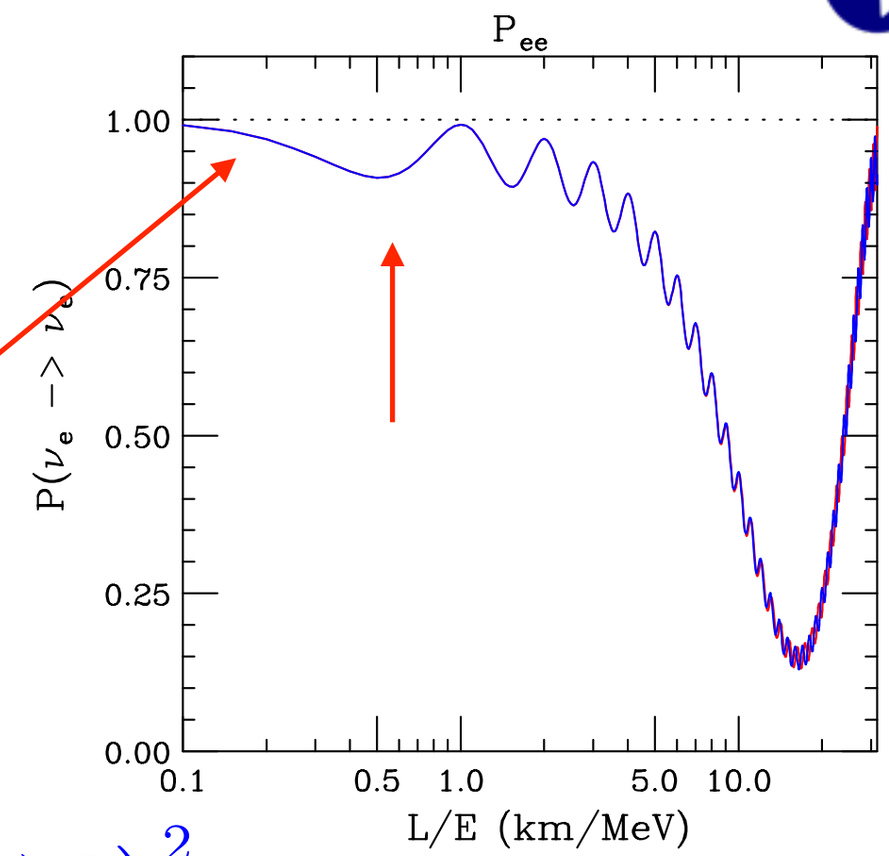
$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2 \Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2 \Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2 \Delta_{32} \\ &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \qquad (< 0.01) \\ &\quad - \sin^2 2\theta_{13}(\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \quad (< 0.1) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{rr})



- Where is the first minima ?

$$\frac{L}{E} \approx \frac{2\pi}{(\ \cos^2\theta_{12}\ \Delta m^2_{31} + \sin^2\theta_{12}\ \Delta m^2_{32}\ )}$$



- As L/E $\rightarrow$ 0

$$P \approx 1 - \sin^2 2\theta_{13} \left(\frac{(\cos^2\theta_{12}\ \Delta m^2_{31} + \sin^2\theta_{12}\ \Delta m^2_{32})\ L}{4E} \right)^2$$

So how about ?

$$\Delta m^2_{ee} = \cos^2\theta_{12}\ \Delta m^2_{31} + \sin^2\theta_{12}\ \Delta m^2_{32}$$

ν_e average !

$$\sqrt{(c_{12}^2\ \Delta m_{31}^4 + s_{12}^2\ \Delta m_{32}^4)} \ \approx\ c_{12}^2\ \Delta m_{31}^2 + s_{12}^2\ \Delta m_{32}^2$$

accurate to 0.02%

- Important ID:

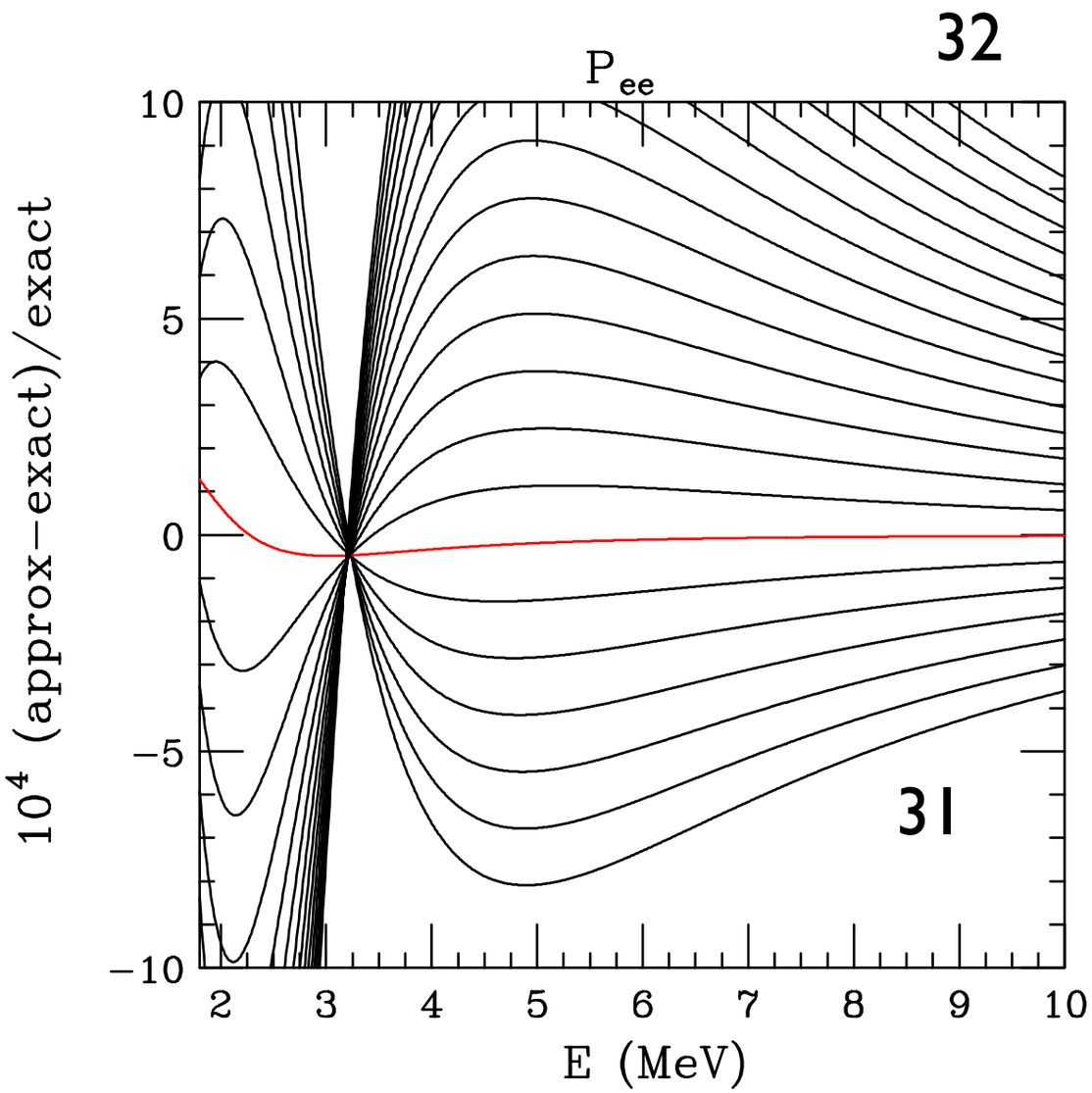
$$(c_{12}^2\ \Delta m_{31}^2 + s_{12}^2\ \Delta m_{32}^2)^2 \ =\ (c_{12}^2\ \Delta m_{31}^4 + s_{12}^2\ \Delta m_{32}^4) - c_{12}^2 s_{12}^2\ \Delta m_{21}^4$$

replace $\Delta m_{21}^2 = \Delta m_{31}^2 - \Delta m_{32}^2$ and it's simple to prove.

(where $\Delta m_{ij}^4 \equiv (\Delta m_{ij}^2)^2$)

$$P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{rr}$$

$$\Delta m^2_{rr} = (1-r)\,\Delta m^2_{31} + r\,\Delta m^2_{32}$$



r=1

L=1.6 km

r=0, 0.05, 0.1 …1.0

r=0

$$r = \sin^2\theta_{12}$$

$$\Delta m^2_{ee} = \cos^2\theta_{12}\,\Delta m^2_{31} + \sin^2\theta_{12}\,\Delta m^2_{32}$$

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Note, the Δ_{21} terms vanishes !

& coeff. of Δ_{21}^2 minimum !

Taylor Series Expansion:

$\Delta_{31} = \Delta_{ee} + s_{12}^2\ \Delta_{21}$ and $\Delta_{32} = \Delta_{ee} - c_{12}^2\ \Delta_{21}$

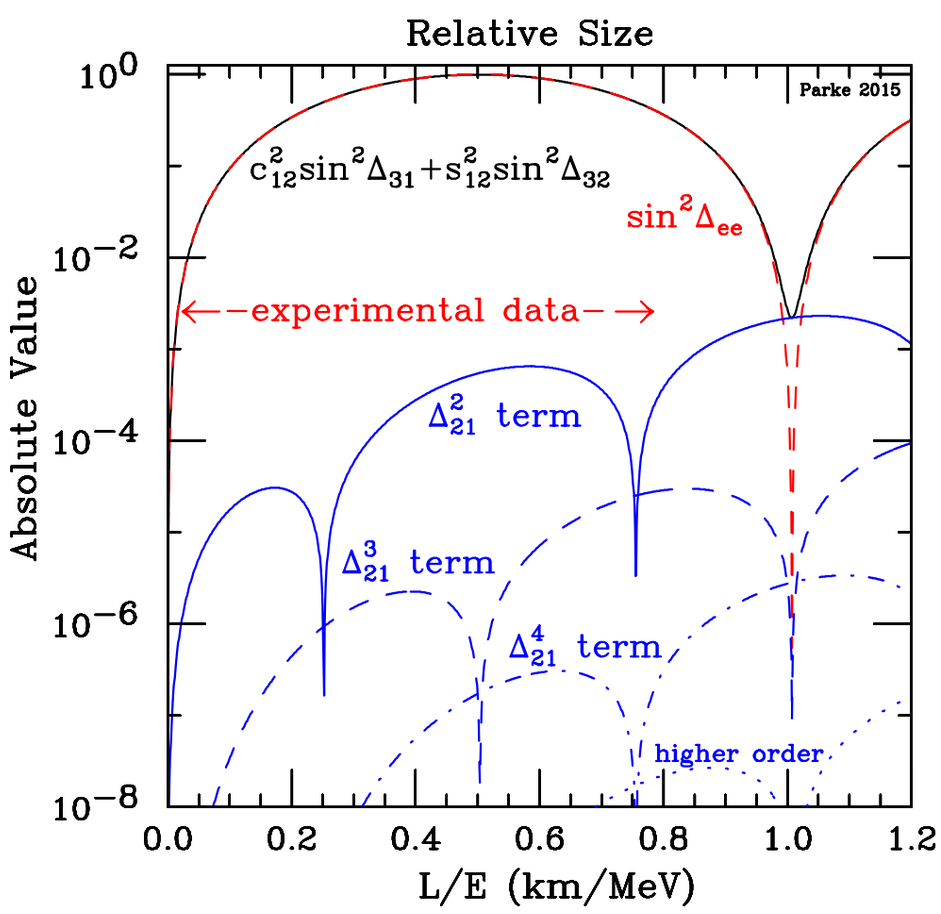
improvement

$$c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{31} = \sin^2 \Delta_{ee} + 0 + s_{12}^2 c_{12}^2 \Delta_{21}^2 \cos(2\Delta_{ee})$$

$$-\frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{12}\ \Delta_{21}^3\ \sin(2\Delta_{ee}) + \mathcal{O}(\Delta_{21}^4)$$

$$= 1 + \mathcal{O}(10^{-3}) \pm \mathcal{O}(10^{-5}) \quad \text{at OM}$$

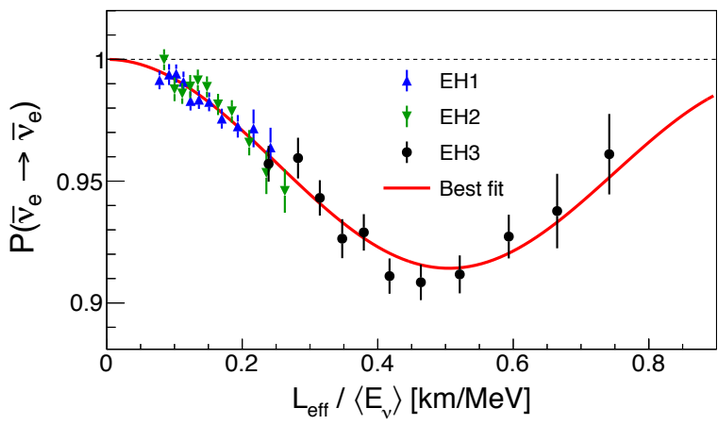
Mass Ordering effects !

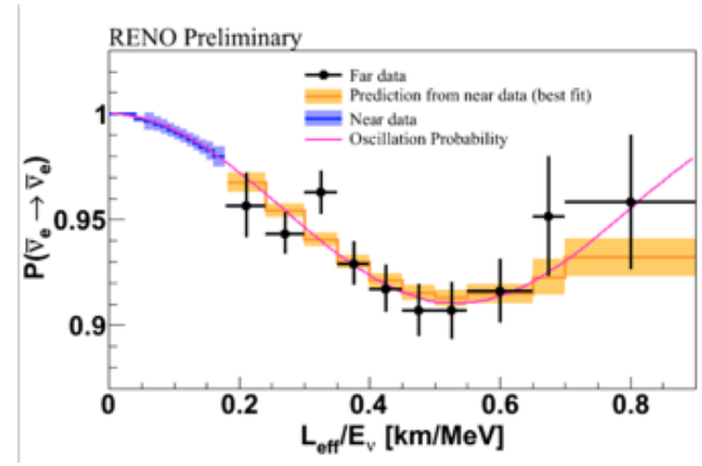


H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, "Another possible way to determine the neutrino mass hierarchy," Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

What Do the Experiments Do ?

What Do the Experiments Do ?





Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$
- using L/E independent θ_{13} and $|\Delta m^2_{ee}|$. (θ_{12} and Δm^2_{21} obtained from other experiments.)
- θ_{13} and $|\Delta m^2_{ee}|$ are only weakly dependent on solar parameters and are INDEPENDENT of mass ordering.
- Trivial to calculate $|\Delta m^2_{31}| = |\Delta m^2_{ee}| \pm \sin^2\theta_{12}\Delta m^2_{21}$ and $|\Delta m^2_{32}| = |\Delta m^2_{ee}| \mp \cos^2\theta_{12}\Delta m^2_{21}$ using mass ordering and solar parameters. But $|\Delta m^2_{31}|$ and $|\Delta m^2_{32}|$ are more dependent on solar parameters than $|\Delta m^2_{ee}|$!

One Possible Way:

$$
\begin{aligned}
P \quad = \quad & 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\
& - \sin^2 2\theta_{13}\ \sin^2|\Delta_{31}| \\
& \pm \frac{1}{2}\sin^2 2\theta_{13}\sin^2\theta_{12}\ \sin 2|\Delta_{31}|\sin 2\Delta_{21} \\
& - \sin^2 2\theta_{13}\sin^2\theta_{12}\ \cos 2|\Delta_{31}|\sin^2\Delta_{21} \qquad (1)
\end{aligned}
$$

with $+$ for NO and - for IO . Run analysis twice: $|\Delta m^2_{31}|$ and $|\Delta m^2_{31}|$.

More sensitive to solar parameters AND

$$|\Delta m^2_{31}| - \sin^2\theta_{12}\Delta m^2_{21} = |\Delta m^2_{ee}| = |\Delta m^2_{31}| + \sin^2\theta_{12}\Delta m^2_{21}$$

(using 32 would be similar)

How Does Daya Bay Define Δm^2_{EE} ?

arXiv:1310.6732

ν_j and ν_i. Since $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ [1], the short-distance (~km) reactor $\overline{\nu}_e$ oscillation is due primarily to the Δ_{3i} terms and naturally leads to the definition of the effective mass-squared difference $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$ [11].

1505.03456v1

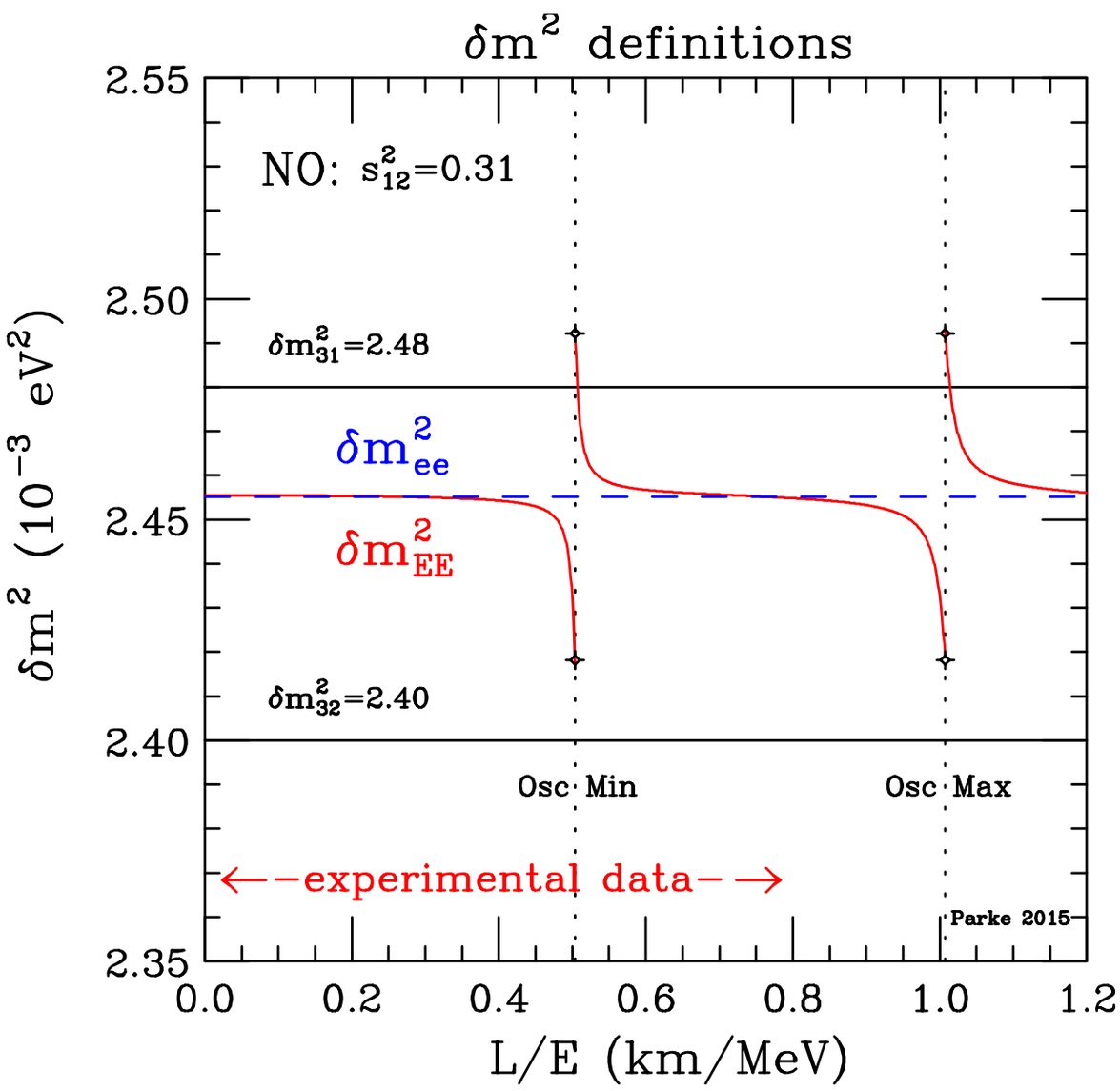
[8] $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$, where $\Delta_{ji} \equiv 1.267 \Delta m^2_{ji}(\mathrm{eV}^2)[L(\mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i.

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32} \quad (1)$$

$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32})}\right]$$



Maladies:

- L/E dependent
- Discontinuous at Osc. Max./Min. (L/E $\approx$ 0.5, 1.0, .. km/MeV) 3% jump
- no simple physical meaning !

Why???

RHS (1) never gets exactly to 1, or back to 0 whereas LHS does !

eg $\sin^2(\frac{\pi}{2} \mp \epsilon) = 1 - \epsilon^2 + \mathcal{O}(\epsilon^4)$
with $\epsilon = s_{12}c_{12}\Delta_{21}$

After I pointing this out Daya Bay changed it's definition of Delta m^2_ee

[8] Δm^2_{ee} is an effective mass splitting that can be obtained by replacing $\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}$ with $\sin^2\Delta_{ee}$, where $\Delta_{ji} \equiv 1.267\Delta m^2_{ji}(\mathrm{eV}^2)[L(\ \mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i. To estimate the values of Δm^2_{31} and Δm^2_{32} from the measured value of Δm^2_{ee}, See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.115.111802.

Supplemental material: Why Δm^2_{ee} is used by Daya Bay

(The Daya Bay Collaboration)

(Dated: July 27, 2015)

This note describes the advantages of reporting the Daya Bay measurement of electron antineutrino disappearance in terms of an effective mass-squared difference Δm^2_{ee}, which is independent of the unknown ordering of neutrino masses and future improvements in our knowledge of the solar oscillation parameters.

INTRODUCTION

In the three-flavor framework, the survival probability of electron antineutrino is given by

$$\begin{aligned} &P(\overline{\nu}_e \to \overline{\nu}_e) \\ &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ &\quad - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}), \end{aligned} \tag{1}$$

where $\Delta_x = \Delta m^2_x \frac{L}{4E}$. The three mass-squared differences are subject to the constraint $|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm |\Delta m^2_{21}|$ where "+"("−") is for the normal(inverted) mass ordering (or hierarchy). Therefore, determination of Δm^2_{32} (or Δm^2_{31}) depends on knowledge of the mass ordering and solar oscillation parameters.

The Daya Bay experiment reports a precise measurement of the effective mass splitting Δm^2_{ee}, which is independent of our knowledge of the ordering and solar parameters. In this approach, we approximate the survival probability using

$$\begin{aligned} P(\overline{\nu}_e \to \overline{\nu}_e) \simeq &1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ &- \sin^2 2\theta_{13}\sin^2\Delta_{ee}. \end{aligned} \tag{2}$$

Despite the advantage of using Δm^2_{ee} for the measurement, it has the disadvantage of not being a fundamental parameter. Therefore, we must determine a relation between Δm^2_{ee} and Δm^2_{32} given knowledge of the mass ordering and solar oscillation parameters.

In the following sections, we are going to address the following two questions:

- Is Eq. 2 good enough at the current experimental precision?
- How can we estimate the value of Δm^2_{32} once the value of Δm^2_{ee} is obtained?

MATHEMATICAL DERIVATION

Using the relation $|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm |\Delta m^2_{21}|$, Eq. 1 can be written as,

$$\begin{aligned} P(\overline{\nu}_e \to \overline{\nu}_e) = &1 - 2s^2_{13}c^2_{13} \\ &+ 2s^2_{13}c^2_{13}\sqrt{1 - 4s^2_{12}c^2_{12}\sin^2\Delta_{21}}\cos(2\Delta_{32} \pm \phi) \\ &- 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21}, \end{aligned} \tag{3}$$

where $s_x = \sin\theta_x$, $c_x = \cos\theta_x$, and $\phi = \arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2\theta_{12}}\right)$. The last term of the above formula is the so-called "solar term" that governs the reactor antineutrino oscillation at O(100) km. For the L/E range covered by Daya Bay, $4s^2_{12}c^2_{12}\sin^2\Delta_{21} \ll 1$. Thus, Eq. 3 can be approximated as,

$$\begin{aligned} &P(\overline{\nu}_e \to \overline{\nu}_e) \\ &\simeq 1 - 4s^2_{13}c^2_{13}\left[\frac{1 - \cos(2\Delta_{32} \pm \phi)}{2}\right] - (solar\ term) \\ &= 1 - \sin^2 2\theta_{13}\sin^2(\Delta_{32} \pm \phi/2) - (solar\ term). \end{aligned} \tag{4}$$

By comparing Eq. 4 with Eq. 2, we obtain the expression relating Δm^2_{ee} to Δm^2_{32} (or Δm^2_{31})

$$|\Delta m^2_{ee}| = |\Delta m^2_{32}| \pm \Delta m^2_{\phi}/2 \tag{5}$$

$$= |\Delta m^2_{31}| \mp (|\Delta m^2_{21}| - \Delta m^2_{\phi}/2), \tag{6}$$

where $\Delta m^2_{\phi} = \phi \times \frac{4E}{L}$.

NUMERICAL EVALUATION

By definition, Δm^2_{ϕ} is a function of L/E. Using the current values of $\Delta m^2_{21} = 7.50 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12} = 0.857$ [1], Fig. 1 shows the value of $\Delta m^2_{\phi}/2$ as a function of energy for L = 1.6 km. We find that $\Delta m^2_{\phi}/2 \simeq 5.17 \times 10^{-5}$ eV^2 is essentially a constant in our L/E region, and numerically identical to $\cos^2\theta_{12}\Delta m^2_{21}$. Thus, this definition of Δm^2_{ee} is similar to the definition introduced in Ref. [2]:

$$\Delta m^2_{\text{eff}}|_e = \cos^2\theta_{12}|\Delta m^2_{31}| + \sin^2\theta_{12}|\Delta m^2_{32}| \tag{7}$$

$$= |\Delta m^2_{32}| \pm \cos^2\theta_{12}\Delta m^2_{21}. \tag{8}$$

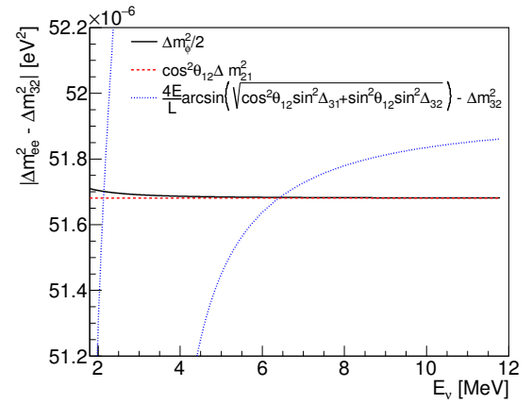


FIG. 1. Values of $\Delta m^2_{\phi}/2 = |\Delta m^2_{ee} - \Delta m^2_{32}|$ (black solid line) at L = 1.6 km as a function of the neutrino energy, with $\Delta m^2_{21} = 7.50 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12} = 0.857$ [1]. For comparison, calculations based on other definitions of Δm^2_{ee}, $\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$ (red dashed line) and $\Delta m^2_{ee} = \frac{4E}{L}\arcsin\left[\sqrt{\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}}\right]$ (blue dotted line) are also shown.

Figure 2 is a comparison of the approximated formula with $\Delta m^2_{\phi}/2 = 5.17 \times 10^{-5}$ eV^2,

$$\begin{aligned} P_{ee} \simeq &1 - \sin^2 2\theta_{13}\sin^2\left[(\Delta m^2_{32} + 5.17 \times 10^{-5}\ \text{eV}^2)\frac{L}{4E}\right] \\ &- (solar\ term), \end{aligned} \tag{9}$$

to the three-flavor formula, Eq. 1. In this comparison, L = 1.6 km, $\sin^2 2\theta_{13} = 0.09$, $\Delta m^2_{32} = 2.44 \times 10^{-3}$ eV^2, and normal mass hierarchy are the inputs. The agreement between the two, better than 10^{-4}, is excellent and exceeds the achievable experimental precision.

This study demonstrates that, once we obtain the value of $|\Delta m^2_{ee}|$ using Eq. 2, we can reliably deduce the values of $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$ using Eqs. 5 and 6 with

$$\Delta m^2_{\phi}/2 \simeq \cos^2\theta_{12}\Delta m^2_{21}. \tag{10}$$

Using the current values of θ_{12} and Δm^2_{21}, $\Delta m^2_{\phi}/2 \simeq 5.17 \times 10^{-5}$eV^2, and $(|\Delta m^2_{21}| - \Delta m^2_{\phi}/2) \simeq 2.33 \times 10^{-5}$ eV^2.

It is important to point out that the exact solution of $\sin^2(\Delta m^2_{ee}\frac{L}{4E}) = \cos^2\theta_{12}\sin^2(\Delta m^2_{31}\frac{L}{4E}) + \sin^2\theta_{12}\sin^2(\Delta m^2_{32}\frac{L}{4E})$ was never used to extract the value of Δm^2_{32} or Δm^2_{31} from the measured Δm^2_{ee} in Daya Bay.

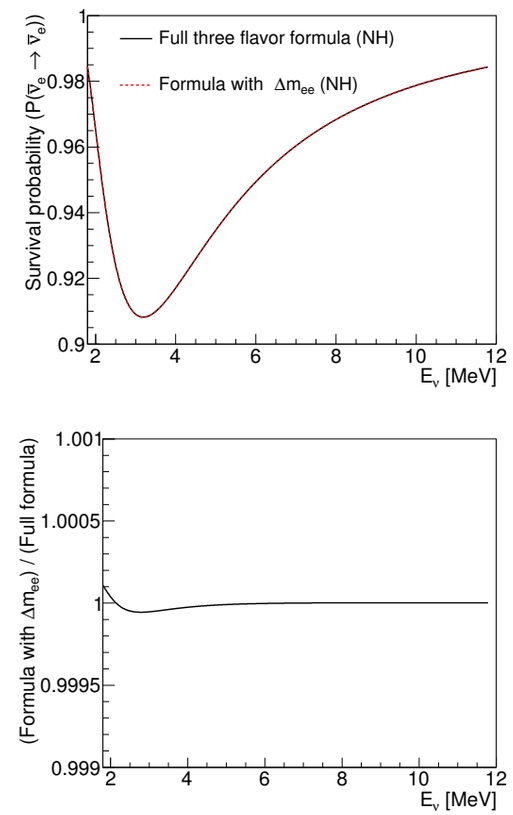
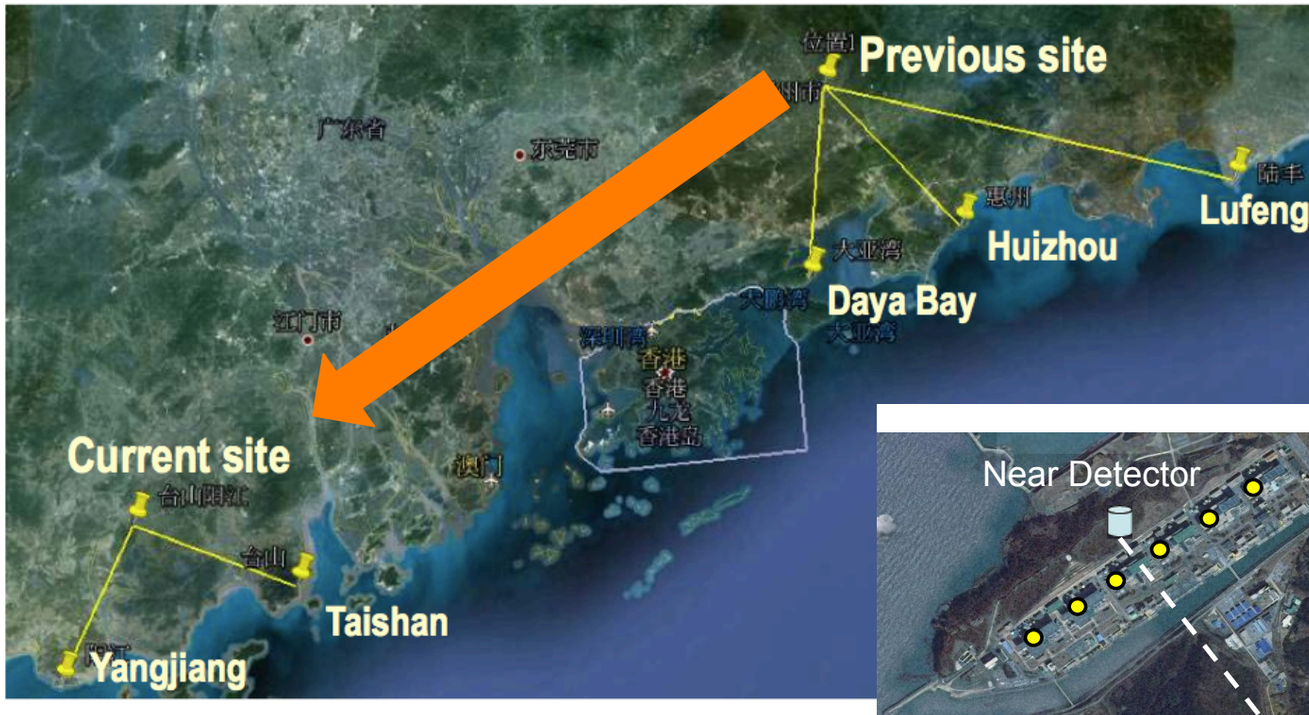


FIG. 2. Comparison of the survival probability at L = 1.6 km between the approximated formula with $\Delta m^2_{ee} = \Delta m^2_{32} + 5.17 \times 10^{-5}$ eV^2 and the exact three-flavor formula (Eq. 1). The oscillation parameters used in this comparison are $\sin^2 2\theta_{13} = 0.09$ and $\Delta m^2_{32} = 2.44 \times 10^{-3}$ eV^2 under the normal mass hierarchy assumption. The top panel shows the survival probabilities calculated with the two formulae, and the bottom panel shows the ratio of the two.

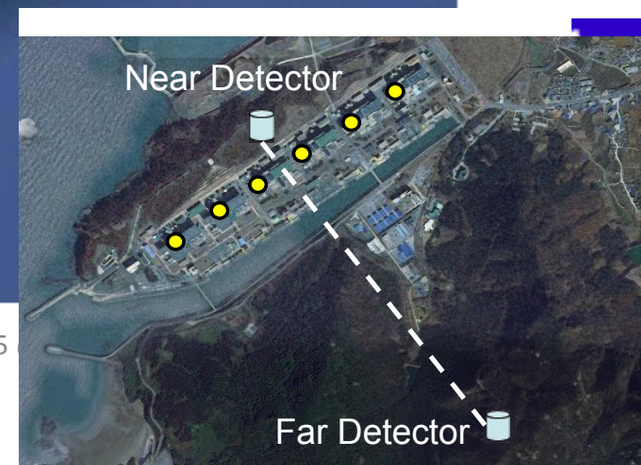
[1] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012), Section 13.
[2] H. Nunokawa, S. J. Parke, and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005).

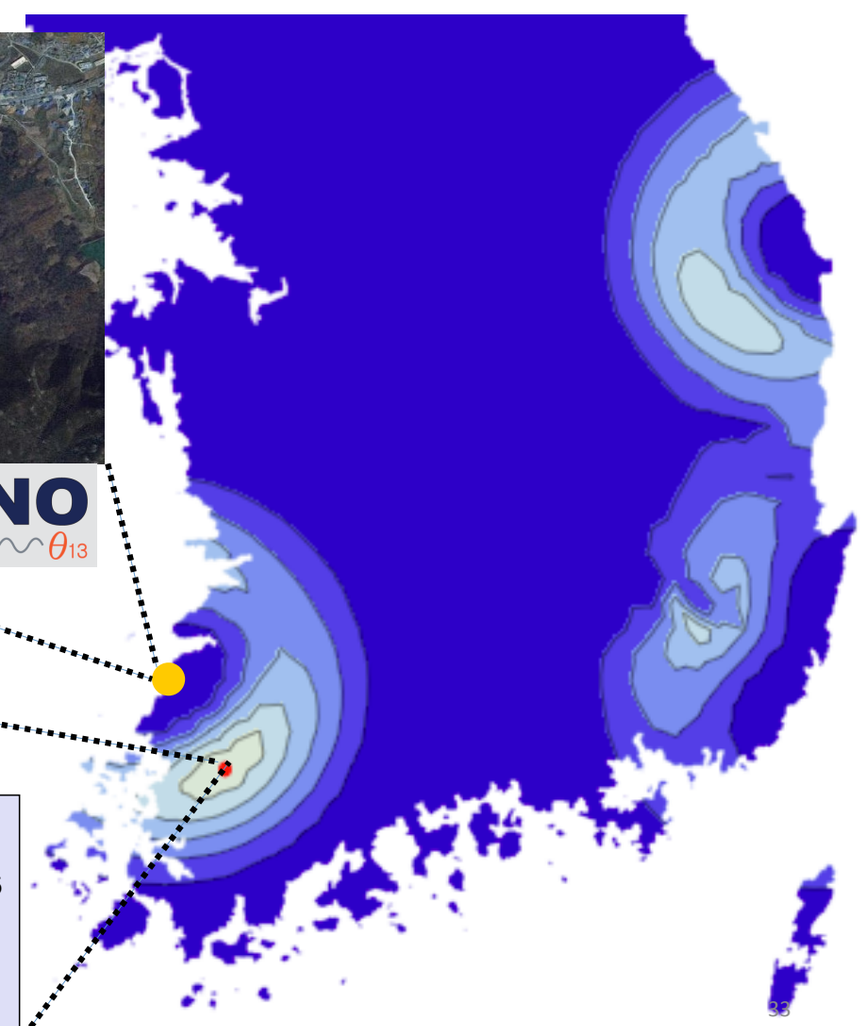
Future Experiments:

JUNO



✧ Baseline difference should be < 500 m.





(NEAR Detector)

RENO

(FAR Detector)

RENO-50

10 kton LS Detector
~47 km from YG reactors

Mt. Guemseong (450 m)
~900 m.w.e. overburden

Seon-Hee Seo, SNU FPCP 2015

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$$= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)$$

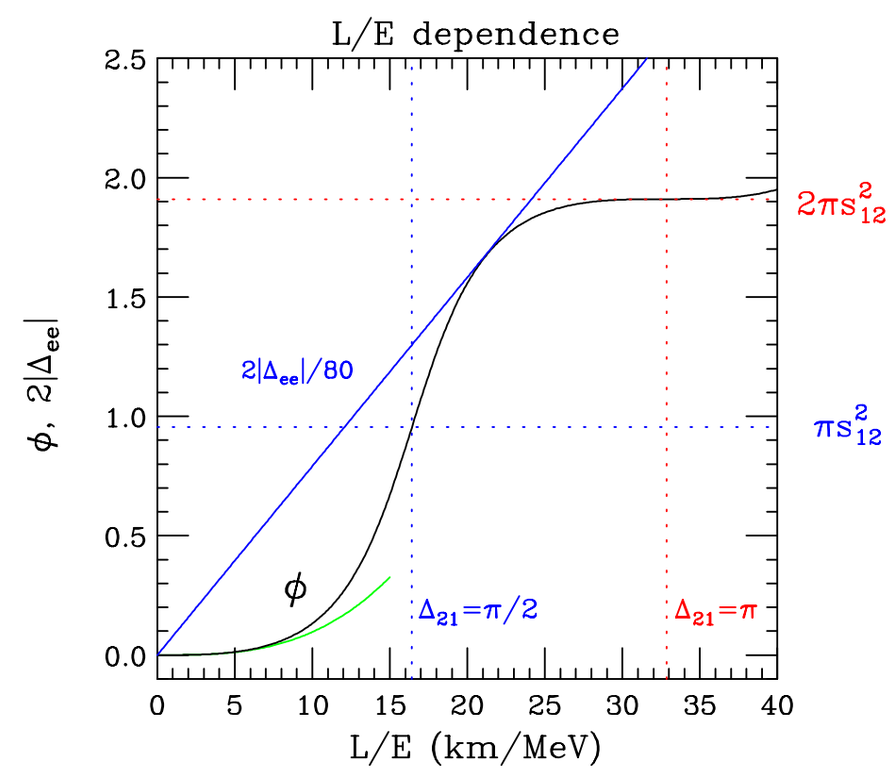
(+/- mass ordering)

where $\Omega = 2|\Delta_{ee}| \pm \phi$

with $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$

Note:

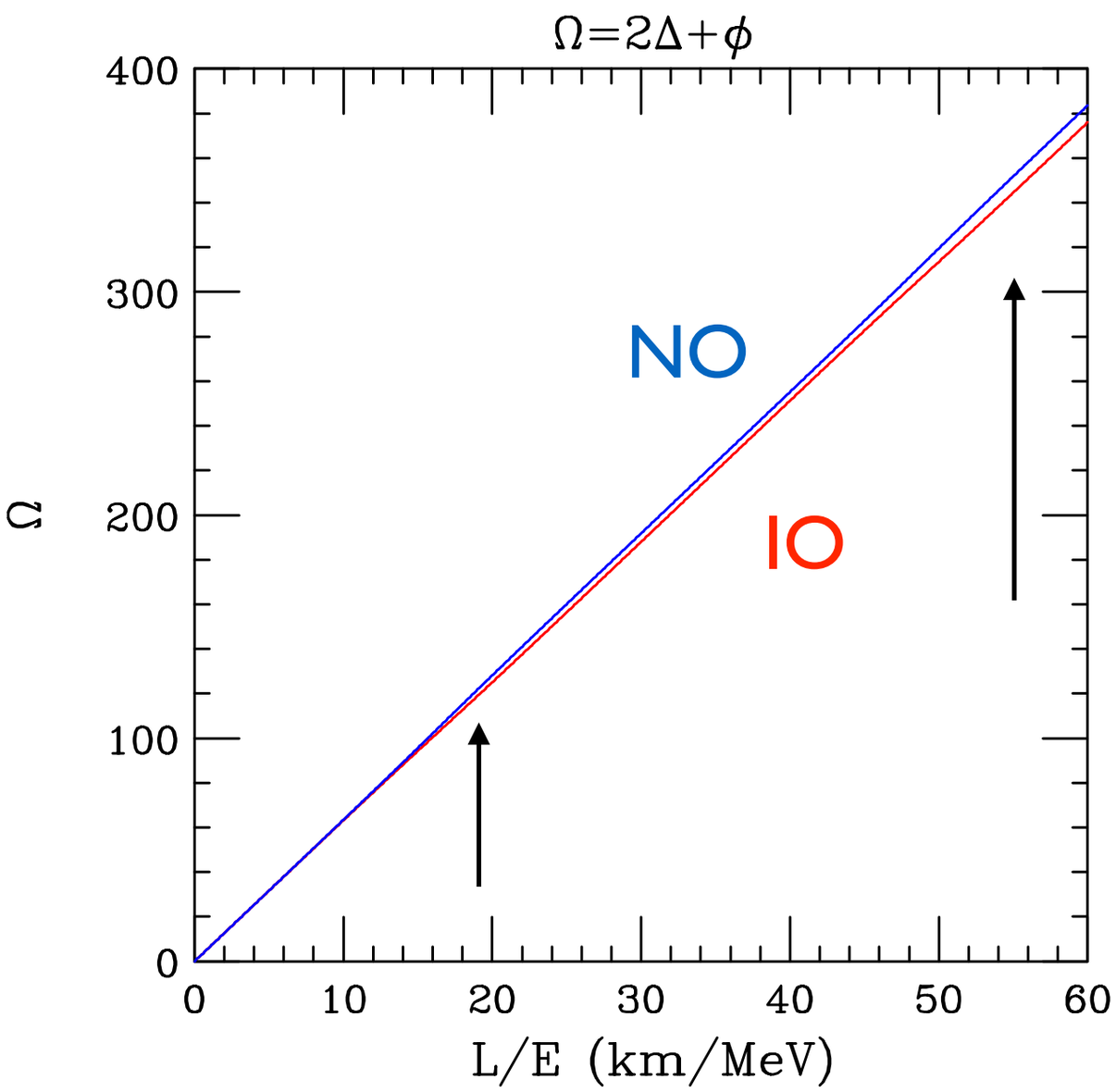
- Separation $2\Delta_{ee}$ and ϕ is unique !
- $2\Delta_{ee}$ is linear in L/E and depends on an atm Δm^2.
- $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$ starts at $\Delta_{21}^3 \sim (L/E)^3$ and depends on only solar parameters, staircase function.

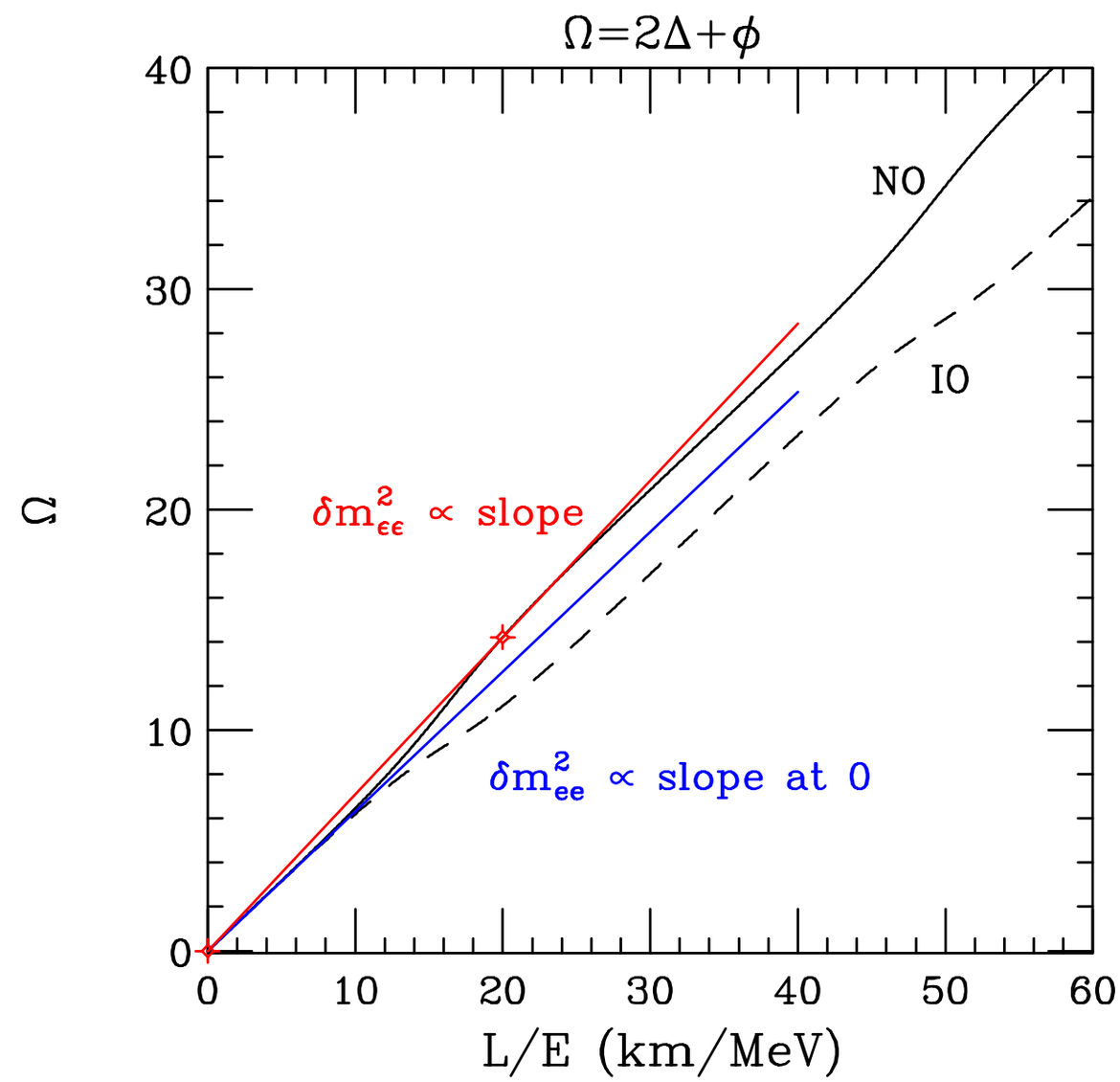


H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)

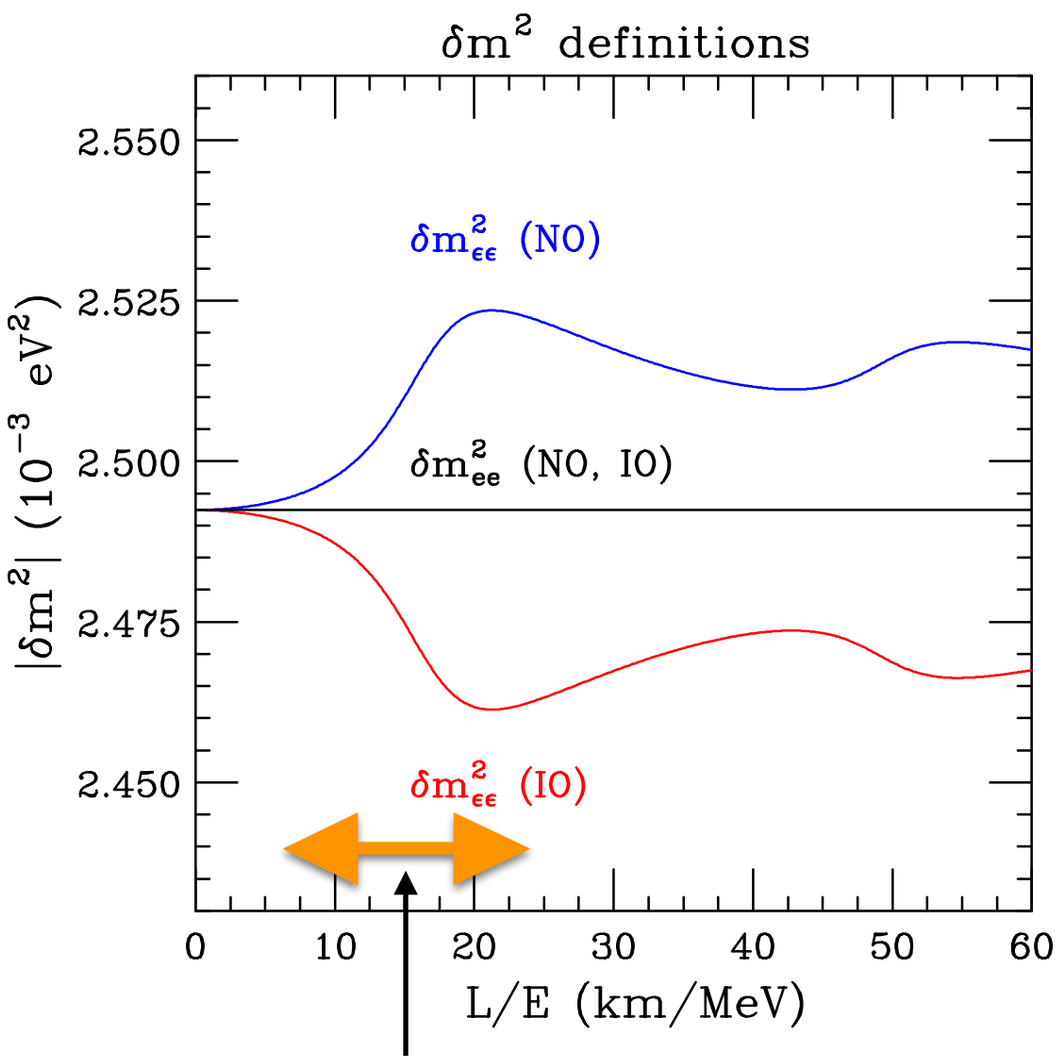




(For this figure, to make the effect visible, the factor 2Δ is reduced by factor 10.)

New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)



JUNO & RENO50

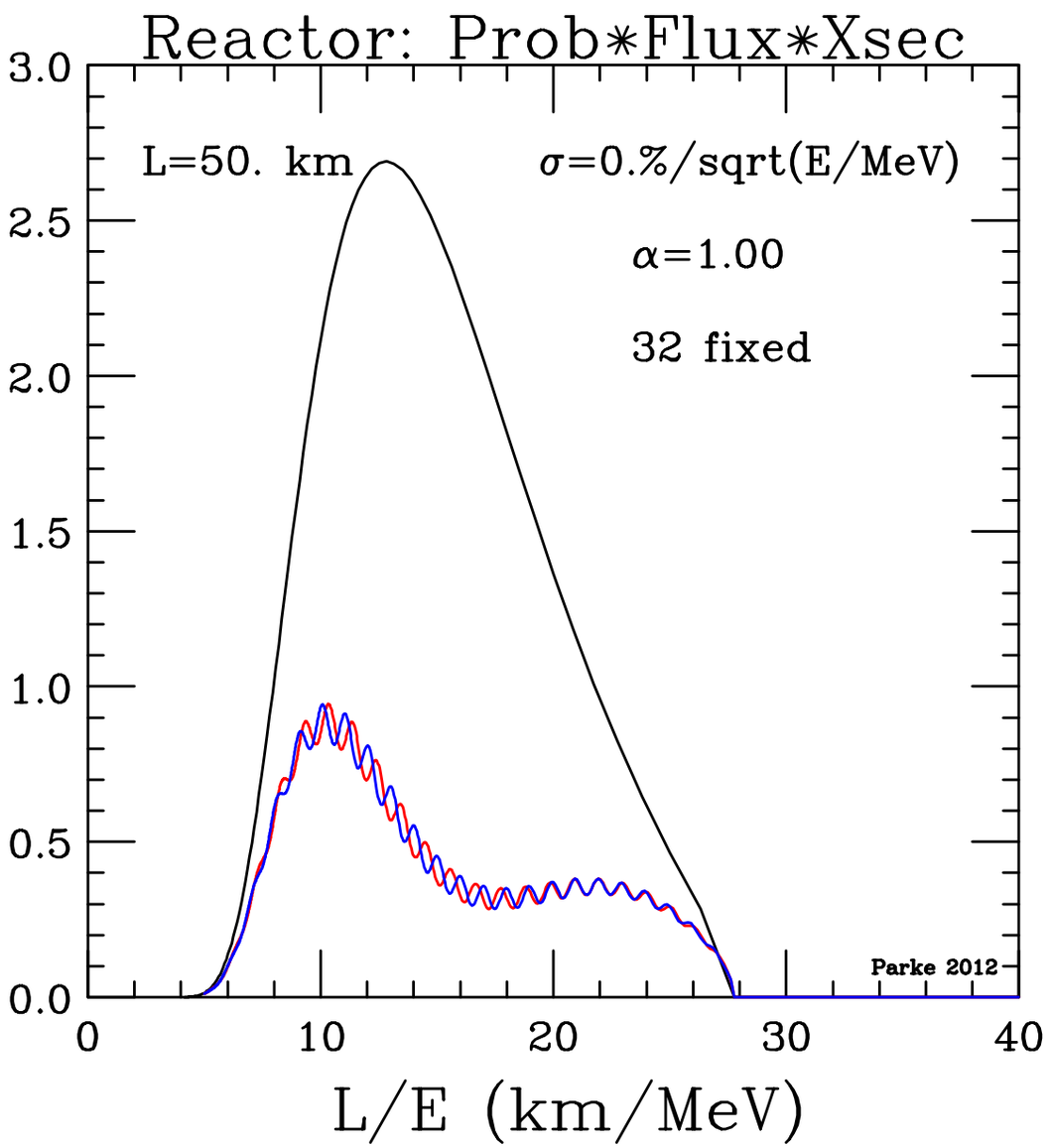
Identical for L/E $<$ 5 -10 km/MeV !

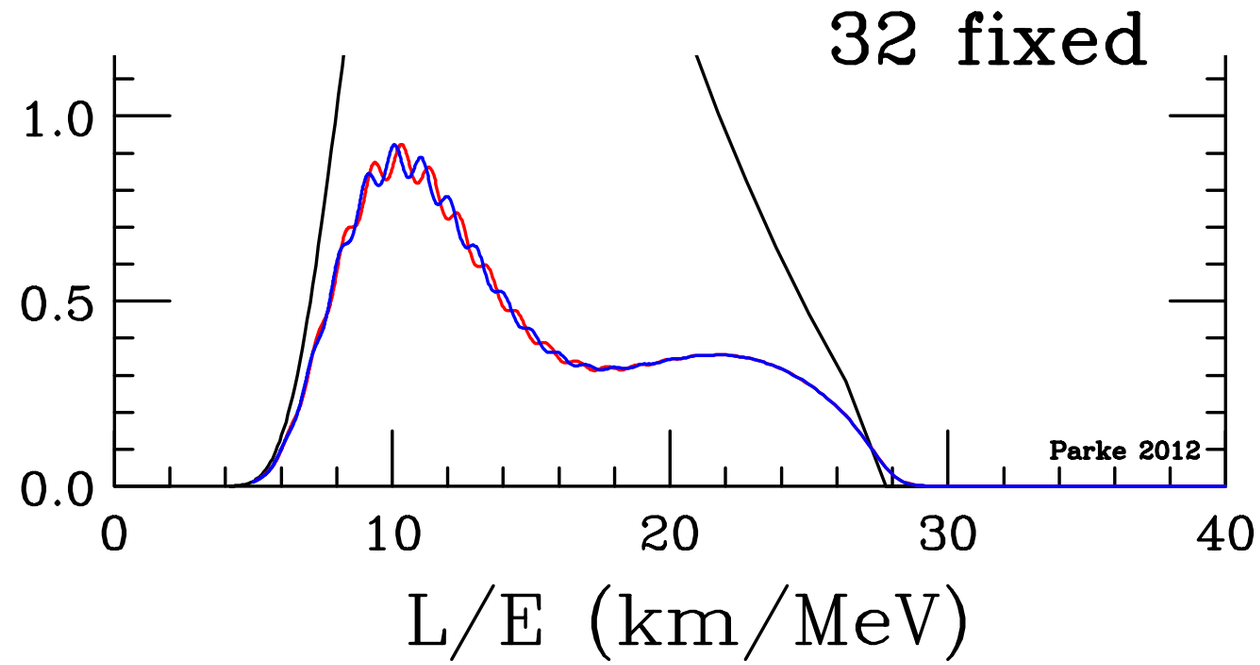
Significant ($\sim 1\%$) L/E
and
Mass Ordering Dependence!

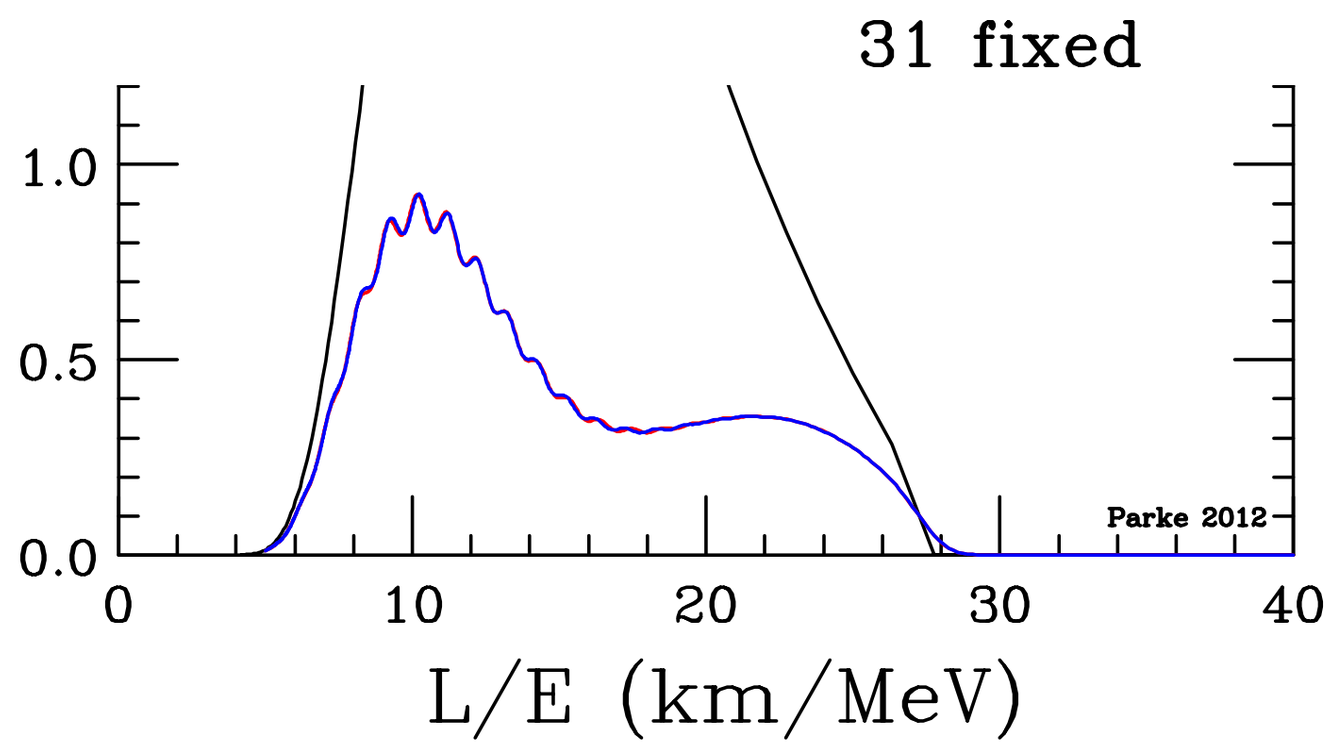
expect ~0.5 % measurement !!!

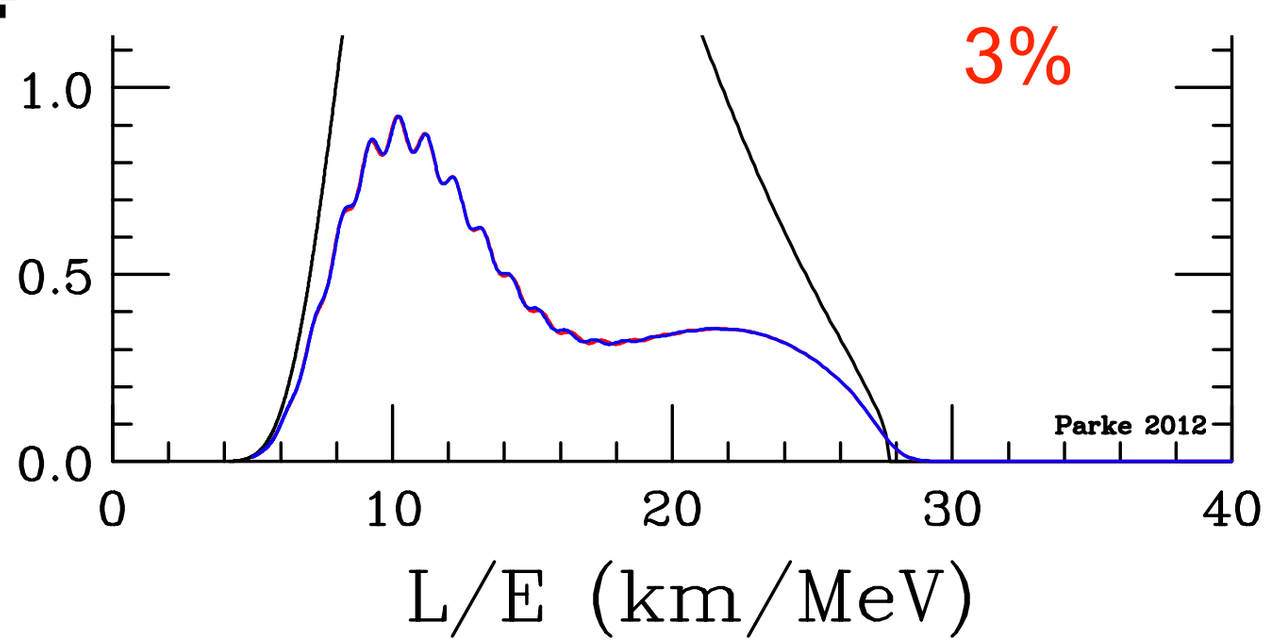
Mass Ordering:

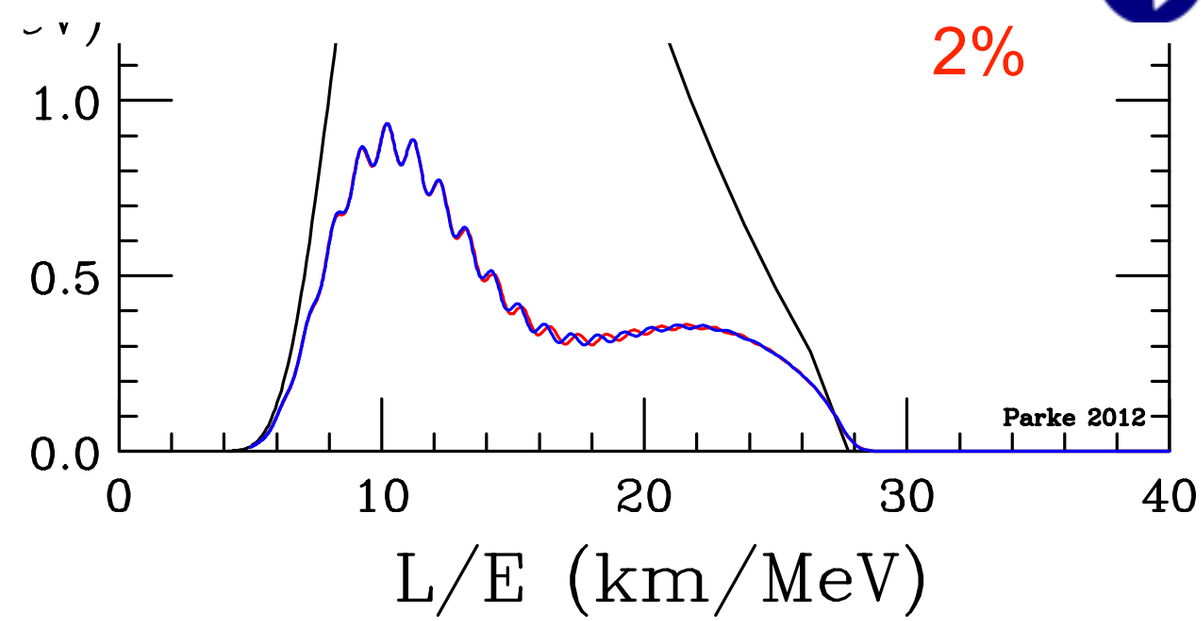
Reactors & the Mass Ordering:



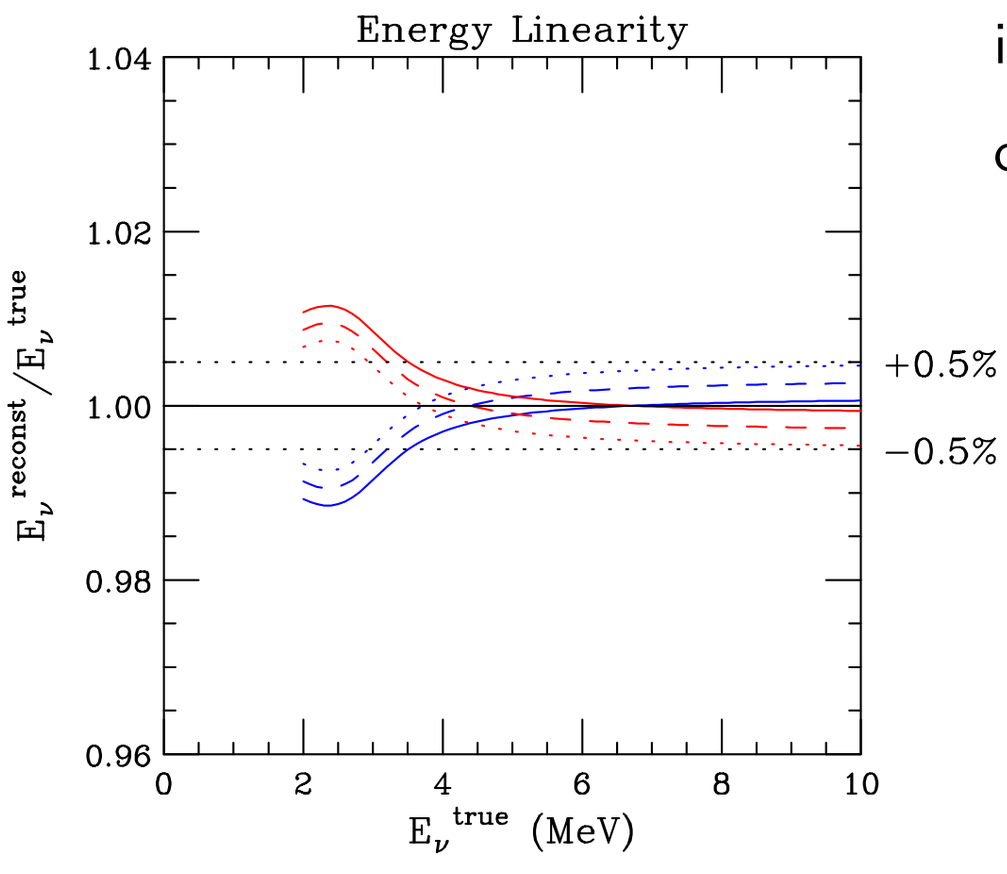








Neutrino Energy



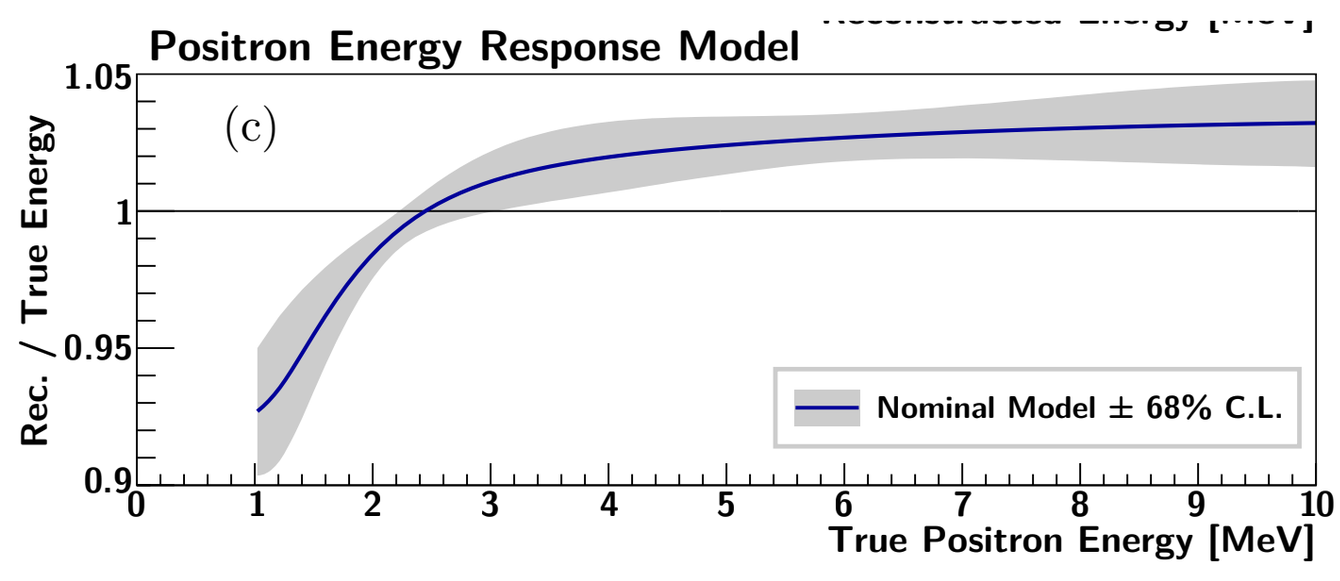
Mass Ordering Indetermined:

if $2\Delta_{ee} + \phi = 2\Delta'_{ee}$

or $2\Delta_{ee} - \phi = 2\Delta'_{ee}$

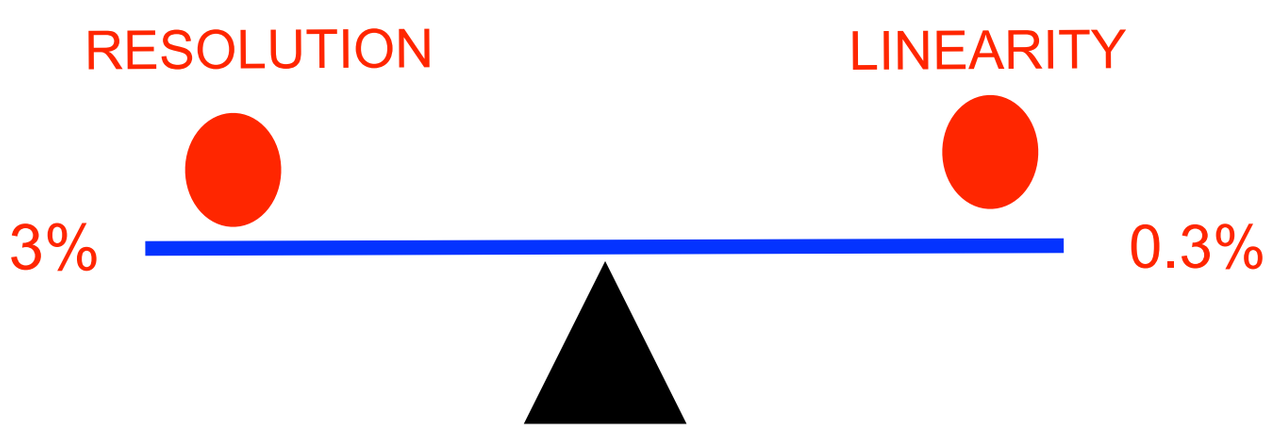
Parke @ NOW 2008

$$\delta\ (\Delta m^2_{ee}) \sim 0.5\%$$



- Energy Resolution to 3% or lower at 1 MeV
- Linearity to sub 1% precision for the reconstructed neutrino energy

	KamLAND	JUNO	RENO-50	
LS mass	~1 kt	20 kt	18 kt	20 x
Energy Resolution	6%/	~3%/	~3%/	
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV	4 x
Linearity	1.9%	< 0.5%	< 0.5%	> 4 x



What is $\Delta m^2_{\mu\mu}$?

"the ν_μ weighted average of Δm^2_{31} and Δm^2_{32}"

$$\Delta m^2_{\mu\mu} \equiv \frac{|U_{\mu 1}|^2}{1-|U_{\mu 3}|^2}\,\Delta m^2_{31} + \frac{|U_{\mu 2}|^2}{1-|U_{\mu 3}|^2}\,\Delta m^2_{32}$$

for vacuum ν_μ-disappearance !

And similarly for ν_τ.

$$\begin{aligned}
\delta m^2_{\rm eff}|_e &= \cos^2\theta_{12}\delta m^2_{31} + \sin^2\theta_{12}\delta m^2_{32} \\
\delta m^2_{\rm eff}|_\mu &= \sin^2\theta_{12}\delta m^2_{31} + \cos^2\theta_{12}\delta m^2_{32} + \cos\delta\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}\delta m^2_{21} \\
\delta m^2_{\rm eff}|_\tau &= \sin^2\theta_{12}\delta m^2_{31} + \cos^2\theta_{12}\delta m^2_{32} - \cos\delta\sin\theta_{13}\sin 2\theta_{12}\cot\theta_{23}\delta m^2_{21}.
\end{aligned}$$

Neutrino Propogation in Matter

if one choses to write the Hamiltonian using Δm^2_{ee} and Δm^2_{21} then the Hamiltonian is simpler than with any other choice !

See Hisakazu Minakata and SP arXiv:1505.01826

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos(\ 2|\Delta_{ee}|\ \pm\phi\ )\right]$$

Daya Bay
Double Chooz
RENO

≤ 1 but $> (1 - 10^{-3})$

$< 10^{-5}$

may as well use: $-\sin^2 2\theta_{13}\sin^2\Delta_{ee}$

JUNO
RENO50

$\sim \cos 2\theta_{12} \approx 0.4$

$< 1\%$ shift in phase
challenging to observe ! ! !

$$\text{remember} \quad \phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation: Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32}," since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

- Can be used in short baseline reactor experiments, $L/E < 1$ km/MeV, using the approximate oscillation probability,

  $$P(\bar{\nu}_e \to \bar{\nu}_e) \ \approx\ 1 - 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21} - 4s^2_{13}c^2_{13}\sin^2\Delta_{ee},$$

  which is accurate to better than one part in 10^4.

- Can be used in medium baseline reactor experiments, $L \approx 50$ km, and determines the Δm^2 of the fundamental oscillation. The advancement or retardation of the phase of this fundamental oscillation determines the neutrino mass ordering.

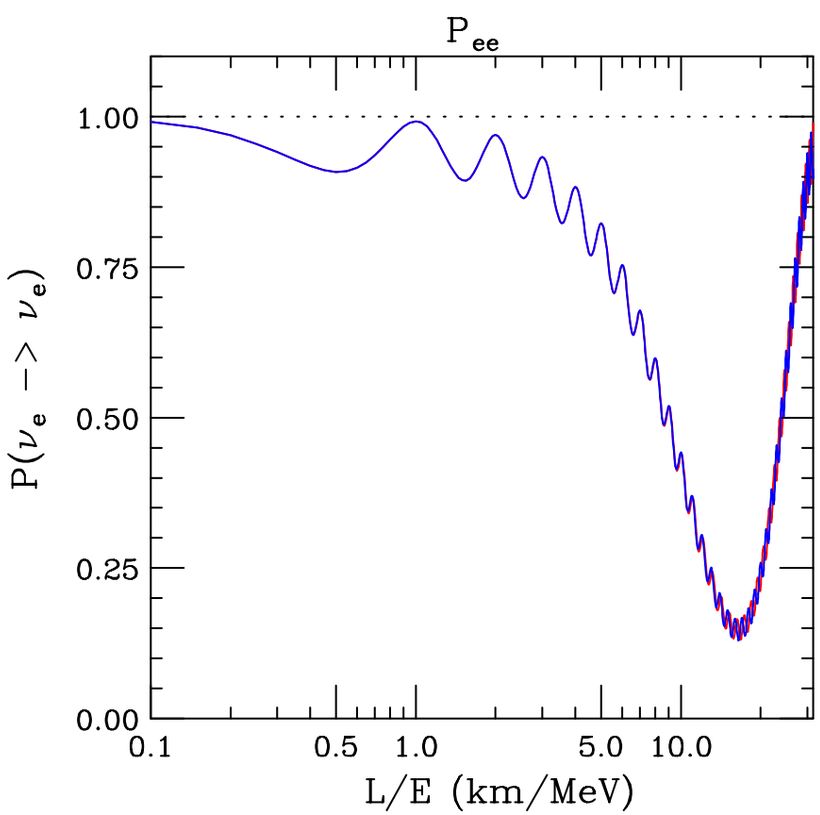
What is δm^2_{ee} ?

$$\delta m^2_{ee} \equiv c^2_{12}\delta m^2_{31} + s^2_{12}\delta m^2_{32}$$

is simpler and has obvious physical meaning:

the ν_e weighted average δm^2_{31} and δm^2_{32}

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283



Solar parameters: $\theta_{12},\ \Delta m^2_{21} > 0$:

Atmospheric parameters: $\theta_{13},\ \Delta m^2_{ee}$:

(sign of Δm^2_{ee} determines the atmospheric mass ordering)

Additional

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

In general, if we use $\Delta m^2_{rr} = (1-r)\ \Delta m^2_{31} + r\ \Delta m^2_{32}$

i.e. $\Delta m^2_{31} = \Delta m^2_{rr} + r\Delta_{21}$ and $\Delta m^2_{32} = \Delta m^2_{rr} - (1-r)\Delta_{21}$

then

$$
\begin{aligned}
& c^2_{12}\sin^2\Delta_{31} + s^2_{12}\sin^2\Delta_{32} \\
= \quad & \sin^2\Delta_{rr} + [c^2_{12}r - s^2_{12}(1-r)]\ \Delta_{21}\sin(2\Delta_{rr}) \\
& + [c^2_{12}r^2 + s^2_{12}(1-r)^2]\ \Delta^2_{21}\cos(2\Delta_{rr}) \\
& - \frac{2}{3}(c^2_{12}r^3 - s^2_{21}(1-r)^3)\Delta^3_{21}\sin(2\Delta_{rr}) + \mathcal{O}(\Delta^4_{21})
\end{aligned}
$$

at $r = \sin^2\theta_{13}$ then coefficient of $\Delta_{21}\sin(2\Delta_{rr})$ is zero and the coefficient of $\Delta^2_{21}\cos(2\Delta_{rr})$ is a minimum ! Makes for an EXCELLENT expansion !

Especially since $\Delta_{21} < 0.1$!