When leptons interact with gluons: implications for τ, μ, and ν

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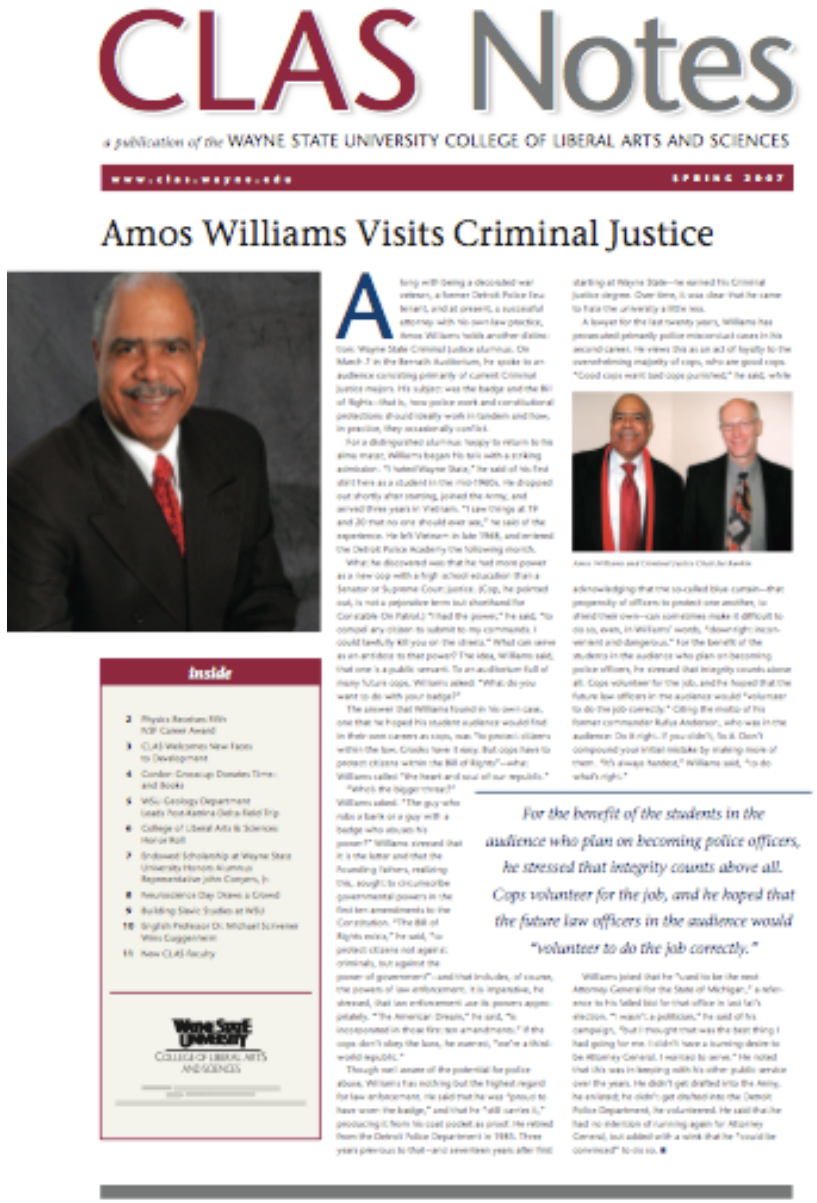
Why flavor?



CLAS Notes

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Amos Williams Visits Criminal Justice

Along with being a decorated war veteran, a former Detroit Police lieutenant, and at present, a successful attorney with his own law practice, Amos Williams holds another distinction: Wayne State Criminal Justice alumnus. On March 7 in the Bernath Auditorium, he spoke to an audience consisting primarily of current Criminal Justice majors. His subject was the badge and the Bill of Rights—that is, how police work and constitutional protections should ideally work in tandem and how, in practice, they occasionally conflict.

For a distinguished alumnus happy to return to his alma mater, Williams began his talk with a striking admission. "I hated Wayne State," he said of his first stint here as a student in the mid-1960s. He dropped out shortly after starting, joined the Army, and served three years in Vietnam. "I saw things at 19 and 20 that no one should ever see," he said of the experience. He left Vietnam in late 1968, and entered the Detroit Police Academy the following month.

What he discovered was that he had more power as a new cop with a high school education than a Senator or Supreme Court justice. (Cop, he pointed out, is not a pejorative term but shorthand for Constable On Patrol.) "I had the power," he said, "to compel any citizen to submit to my commands. I could lawfully kill you on the streets." What can serve as an antidote to that power? The idea, Williams said, that one is a public servant. To an auditorium full of many future cops, Williams asked: "What do you want to do with your badge?"

The answer that Williams found in his own case, one that he hoped his student audience would find in their own careers as cops, was "to protect citizens within the law. Crooks have it easy. But cops have to protect citizens within the Bill of Rights"—what Williams called "the heart and soul of our republic."

"Which the bigger threat?" Williams asked. "The guy who robs a bank or a guy with a badge who abuses his power?" Williams stressed that it is the latter and that the founding fathers, realizing this, sought to circumscribe governmental powers in the first ten amendments to the Constitution. "The Bill of Rights exists," he said, "to protect citizens not against criminals, but against the power of government"—and that includes, of course, the powers of law enforcement. It is imperative, he stressed, that law enforcement use its powers appropriately. "The American Dream," he said, "is incorporated in those first ten amendments." If the cops don't obey the laws, he warned, "we're a third-world republic."

Though well aware of the potential for police abuse, Williams has nothing but the highest regard for law enforcement. He said that he was "proud to have worn the badge," and that he "still carries it," producing it from his coat pocket as proof. He retired from the Detroit Police Department in 1985. Three years previous to that—and seventeen years after first starting at Wayne State—he earned his Criminal Justice degree. Over time, it was clear that he came to hate the university a little less.

A lawyer for the last twenty years, Williams has prosecuted primarily police misconduct cases in his second career. He views this as an act of loyalty to the overwhelming majority of cops, who are good cops. "Good cops want bad cops punished," he said, while

Amos Williams and Criminal Justice Chair Joe Rankin

acknowledging that the so-called blue curtain—that propensity of officers to protect one another, to shield their own—can sometimes make it difficult to do so, even, in Williams' words, "downright inconvenient and dangerous." For the benefit of the students in the audience who plan on becoming police officers, he stressed that integrity counts above all. Cops volunteer for the job, and he hoped that the future law officers in the audience would "volunteer to do the job correctly." Citing the motto of his former commander Rufus Anderson, who was in the audience: Do it right. If you didn't, fix it. Don't compound your initial mistake by making more of them. "It's always hardest," Williams said, "to do what's right."

For the benefit of the students in the audience who plan on becoming police officers, he stressed that integrity counts above all. Cops volunteer for the job, and he hoped that the future law officers in the audience would "volunteer to do the job correctly."

Williams joined that he "used to be the next Attorney General for the State of Michigan," a reference to his failed bid for that office in last fall's election. "I wasn't a politician," he said of his campaign, "but I thought that was the best thing I had going for me. I didn't have a burning desire to be Attorney General. I wanted to serve." He noted that this was in keeping with his other public service over the years. He didn't get drafted into the Army, he enlisted; he didn't get drafted into the Detroit Police Department, he volunteered. He said that he had no intention of running again for Attorney General, but added with a wink that he "could be convinced" to do so. ■

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Chemistry	Jin Cha	NUTRITION AND FOOD SCIENCE
Chemistry	Arthur Suits	NUTRITION AND FOOD SCIENCE
Computer Science	Monica Brockmeyer	NUTRITION AND FOOD SCIENCE
Computer Science	Hasan Jamil	NUTRITION AND FOOD SCIENCE
English	Anthony Manuel Aristar	NUTRITION AND FOOD SCIENCE
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Mathematics	Paoliu Chow	NUTRITION AND FOOD SCIENCE
Mathematics	Zhimin Zhang	NUTRITION AND FOOD SCIENCE
Mathematics	Gang Yin	NUTRITION AND FOOD SCIENCE
Mathematics	Boris Mordukhovich	NUTRITION AND FOOD SCIENCE
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Geology	Mark Baskaran	U.S. GEOLOGICAL SURVEY
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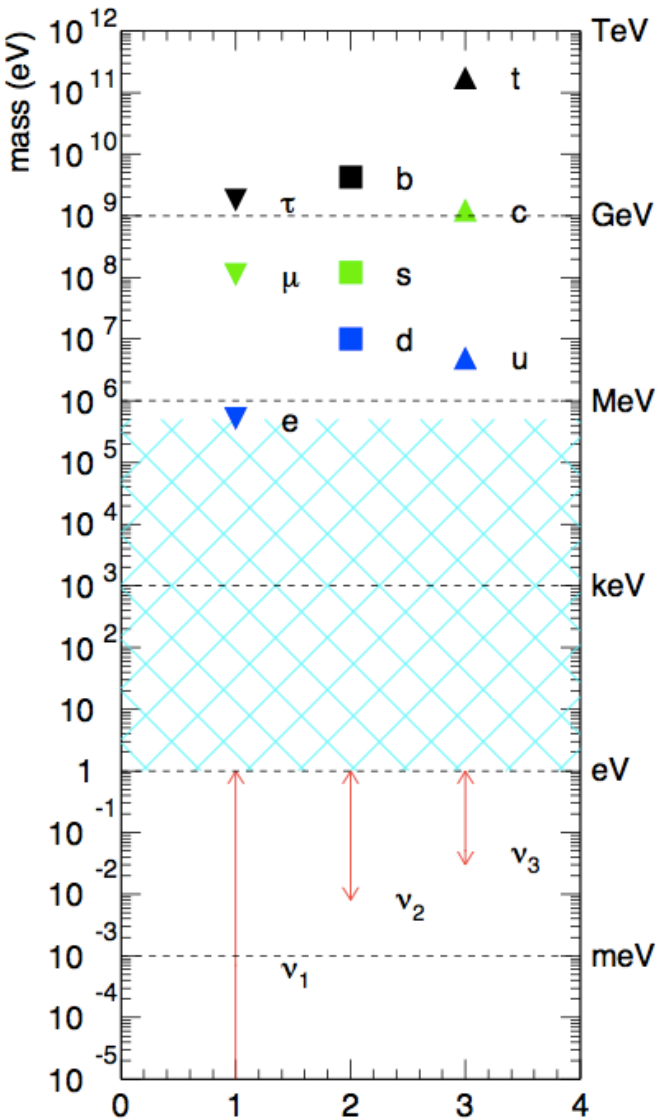
CLAS Notes 3

Quark
Quark
Quark

Matter sector: experimental data



★ Ratios of masses of quarks and leptons

- quarks

$$\frac{m_d}{m_u} \simeq 2 \ , \quad \frac{m_s}{m_d} \simeq 21 \ ,$$
$$\frac{m_t}{m_c} \simeq 267 \ , \quad \frac{m_c}{m_u} \simeq 431 \ , \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5 \ .$$

- leptons

$$\frac{m_\tau}{m_\mu} \simeq 17 \ , \quad \frac{m_\mu}{m_e} \simeq 207 \ .$$

★ Quark mixing (Cabibbo-Kobayashi-Maskawa) matrix parameters

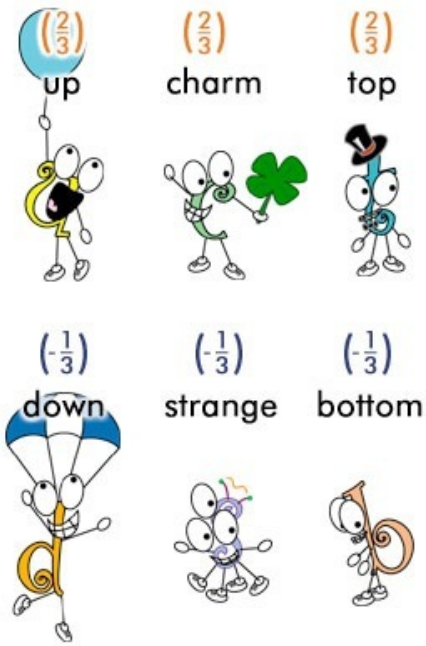
$$V_{\rm CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud} \sim 1, V_{us} \sim 0.2, \ V_{cb} \sim 0.04, \ V_{ub} \sim 0.004$$

A couple of questions...

Gauge forces in SM do not distinguish between fermions of different generations:

- e, μ, τ have same electrical charge
- quarks have same color charge



★ Why generations? Why only 3? Are there only 3?

★ Why hierarchies of masses and mixings?

★ Can there be transitions between quarks/leptons of the same charge but different generations?

A couple of questions...

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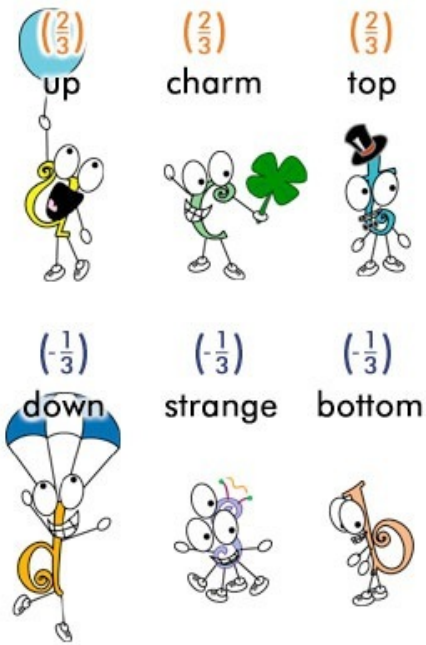
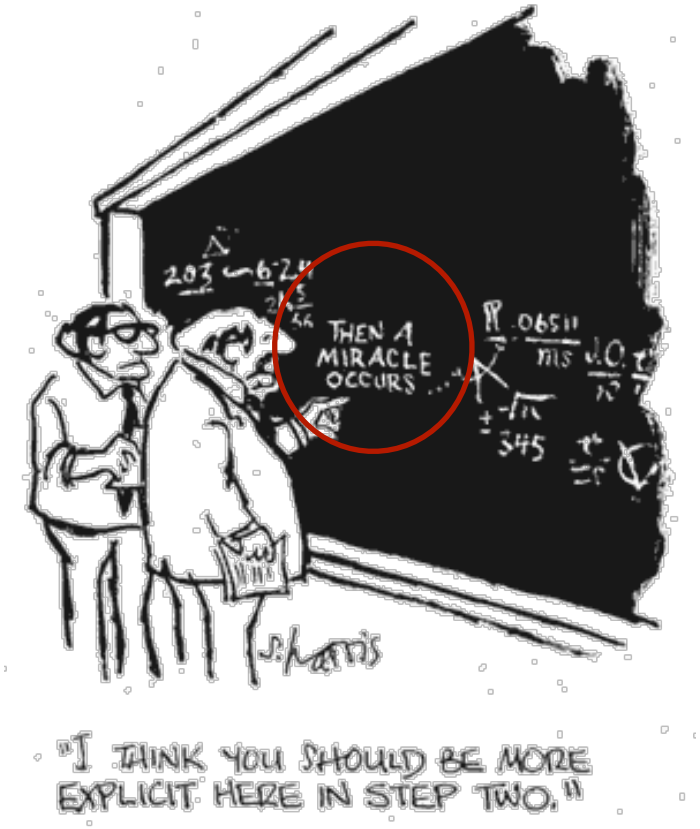


- ★ Why generations? Why only 3? Are there only 3?
- ★ Why hierarchies of masses and mixings?
- ★ Can there be transitions between quarks/leptons of the same charge but different generations?

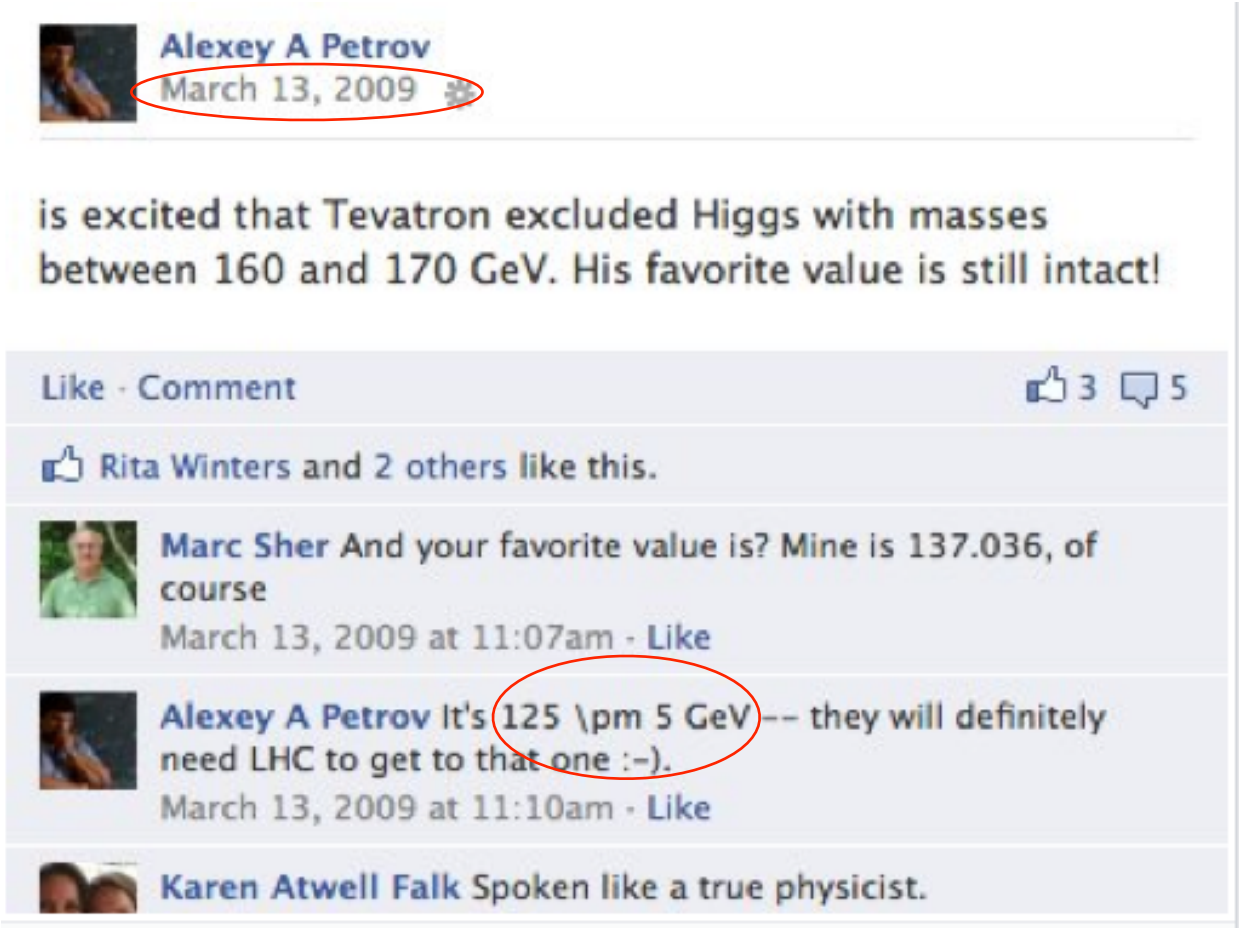
The flavor puzzle

Standard Model "solution"



S. Harris

… if you accept that, you should probably also accept this:



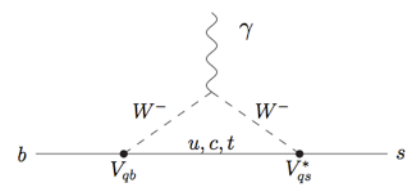
1. Introduction: quarks

★ Why study flavor-changing neutral currents?

★ No trivial FCNC vertices in the Standard Model: sensitive NP

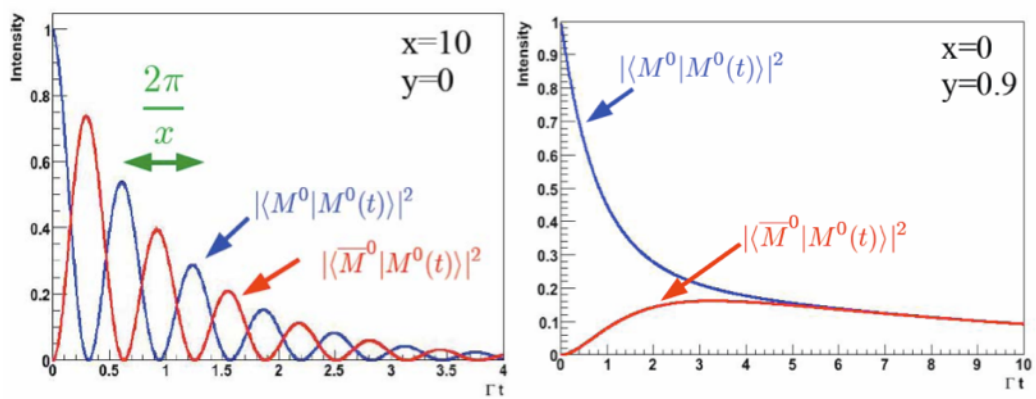
★ Possible experimental studies in quark and lepton sectors

- $\Delta F = 1$ processes ($b \to s\gamma$, $c \to u\gamma$, etc)



$$\mathcal{B}r^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} = \frac{3}{4\pi^3}\cdot\frac{\Delta M^{(\mathrm{Expt})}_{B_s}}{\Gamma_{B_s}}\cdot\frac{(G_F M_W m_\mu \eta_Y Y)^2}{\hat{\eta}\hat{B}_{B_s} S_0(\bar{x}_t)}\left[1-4\frac{m_\mu^2}{M_{B_s}^2}\right]^{1/2} = (3.3\pm 0.2)\times 10^{-9}$$

- $\Delta F = 2$ processes ($B\bar{B}$-mixing, $K\bar{K}$-mixing, $D\bar{D}$-mixing)



M. Purohit

Introduction: leptons

★ Why study flavor-changing neutral currents?

★ No trivial FCNC vertices in the Standard Model: sensitive NP tests

★ Possible experimental studies in a lepton sector

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+ e^- \to e^- \mu^+$
  - $Z^0 \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc. (if kinematically allowed)
  - K^+ (B^+, D^+, ...) $\to \pi^+ \mu e$, $\pi^+ \tau e$, etc. (if kinematically allowed)
  - $\mu^- + (A, Z) \to e^- + (A, Z)$

- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z \pm 2) + e^{\mp} e^{\mp}$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

Introduction: leptons

★ Strong constraints exist on many of those processes

- some processes, will talk about others shortly

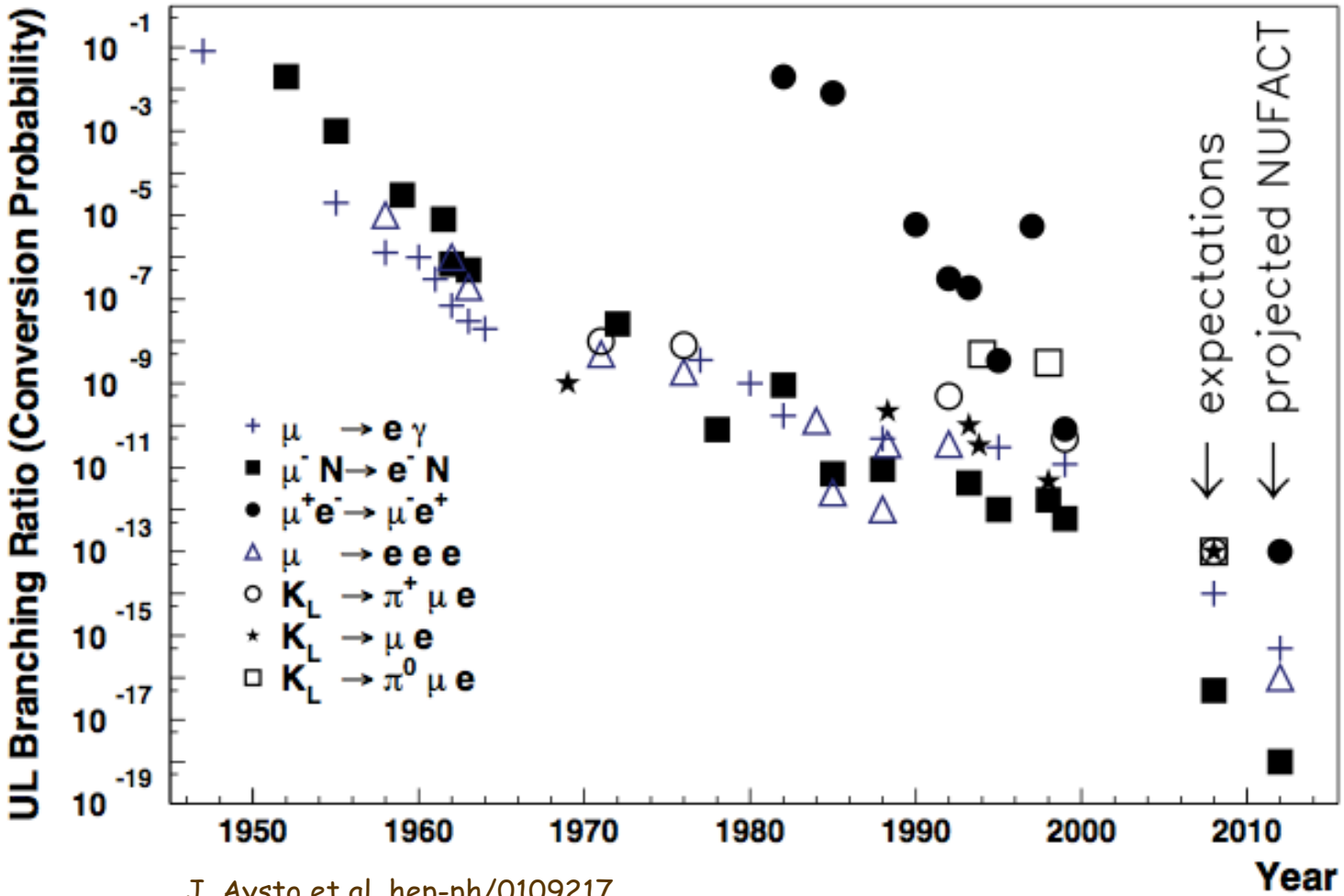
Channel	Babar $\mathcal{L}$ (fb^{-1})	Babar $\mathcal{B}_{\mathrm{UL}}$ (10^{-8})	BELLE $\mathcal{L}$ (fb^{-1})	BELLE $\mathcal{B}_{\mathrm{UL}}$ (10^{-8})
$\tau^\pm \to e^\pm\gamma$	232	11	535	12
$\tau^\pm \to \mu^\pm\gamma$	232	6.8	535	4.5
$\tau^\pm \to \ell^\pm\ell^\mp\ell^\pm$	92	11 - 33	535	2 - 4
$\tau^\pm \to e^\pm\pi^0$	339	13	401	8.0
$\tau^\pm \to \mu^\pm\pi^0$	339	11	401	12
$\tau^\pm \to e^\pm\eta$	339	16	401	9.2
$\tau^\pm \to \mu^\pm\eta$	339	15	401	6.5
$\tau^\pm \to e^\pm\eta'$	339	24	401	16
$\tau^\pm \to \mu^\pm\eta'$	339	14	401	13

$$\mathrm{BR}\,(K_L^0 \to \mu e) \;<\; 4.7 \times 10^{-12}$$
$$\mathrm{BR}\,(B_d^0 \to \mu e) \;<\; 1.7 \times 10^{-7} \qquad [\mathrm{Belle}]$$
$$\mathrm{BR}\,(B_s^0 \to \mu e) \;<\; 6.1 \times 10^{-6} \qquad [\mathrm{CDF}]$$

- very much suppressed in the Standard Model, e.g.

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U_{\mu i}^* U_{ei}\frac{m_{\nu_i}^2}{M_W^2}\right|^2 < 10^{-54}$$

Searches for Lepton Number Violation



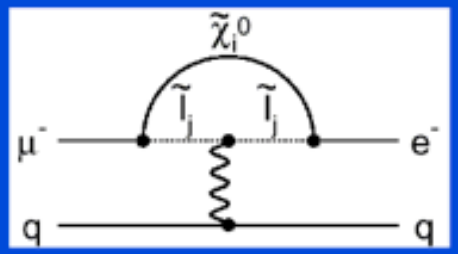
J. Aysto et al, hep-ph/0109217

★ Let's concentrate on **muon conversion** (we are at FNAL after all)

Sensitivity to Different Muon Conversion Mechanisms

Supersymmetry

Predictions at 10^{-15}

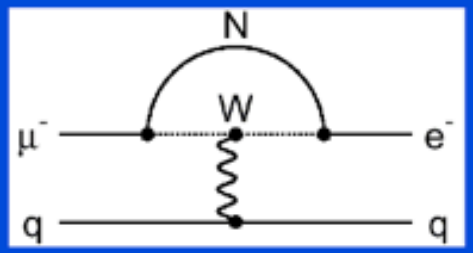


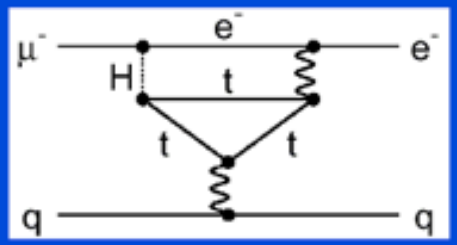
Compositeness

$\Lambda_C = 3000$ TeV

Heavy Neutrinos

$|U^*_{\mu N} U_{eN}|^2 = 8 \times 10^{-13}$



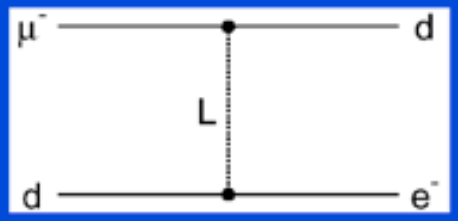


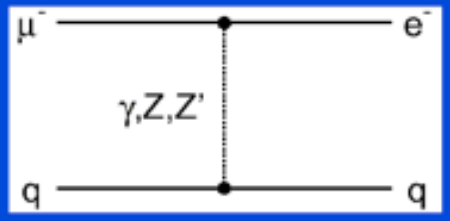
Second Higgs doublet

$g_{H_{\mu e}} = 10^{-4} \times g_{H_{\mu\mu}}$

Leptoquarks

$M_L = 3000\sqrt{\lambda_{\mu d}\lambda_{ed}}$ TeV/c²





Heavy Z', Anomalous Z coupling

$M_{Z'} = 3000$ TeV/c²

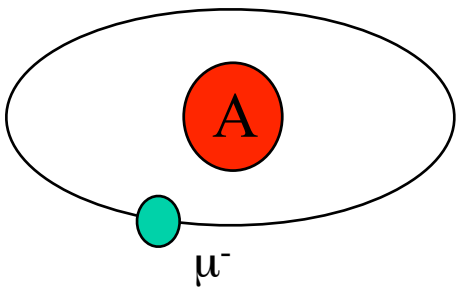
$B(Z \to \mu e) < 10^{-17}$

After W. Marciano

James Miller, 2006

Muonic atoms

★ Basic idea for the muon conversion experiment



★ take low energy muons (~ 30 MeV) to be stopped in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$

overlap probability: $p \sim r^{2\ell} Z^3$

large overlap for an s-wave and high-Z nucleus

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

Experimental ideas

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

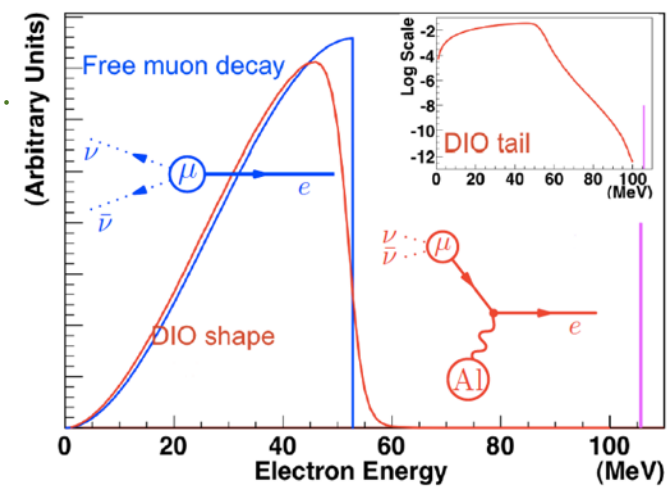
J. Miller, 2006

★ The experiment is tricky

✓ Muon conversion gives monoenergetic electrons...

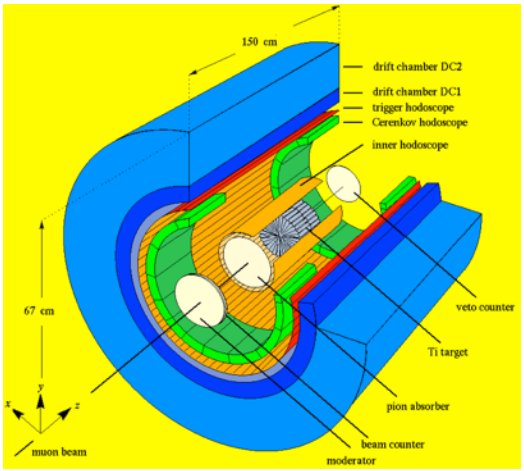
✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

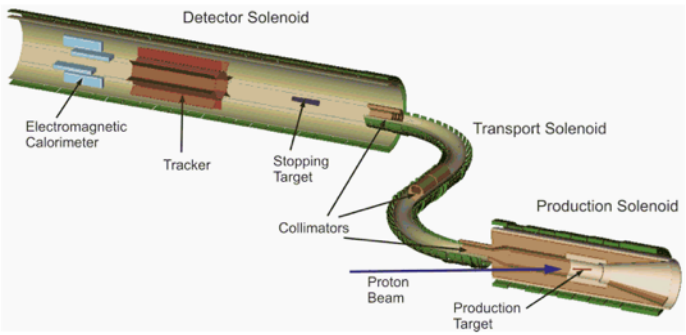


Czarnecki, Marciano, Tormo

Muon conversion experiments



SINDRUM II (PSI, Switzerland)



Mu2e (FNAL) -- hopefully…

Experimental considerations

$\mu \to e\gamma$, $\mu \to eee$

- signal is combination of final state particles
- accidental coincidences limit usable μ rate

$\mu N \to eN$

- signal is single electron track
  - at 105 MeV
- Can take more muons/second
- Only need to see high momentum tracks ($\gtrsim$ 90 MeV/c)

Limits

$Br(\mu^+ \to e\gamma) < 2.4 \times 10^{-12}$
[MEG, 2011 (link)]

$Br(\mu^+ \to e^+e^-e^+) < 1 \times 10^{-12}$
[SINDRUM-I, 1988 (link)]

$R_{\mu e} < 7 \times 10^{-13}$
[SINDRUM-II, 2006 (link)]

Mu2e goal: single event sensitivity few $\times 10^{-17}$

A. Gaponenko, CIPANP-2012

Phenomenology of muon conversion I

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Let's look at the vector/axial vector interactions as an example

Czarnecki, Marciano, Melnikov

$$A(\mu \to e) \sim \sqrt{4\pi\alpha}\, \bar{e} \left[\gamma_\alpha \left(f_{E0} - f_{M0}\gamma_5 \right) \frac{q^2}{m} + i\sigma_{\alpha\beta} \frac{q^\beta}{m} \left(f_{M1} + f_{E1}\gamma_5 \right) \right] \mu\, A^\alpha(q)$$

$$+ \quad \frac{G_F}{\sqrt{2}} \bar{e}\gamma_\alpha \left(a - b\gamma_5 \right) \mu \left[\bar{u}\gamma^\alpha u + c_d\, \bar{d}\gamma^\alpha d \right]$$

need to calculate averages over nuclear states

★ Nuclear averages are often done as an approximation. For a general operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z\rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle \right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \quad \int d^3\rho_{p(n)}(r) = 1$$

Phenomenology of muon conversion II

★ Matrix elements of light quark currents are easily computed

- since (m_μ-m_e) << m_N we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

★ ... which gives for the effective Hamiltonian for the coherent muon conversion

$$H_{int} = H_1 + H_2 + H_3$$
$$H_1 = 4\pi e Z \int d^3r \bar{\psi}_e(r)\gamma_0 \left(f_{E0} - f_{M0}\gamma_5\right)\psi_\mu(r)\rho_p(r)$$
$$H_2 = e \int d^3r \bar{\psi}_e(r) i\sigma_{\alpha\beta}\left(f_{M1} + f_{E1}\gamma_5\right)\psi_\mu(r)F^{\alpha\beta}(r)$$
$$H_3 = \frac{G_F}{\sqrt{2}} \int d^3r \bar{\psi}_e(r)\gamma_0 \left(a - b\gamma_5\right)\psi_\mu(r)$$
$$\times \left[Z(2+c_d)\rho_p(r) + (A-Z)(1+2c_d)\rho_n(r)\right]$$

integrate over lepton wavefunctions to get conversion probability

Phenomenology of muon conversion III

★ Lepton wave functions are taken as solutions of Dirac equation

- with usual substitutions $u_1(r) = r\, g(r)$ and $u_2(r) = r\, f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi^{\mu}_{\kappa} = \begin{pmatrix} g(r)\chi^{\mu}_{\kappa}(\theta,\phi) \\ i f(r)\chi^{\mu}_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^{\infty} E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

Finally, the answer!

★ The conversion rate can be written as $B^N_{\mu e} \equiv \Gamma_{\rm conv}(\mu^- N \to e^- N_{\rm g.s.})/\Gamma_{\rm capture}(\mu^- N)$

$$\Gamma_{conv} = \left| \int dr r^2 \left[\widetilde{f}_{E0}(r) \left(g_e^- g_\mu^- + f_e^- f_\mu^- \right) + \frac{f_{M1}}{m} \left(g_e^- f_\mu^- + f_e^- g_\mu^- \right) \frac{dV}{dr} \right] \right|^2$$
$$+ \left| \int dr r^2 \left[\widetilde{f}_{M0}(r) \left(g_e^- g_\mu^- + f_e^- f_\mu^- \right) + \frac{f_{E1}}{m} \left(g_e^- f_\mu^- + f_e^- g_\mu^- \right) \frac{dV}{dr} \right] \right|^2$$

where we defined

Czarnecki, Marciano, Melnikov

$$\widetilde{f}_{E0} = -f_{E0} \frac{4\pi Z\alpha}{m^2} \rho_p(r) + \frac{G_F}{\sqrt{2}} a \left[Z(2+c_d)\rho_p(r) + N(1+2c_d)\rho_n(r) \right]$$

$$\widetilde{f}_{M0} = -f_{E0} \frac{4\pi Z\alpha}{m^2} \rho_p(r) + \frac{G_F}{\sqrt{2}} b \left[Z(2+c_d)\rho_p(r) + N(1+2c_d)\rho_n(r) \right]$$

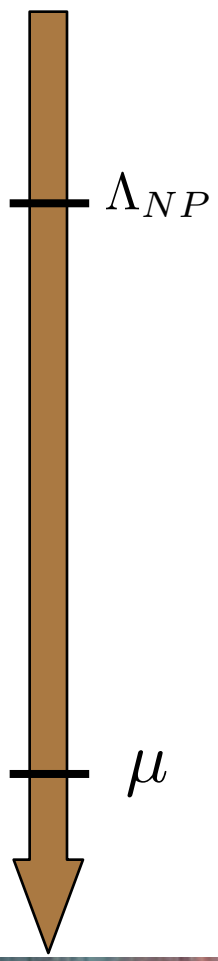
★ Now, for a particular model (Z' exchange): $f_{E(M)i}=0$, $b=a$, $ac_d=(g_v^0-g_v^1)/2$, $a=(g_v^0+g_v^1)/2$

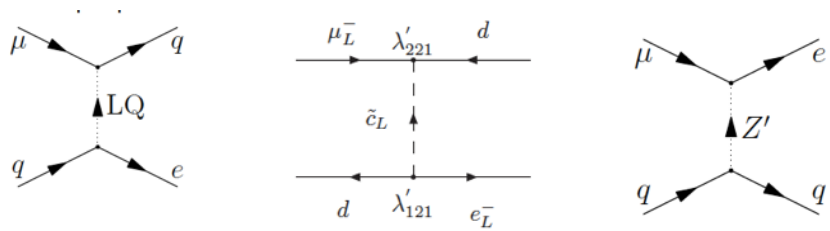
$|g_v^0| < 3.9\cdot 10^{-7}$, $|g_v^1| < 9.7\cdot 10^{-6}$

2. Muon conversion: Effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

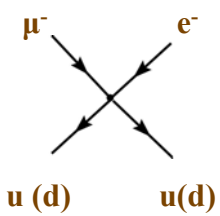
★ It is important to understand relevant energy scales for the problem at hand

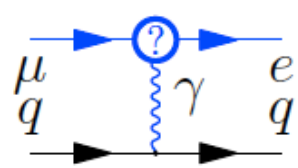




...

other New Physics models





Scale of the conversion experiment

Effective Lagrangians: dim 6

★ Naive power counting: largest contribution from lowest dimensional operators

★ Can write the most general dim-6 Lagrangian $\mathcal{L}^{(6)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{12}\sum_q C_i^{q\ell_1\ell_2} Q_i^{q\ell_1\ell_2} + \text{H.c.},$

- scalar operators

$$Q_1^{q\ell_1\ell_2} = (\bar{\ell}_{1R}\ell_{2L})\ (\bar{q}_R q_L),$$
$$Q_2^{q\ell_1\ell_2} = (\bar{\ell}_{1R}\ell_{2L})\ (\bar{q}_L q_R),$$
$$Q_3^{q\ell_1\ell_2} = (\bar{\ell}_{1L}\ell_{2R})\ (\bar{q}_R q_L),$$
$$Q_4^{q\ell_1\ell_2} = (\bar{\ell}_{1L}\ell_{2R})\ (\bar{q}_L q_R),$$

- vector operators

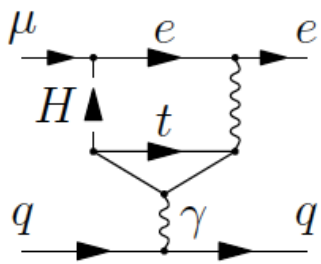
$$Q_5^{q\ell_1\ell_2} = (\bar{\ell}_{1L}\gamma^\mu\ell_{2L})(\bar{q}_L\gamma_\mu q_L),$$
$$Q_6^{q\ell_1\ell_2} = (\bar{\ell}_{1L}\gamma^\mu\ell_{2L})(\bar{q}_R\gamma_\mu q_R),$$
$$Q_7^{q\ell_1\ell_2} = (\bar{\ell}_{1R}\gamma^\mu\ell_{2R})(\bar{q}_L\gamma_\mu q_L),$$
$$Q_8^{q\ell_1\ell_2} = (\bar{\ell}_{1R}\gamma^\mu\ell_{2R})(\bar{q}_R\gamma_\mu q_R),$$

- tensor operators (do not contribute to considered processes)

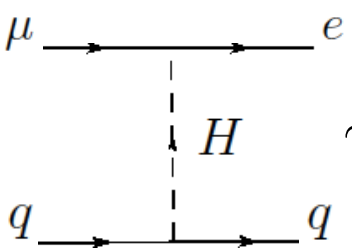
An example in a particular model:

★ Consider an example: FCNC Higgs model

e.g.



$\sim \mathcal{O}(\alpha^2)$

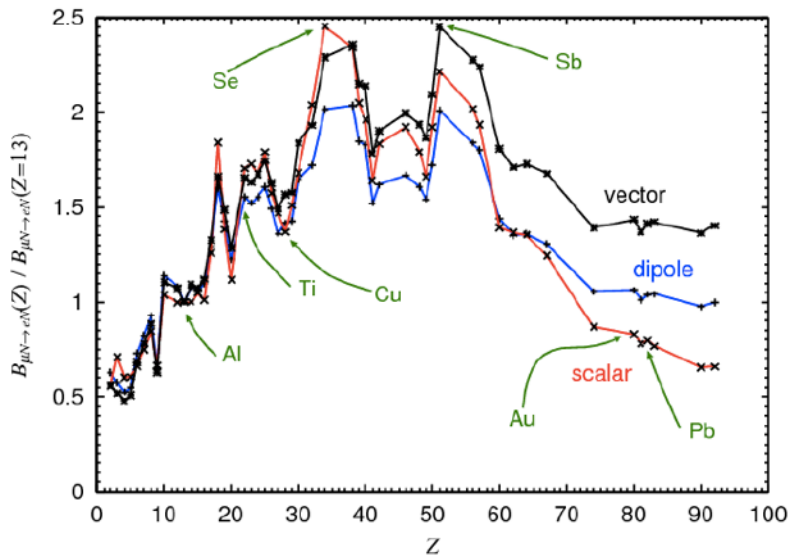


$\sim \mathcal{O}(m_u \text{ or } m_d)$

★ Couplings of new physics to light quarks are suppressed...

Effective Lagrangians: dim 6

★ In principle, can separate contributions of different operators by looking at different nuclei



Kitano, Koike, Okada, 2002

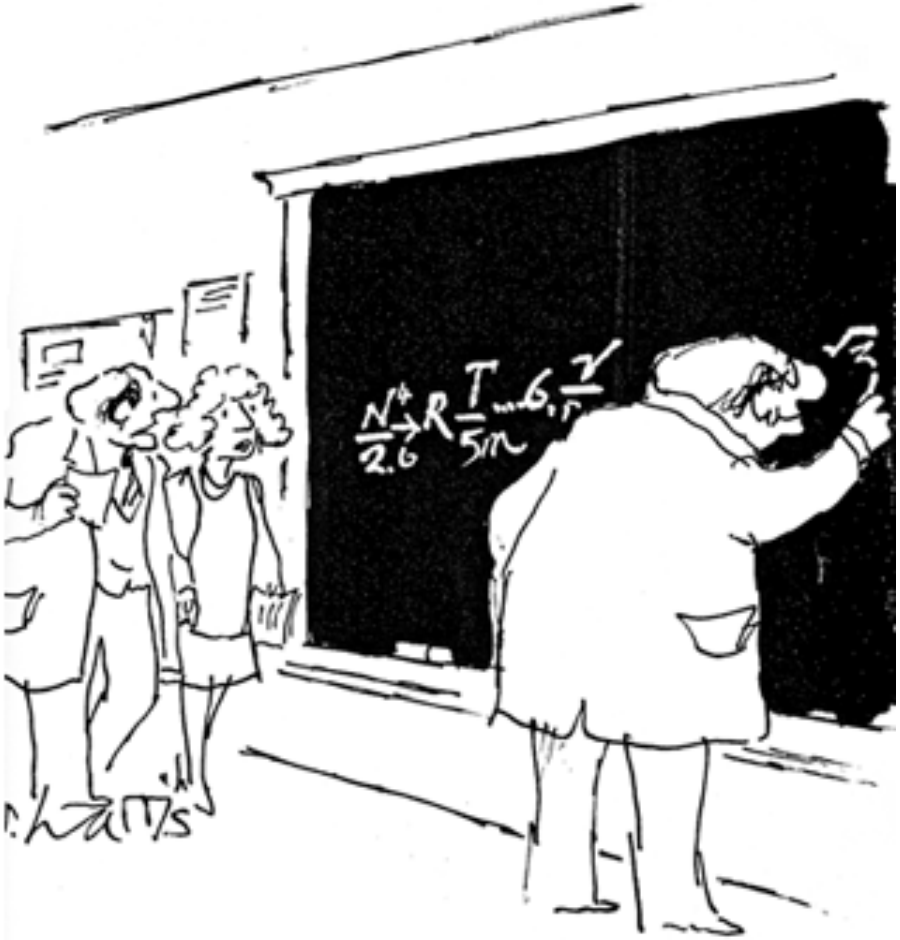
★ It appears that some models are not probed well by dim 6 operators

★ What about higher-dimensional operators? Just like in Fermi theory of weak interactions

$$J_{1\mu}\frac{1}{q^2-M_W^2}J_2^\mu \to -\frac{1}{M_W^2}J_{1\mu}J_2^\mu - \frac{q^2}{M_W^4}J_{1\mu}J_2^\mu + \ldots$$

suppressed by the highest scale

This is not entirely correct!



"I THINK HE'S CROSSED THAT THIN LINE FROM SPECULATIVE FICTION INTO OUTRIGHT FANTASY."

S. Harris

A counter-example

★ Consider other FCNC; e.g. in kaon decays:

★ Main contribution is from dim-6 operators:

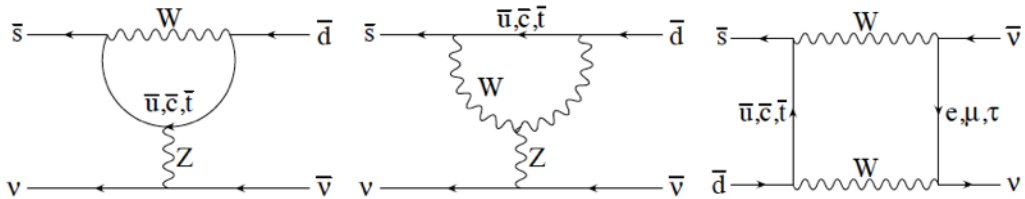


FIG. 1. Penguin and box diagrams responsible for $K^+ \to \pi^+ \nu\bar{\nu}$.

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ \left[\left(\frac{\mathrm{Im}\,\xi_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\mathrm{Re}\,\xi_c}{3\lambda^5} \sum_l X^l_{NL}(x_c) + \frac{\mathrm{Re}\,\xi_t}{\lambda^5} X(x_t) \right)^2 \right]$$

★ ... but there is also contribution from dim-8 operators:

a chunk of uncertainty due to charm mass

Falk, Lewandowski, AAP (1998)

Remember to take into account charm scale!

A counter-example

★ Consider other FCNC; e.g. in kaon decays: $K^+ \to \pi^+ \nu\bar{\nu}$

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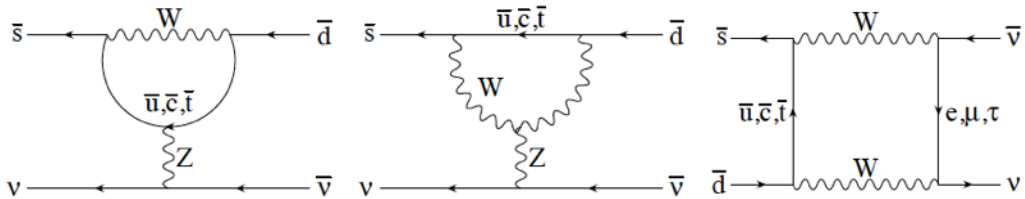

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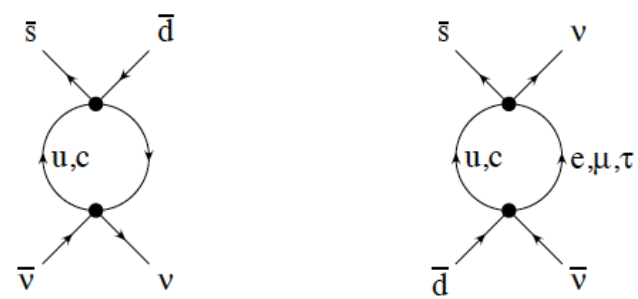


$$O_1^l = \bar{s}\gamma^\nu(1-\gamma^5)d\,(i\partial)^2\left[\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l\right],$$

$$O_2^l = \bar{s}\gamma^\nu(1-\gamma^5)(iD)^2 d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l + 2\bar{s}\gamma^\nu(1-\gamma^5)(iD^\mu)d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial_\mu)\nu_l$$
$$+ \bar{s}\gamma^\nu(1-\gamma^5)d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial)^2\nu_l\,.$$

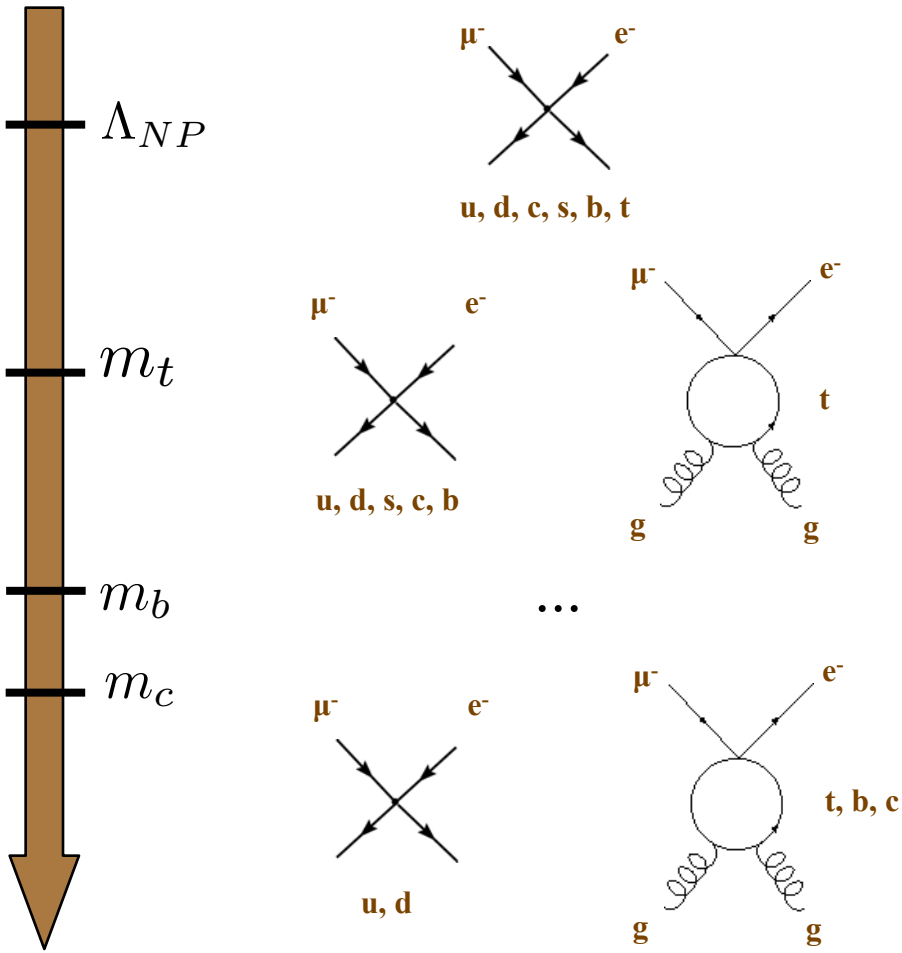
Falk, Lewandowski, AAP (1998)

Remember to take into account charm scale!

Muon conversion: Effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

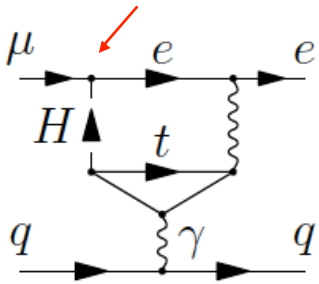
heavy quarks decouple

Scale of the conversion experiment

An example in a FCNC Higgs model:

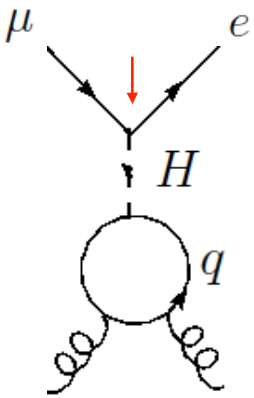
★ Contribution of heavy quarks can, in principle, be large

★ Two-loop sensitivity to NP in conversion experiment...



$$\sim \mathcal{O}(\alpha^2)$$

★ ... becomes one-loop!



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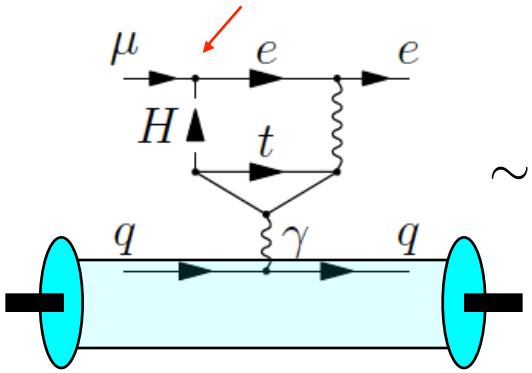
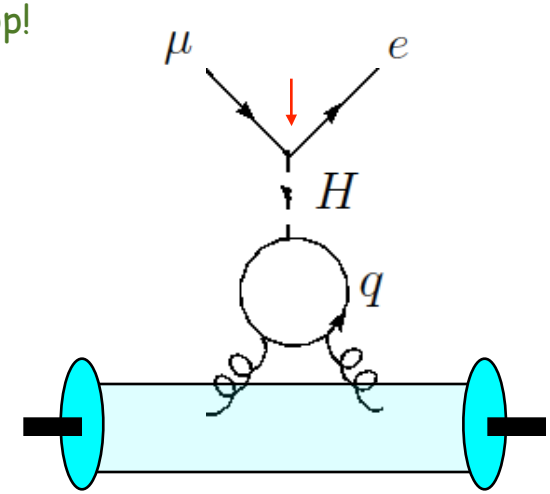


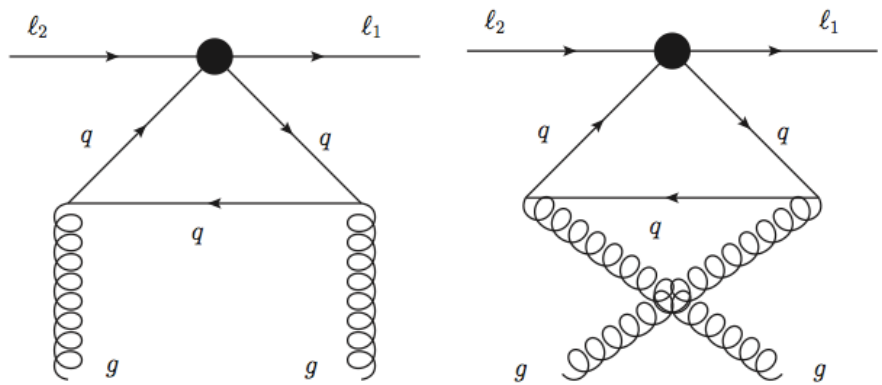
$\sim \mathcal{O}(\alpha^2)$

★ ... becomes one-loop!



Effective Lagrangians: dim 7

★ Let's integrate out heavy quarks and concentrate on gluonic operators



$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \text{H.c.},$$

★ we can calculate their contribution to tau decay rates!
★ c_i probe couplings of heavy quarks to New Physics

AAP and D. Zhuridov
PRD89 (2014) 033005

Effective Lagrangians: dim 7

★ ... get an effective Lagrangian

$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \text{H.c.},$$

...where we defined operators

$$O_1^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu},$$

$$O_2^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu},$$

$$O_3^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu},$$

$$O_4^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu},$$

...and Wilson coefficients

$$c_1^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}(C_1^{q\ell_1\ell_2} + C_2^{q\ell_1\ell_2}),$$

$$c_2^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}(C_1^{q\ell_1\ell_2} - C_2^{q\ell_1\ell_2}),$$

$$c_3^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}(C_3^{q\ell_1\ell_2} + C_4^{q\ell_1\ell_2}),$$

$$c_4^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}(C_3^{q\ell_1\ell_2} - C_4^{q\ell_1\ell_2}),$$

AAP and D. Zhuridov
PRD89 (2014) 033005

Effective Lagrangians: dim 7

★ Just like before, let's calculate the conversion amplitude (lepton part is identical)

$$M^{\mu e}_{NN'} = \frac{1}{\Lambda^2 M_Q} \int d^3x \left[\left(c_1 \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} + c_3 \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} \right) \langle N' | \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} | N \rangle \right.$$
$$\left. + \left(c_2 \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} + c_4 \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} \right) \langle N' | \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} | N \rangle \right]$$

★ Use the same approximation to relate nucleon and nuclear matrix elements…

$$\left\langle N \left| \frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} \right| N \right\rangle$$
$$= -\frac{9}{2} \left[Z G^{(g,p)} \rho^{(p)} + (A - Z) G^{(g,n)} \rho^{(n)} \right]$$

★ … and calculate (relevant) parity-conserving nucleon matrix element

$$G^{(g,\mathcal{N})} = \left\langle \mathcal{N} \left| \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} \right| \mathcal{N} \right\rangle = -189 \text{ MeV}$$

Numerical estimates

★ Conversion probability (factoring out lightest heavy quark mass)

$$\Gamma_{\rm conv}(\mu N \to eN) = \frac{4}{\Lambda^4}(|c_1|^2 + |c_3|^2) a_N^2,$$

★ ... where we defined $a_N = G^{(g,p)} S^{(p)} + G^{(g,n)} S^{(n)}$

★ ... and also

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} (g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A - Z) \rho^{(n)} (g_e^- g_\mu^- - f_e^- f_\mu^-)$$

$$\rho_{p(n)}(r) = \frac{\rho_0}{1 + \exp[(r-c)/z]}$$

Nucleus	Model	c, fm	z, fm	$S^{(p)}$	$S^{(n)}$
${}^{48}_{22}$Ti	FB	–	–	0.0368	0.0435
${}^{197}_{79}$Au	2pF	6.38	0.535	0.0614	0.0918

TABLE I. Nucleon densities model parameters and the overlap integrals in the unit of $m_\mu^{5/2}$ for several nuclei.

Numerical estimates

★ Conversion probability

$$B^N_{\mu e} \equiv \Gamma_{\rm conv}(\mu^- N \to e^- N_{\rm g.s.})/\Gamma_{\rm capture}(\mu^- N)$$

★ ... results in constraints on scale/Wilson coefficient

Coefficient	Bound on $\lvert c_i^{e\mu}\rvert/\Lambda^2$, GeV^{-3}	
	Conversion on $^{48}_{22}$Ti	Conversion on $^{197}_{79}$Au
c_1	5.7×10^{-12}	2.6×10^{-12}
c_2	N/A	N/A
c_3	5.7×10^{-12}	2.6×10^{-12}
c_4	N/A	N/A

★ Important: can only probe parity-conserving operators!!!

2a. Leptoquarks as an example

★ Leptoquark interactions

- scalar leptoquarks

$$\mathcal{L}_S = \left(\lambda_{LS_0}\bar{q}_L^c i\tau_2 \ell_L + \lambda_{RS_0}\bar{u}_R^c e_R\right) S_0^\dagger + \left(\lambda_{LS_{1/2}}\bar{u}_R \ell_L + \lambda_{RS_{1/2}}\bar{q}_L i\tau_2 e_R\right) S_{1/2}^\dagger + \text{H.c.},$$

- vector leptoquarks

$$\mathcal{L}_V = \left(\lambda_{LV_0}\bar{q}_L\gamma_\mu \ell_L + \lambda_{RV_0}\bar{d}_R\gamma_\mu e_R\right) V_0^{\mu\dagger} + \left(\lambda_{LV_{1/2}}\bar{d}_R^c\gamma_\mu \ell_L + \lambda_{RV_{1/2}}\bar{q}_L^c\gamma_\mu e_R\right) V_{1/2}^{\mu\dagger} + \text{H.c.},$$

Davidson, Bailey, Campbell

Leptoquarks (cont)

★ Integrating out **scalar** LQ, we get the following dim 6 operators

AAP and D. Zhuridov
PRD89 (2014) 033005

$$\frac{\lambda^e_{RS_0}\lambda^\mu_{LS_0}}{2M^2_{S_0}}(\bar{e}_R\mu_L)(\bar{t}^c_L t^c_R) + \frac{\lambda^e_{RS_{1/2}}\lambda^\mu_{LS_{1/2}}}{2M^2_{S_{1/2}}}(\bar{e}_R\mu_L)(\bar{t}_R t_L)$$

$$+ \quad \frac{\lambda^\mu_{RS_0}\lambda^e_{LS_0}}{2M^2_{S_0}}(\bar{e}_L\mu_R)(\bar{t}^c_R t^c_L) + \frac{\lambda^\mu_{RS_{1/2}}\lambda^e_{LS_{1/2}}}{2M^2_{S_{1/2}}}(\bar{e}_L\mu_R)(\bar{t}_L t_R) + \text{H.c.},$$

★ Integrating out **vector** LQ, we get the following dim 6 operators

$$\frac{\lambda^e_{LV_0}\lambda^\mu_{RV_0}}{M^2_{V_0}}(\bar{e}_L\mu_R)(\bar{b}_R b_L) + \frac{\lambda^e_{LV_{1/2}}\lambda^\mu_{RV_{1/2}}}{M^2_{V_{1/2}}}(\bar{e}_L\mu_R)(\bar{b}^c_L b^c_R)$$

$$+ \quad \frac{\lambda^\mu_{LV_0}\lambda^e_{RV_0}}{M^2_{V_0}}(\bar{e}_R\mu_L)(\bar{b}_L b_R) + \frac{\lambda^\mu_{LV_{1/2}}\lambda^e_{RV_{1/2}}}{M^2_{V_{1/2}}}(\bar{e}_R\mu_L)(\bar{b}^c_R b^c_L) + \text{H.c.}$$

using $\bar{\psi}^c_{1L}\psi^c_{2R} = -\psi^T_1 P_R \bar{\psi}^T_2 = \bar{\psi}_{2L}\psi_{1R}$ we get our old operators

Leptoquarks (cont)

★ Thus, we get the operators from our general basis

$$\frac{\lambda^e_{RS_0}\lambda^\mu_{LS_0}}{2M^2_{S_0}}(\bar{e}_R\mu_L)(\bar{t}_L t_R) + \frac{\lambda^e_{RS_{1/2}}\lambda^\mu_{LS_{1/2}}}{2M^2_{S_{1/2}}}(\bar{e}_R\mu_L)(\bar{t}_R t_L)$$
$$+ \quad \frac{\lambda^\mu_{RS_0}\lambda^e_{LS_0}}{2M^2_{S_0}}(\bar{e}_L\mu_R)(\bar{t}_R t_L) + \frac{\lambda^\mu_{RS_{1/2}}\lambda^e_{LS_{1/2}}}{2M^2_{S_{1/2}}}(\bar{e}_L\mu_R)(\bar{t}_L t_R) + \text{H.c.},$$

and

$$\frac{\lambda^e_{LV_0}\lambda^\mu_{RV_0}}{M^2_{V_0}}(\bar{e}_L\mu_R)(\bar{b}_R b_L) + \frac{\lambda^e_{LV_{1/2}}\lambda^\mu_{RV_{1/2}}}{M^2_{V_{1/2}}}(\bar{e}_L\mu_R)(\bar{b}_L b_R)$$
$$+ \quad \frac{\lambda^\mu_{LV_0}\lambda^e_{RV_0}}{M^2_{V_0}}(\bar{e}_R\mu_L)(\bar{b}_L b_R) + \frac{\lambda^\mu_{LV_{1/2}}\lambda^e_{RV_{1/2}}}{M^2_{V_{1/2}}}(\bar{e}_R\mu_L)(\bar{b}_R b_L) + \text{H.c.}$$

★ ... and so we can get constraints on leptoquark parameters

Leptoquarks (fin.)

★ Correspondence table

Factor in $\mathcal{L}^{gg}_{e\mu}$	Expression through LQ parameters
$-\frac{c_1}{\Lambda^2 M_Q}$	$\frac{I_1^t}{m_t}\left(\frac{\lambda^e_{RS_0}\lambda^\mu_{LS_0}}{M^2_{S_0}}+\frac{\lambda^e_{RS_{1/2}}\lambda^\mu_{LS_{1/2}}}{M^2_{S_{1/2}}}\right)+2\frac{I_1^b}{m_b}\left(\frac{\lambda^\mu_{LV_0}\lambda^e_{RV_0}}{M^2_{V_0}}+\frac{\lambda^\mu_{LV_{1/2}}\lambda^e_{RV_{1/2}}}{M^2_{V_{1/2}}}\right)$
$-\frac{c_2}{\Lambda^2 M_Q}$	$\mathrm{i}\frac{I_2^t}{m_t}\left(-\frac{\lambda^e_{RS_0}\lambda^\mu_{LS_0}}{M^2_{S_0}}+\frac{\lambda^e_{RS_{1/2}}\lambda^\mu_{LS_{1/2}}}{M^2_{S_{1/2}}}\right)+2\mathrm{i}\frac{I_2^b}{m_b}\left(-\frac{\lambda^\mu_{LV_0}\lambda^e_{RV_0}}{M^2_{V_0}}+\frac{\lambda^\mu_{LV_{1/2}}\lambda^e_{RV_{1/2}}}{M^2_{V_{1/2}}}\right)$
$-\frac{c_3}{\Lambda^2 M_Q}$	$\frac{I_1^t}{m_t}\left(\frac{\lambda^\mu_{RS_0}\lambda^e_{LS_0}}{M^2_{S_0}}+\frac{\lambda^\mu_{RS_{1/2}}\lambda^e_{LS_{1/2}}}{M^2_{S_{1/2}}}\right)+2\frac{I_1^b}{m_b}\left(\frac{\lambda^e_{LV_0}\lambda^\mu_{RV_0}}{M^2_{V_0}}+\frac{\lambda^e_{LV_{1/2}}\lambda^\mu_{RV_{1/2}}}{M^2_{V_{1/2}}}\right)$
$-\frac{c_4}{\Lambda^2 M_Q}$	$\mathrm{i}\frac{I_2^t}{m_t}\left(\frac{\lambda^\mu_{RS_0}\lambda^e_{LS_0}}{M^2_{S_0}}-\frac{\lambda^\mu_{RS_{1/2}}\lambda^e_{LS_{1/2}}}{M^2_{S_{1/2}}}\right)+2\mathrm{i}\frac{I_2^b}{m_b}\left(\frac{\lambda^e_{LV_0}\lambda^\mu_{RV_0}}{M^2_{V_0}}-\frac{\lambda^e_{LV_{1/2}}\lambda^\mu_{RV_{1/2}}}{M^2_{V_{1/2}}}\right)$

Leptoquarks (fin.)

★ ... or numerically

from c_2

$$\frac{|\lambda^e_{RS_0}\lambda^\mu_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^e_{RS_{1/2}}\lambda^\mu_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.63\times 10^{-15}\ \mathrm{MeV}^{-2},$$

$$\frac{|\lambda^\mu_{LV_0}\lambda^e_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^\mu_{LV_{1/2}}\lambda^e_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 3.52\times 10^{-17}\ \mathrm{MeV}^{-2},$$

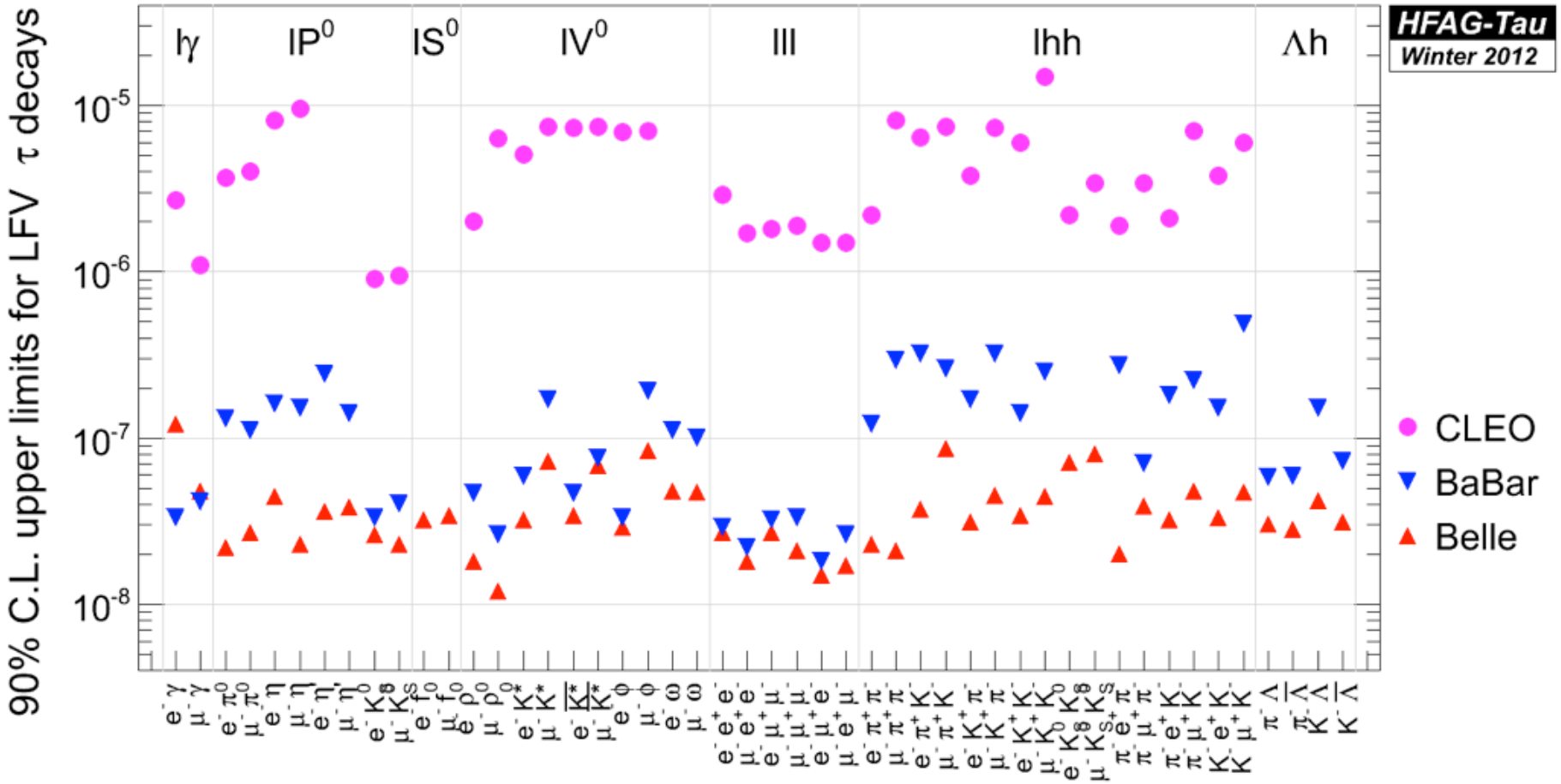
from c_4

$$\frac{|\lambda^\mu_{RS_0}\lambda^e_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^\mu_{RS_{1/2}}\lambda^e_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.63\times 10^{-15}\ \mathrm{MeV}^{-2},$$

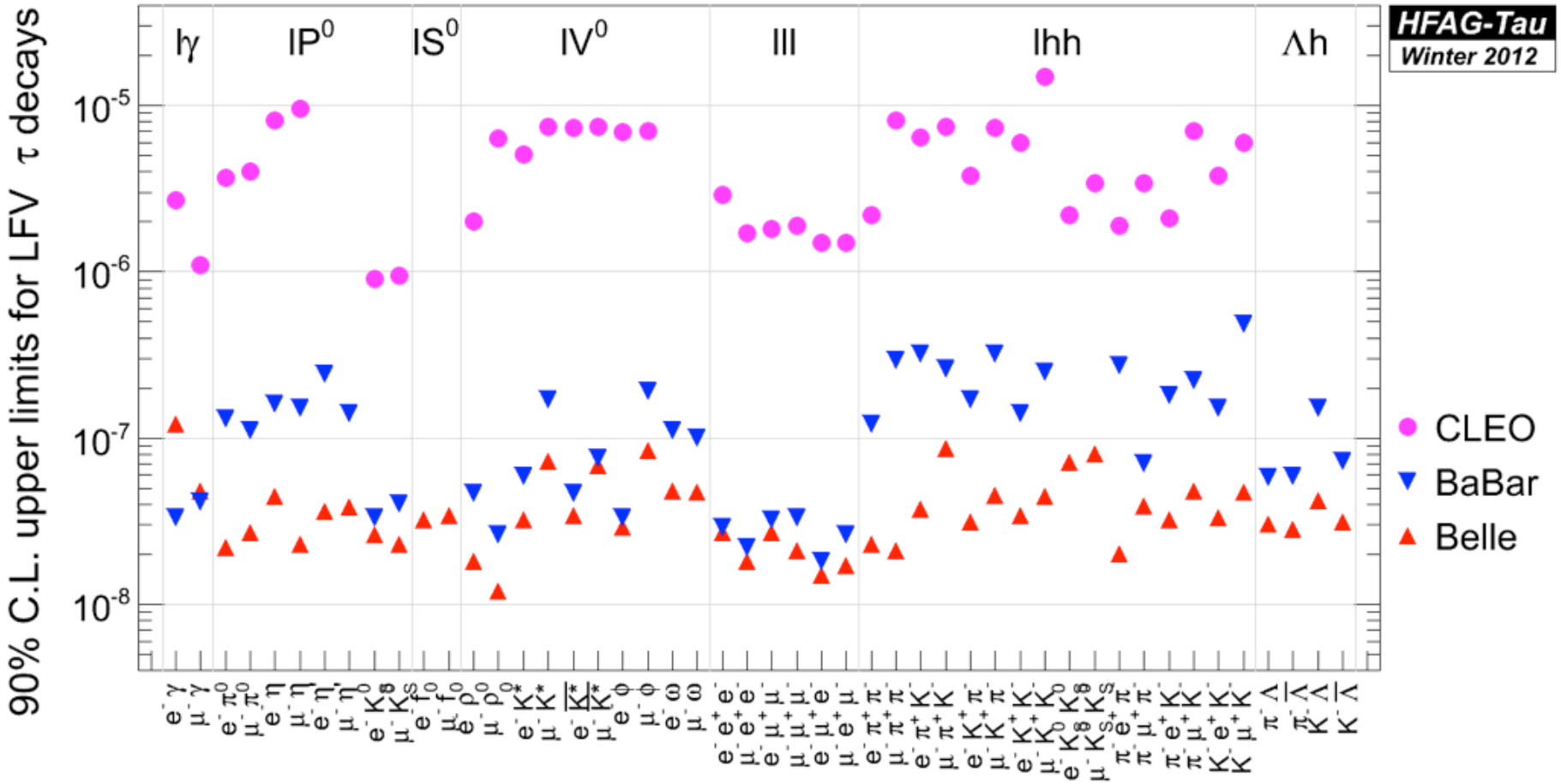
$$\frac{|\lambda^e_{LV_0}\lambda^\mu_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^e_{LV_{1/2}}\lambda^\mu_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 3.52\times 10^{-17}\ \mathrm{MeV}^{-2}.$$

★ Important: can only probe parity-conserving operators!!!

What about taus?



What about taus?



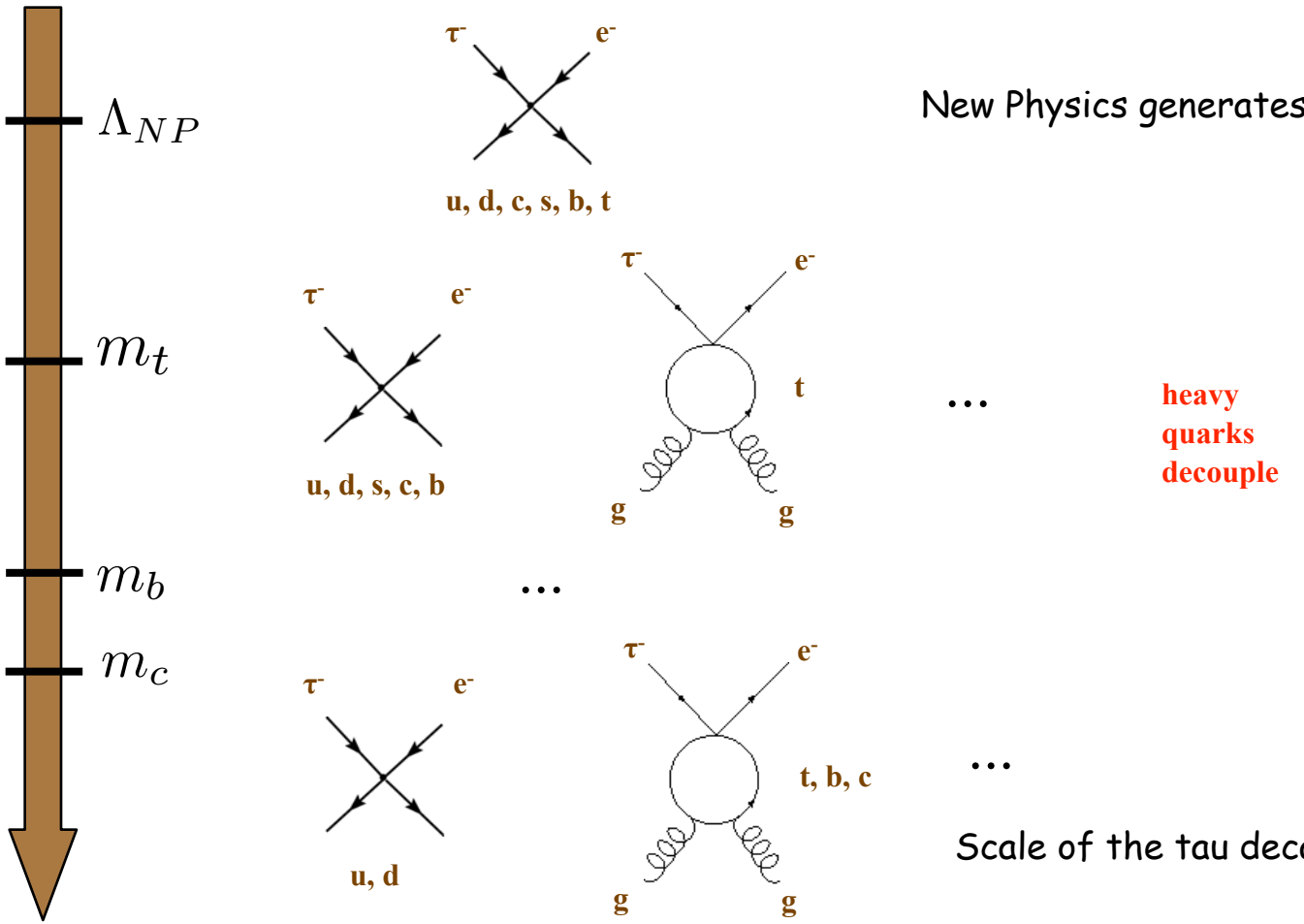
What can we learn about NP from this data?

Can we probe parity-violating operators?

3. Effective Lagrangians for tau decay

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

heavy quarks decouple

Scale of the tau decay experiment

Effective Lagrangians: dim 7

★ ... get an effective Lagrangian

$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \text{H.c.},$$

...where we defined operators

$$O_1^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu},$$

$$O_2^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu},$$

$$O_3^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu},$$

$$O_4^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu},$$

...and Wilson coefficients

$$c_1^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}(C_1^{q\ell_1\ell_2} + C_2^{q\ell_1\ell_2}),$$

$$c_2^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}(C_1^{q\ell_1\ell_2} - C_2^{q\ell_1\ell_2}),$$

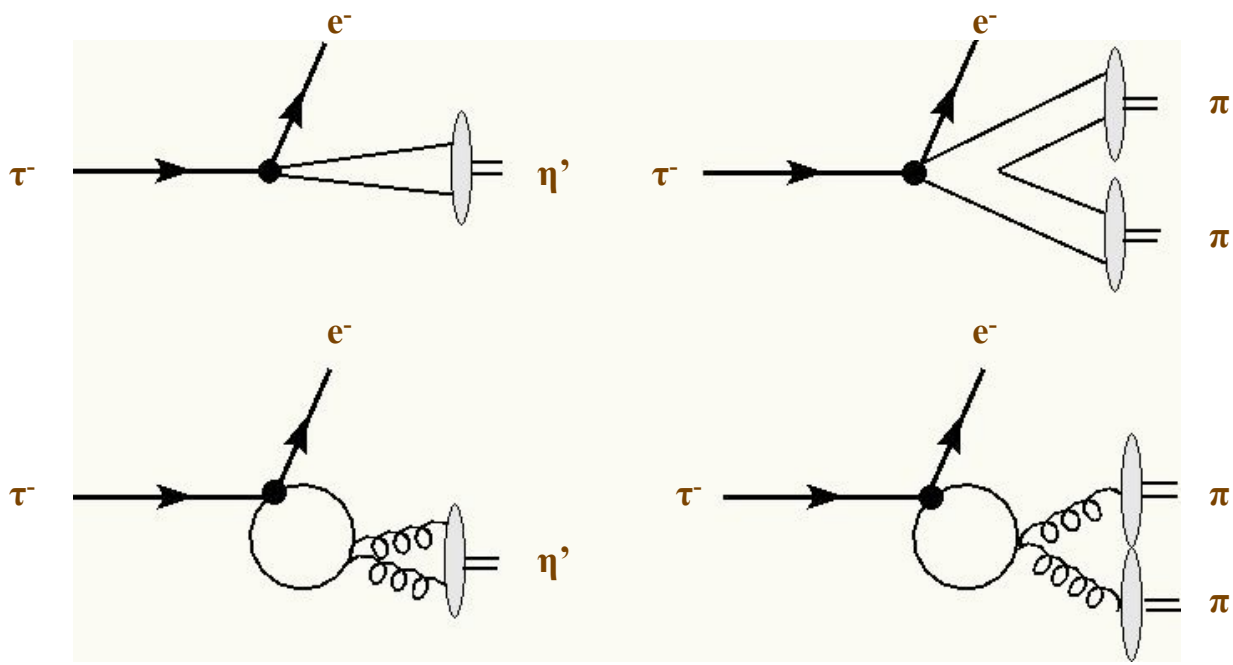
$$c_3^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}(C_3^{q\ell_1\ell_2} + C_4^{q\ell_1\ell_2}),$$

$$c_4^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}(C_3^{q\ell_1\ell_2} - C_4^{q\ell_1\ell_2}),$$

AAP and D. Zhuridov
PRD89 (2014) 033005

Tau decays and heavy quarks

★ Let's compute FCNC tau decays



Hadronic physics I

★ To calculate FV tau decays we need a bit of hadronic physics

- matrix elements of parity-conserving operators

$$\langle \pi^+\pi^- | \bar{q}q | 0 \rangle = \langle K^+K^- | \bar{q}q | 0 \rangle = \delta_q^M B_0$$

$$\langle M^+M^- | \bar{q}\gamma_\mu q | 0 \rangle = \delta_q^M G_M^{(q)}(Q^2)\,(p_+ - p_-)_\mu$$

$$\langle M^+M^- | \frac{\alpha_s}{4\pi} G^{a\mu\nu} G^a_{\mu\nu} | 0 \rangle = -\frac{2}{9} q^2,$$

- ... where B_0=1.96 GeV from $m_\pi^2 = (m_u + m_d)\, B_0$

Black, Han, He, Sher

- ... and we used $\theta^\mu_\mu = -\frac{b\alpha_s}{8\pi} G^{\mu\nu a} G^a_{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} q$

Voloshin

Hadronic physics II

★ To calculate FV tau decays we need a bit of hadronic physics

- matrix elements of parity-violating operators

$$\langle M(p)|\bar{q}\gamma^\mu\gamma_5 q|0\rangle = -ib_q f_M^q p^\mu,$$
$$\langle M(p)|\bar{q}\gamma_5 q|0\rangle = -ib_q h_M^q,$$
$$\langle M(p)|\frac{\alpha_s}{4\pi}G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}|0\rangle = a_M,$$

- ... where q=u,d,s and $b_{u,d}=1/2^{1/2}$, while $b_s=1$

- ... and in the FKS scheme of eta-eta' mixing

$$a_\eta = -\frac{m_{\eta'}^2 - m_\eta^2}{2}\sin 2\phi\left(-f_q b_q \sin\phi + f_s\cos\phi\right),$$
$$a_{\eta'} = -\frac{m_{\eta'}^2 - m_\eta^2}{2}\sin 2\phi\left(f_q b_q \sin\phi + f_s\cos\phi\right),$$

Bounds: parity conserving

★ Looking at the scalar operators only

$$\frac{d\Gamma(\tau \to \ell M^+ M^-)}{dq^2} = \frac{m_\tau}{32(2\pi)^3\Lambda^4}\left[|A_{MM}|^2 + |B_{MM}|^2\right] \times \sqrt{1-\frac{4m_M^2}{q^2}}\left(1-\frac{q^2}{m_\tau^2}\right)^2,$$

- … with the following coefficients

$$A_{MM} = -\frac{2c_1^{\ell\tau}}{9}q^2 + \frac{1}{2}\sum_{q=u,d,s}\left(C_1^{q\ell\tau} + C_2^{q\ell\tau}\right)\delta_q^M B_0,$$

$$B_{MM} = -\frac{2c_3^{\ell\tau}}{9}q^2 + \frac{1}{2}\sum_{q=u,d,s}\left(C_3^{q\ell\tau} + C_4^{q\ell\tau}\right)\delta_q^M B_0.$$

Coef	Bound on $\lvert c_i^{\ell\tau}\rvert/\Lambda^2$, GeV^{-3}							
	$\mathcal{B}(\tau \to \mu\,\pi^+\pi^-)$ $< 2.1\times10^{-8}$	$\mathcal{B}(\tau \to e\,\pi^+\pi^-)$ $< 2.3\times10^{-8}$	$\mathcal{B}(\tau \to \mu K^+K^-)$ $< 4.4\times10^{-8}$	$\mathcal{B}(\tau \to eK^+K^-)$ $< 3.3\times10^{-8}$	$\mathcal{B}(\tau \to \mu\eta')$ $< 1.3\times10^{-7}$	$\mathcal{B}(\tau \to e\eta')$ $< 1.6\times10^{-7}$	$\mathcal{B}(\tau \to \mu\eta)$ $< 1.3\times10^{-7}$	$\mathcal{B}(\tau \to e\eta)$ $< 1.6\times10^{-7}$
c_1	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_2	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}
c_3	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_4	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}

Bounds: parity violating

★ Again, looking at the scalar operators only

$$\Gamma(\tau \to \mu M) = \frac{m_\tau}{8\pi\Lambda^4}\left[|A_M|^2 + |B_M|^2\right]\left(1 - \frac{m_M^2}{m_\tau^2}\right)^2$$

- … with the following coefficients

$$A_M = -\frac{2i}{9}c_2^{\ell\tau}a_M + \sum_{q=u,d,s}\left(C_2^{q\ell\tau} - C_1^{q\ell\tau}\right)\frac{b_q h_M^q}{4m_q} + \frac{1}{2}m_\mu \sum_{q=u,d,s}\left(C_5^{q\ell\tau} - C_6^{q\ell\tau}\right) b_q f_M^q - \frac{1}{2}m_\tau \sum_{q=u,d,s}\left(C_7^{q\ell\tau} - C_8^{q\ell\tau}\right) b_q f_M^q$$

$$B_M = -\frac{2i}{9}c_4^{\ell\tau}a_M + \sum_{q=u,d,s}\left(C_4^{q\ell\tau} - C_3^{q\ell\tau}\right)\frac{b_q h_M^q}{4m_q} - \frac{1}{2}m_\tau \sum_{q=u,d,s}\left(C_5^{q\ell\tau} - C_6^{q\ell\tau}\right) b_q f_M^q + \frac{1}{2}m_\mu \sum_{q=u,d,s}\left(C_7^{q\ell\tau} - C_8^{q\ell\tau}\right) b_q f_M^q$$

Coef	Bound on $\lvert c_i^{\ell\tau}\rvert/\Lambda^2$, GeV^{-3}							
	$\mathcal{B}(\tau \to \mu\,\pi^+\pi^-)$ $< 2.1 \times 10^{-8}$	$\mathcal{B}(\tau \to e\,\pi^+\pi^-)$ $< 2.3 \times 10^{-8}$	$\mathcal{B}(\tau \to \mu K^+K^-)$ $< 4.4 \times 10^{-8}$	$\mathcal{B}(\tau \to eK^+K^-)$ $< 3.3 \times 10^{-8}$	$\mathcal{B}(\tau \to \mu\eta')$ $< 1.3 \times 10^{-7}$	$\mathcal{B}(\tau \to e\eta')$ $< 1.6 \times 10^{-7}$	$\mathcal{B}(\tau \to \mu\eta)$ $< 1.3 \times 10^{-7}$	$\mathcal{B}(\tau \to e\eta)$ $< 1.6 \times 10^{-7}$
c_1	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_2	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}
c_3	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_4	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}

3a. Leptoquarks as an example

★ Leptoquark interactions

- scalar leptoquarks

$$\mathcal{L}_S = \left(\lambda_{LS_0}\bar{q}_L^c \mathrm{i}\tau_2 \ell_L + \lambda_{RS_0}\bar{u}_R^c e_R\right) S_0^\dagger + \left(\lambda_{LS_{1/2}}\bar{u}_R \ell_L + \lambda_{RS_{1/2}}\bar{q}_L \mathrm{i}\tau_2 e_R\right) S_{1/2}^\dagger + \mathrm{H.c.},$$

- vector leptoquarks

$$\mathcal{L}_V = \left(\lambda_{LV_0}\bar{q}_L \gamma_\mu \ell_L + \lambda_{RV_0}\bar{d}_R \gamma_\mu e_R\right) V_0^{\mu\dagger} + \left(\lambda_{LV_{1/2}}\bar{d}_R^c \gamma_\mu \ell_L + \lambda_{RV_{1/2}}\bar{q}_L^c \gamma_\mu e_R\right) V_{1/2}^{\mu\dagger} + \mathrm{H.c.},$$

Davidson, Bailey, Campbell

★ Matching to the general result above, get

C_i^u/Λ^2	Expression	C_i^d/Λ^2	Expression
$\frac{C_1^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_1 u}\lambda_{LS_{1/2}}^{\ell_2 u}}{2M_{S_{1/2}}^2}$	$\frac{C_1^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_2 b}\lambda_{RV_{1/2}}^{\ell_1 b}}{M_{V_{1/2}}^2}$
$\frac{C_2^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_1 u}\lambda_{LS_0}^{\ell_2 u}}{2M_{S_0}^2}$	$\frac{C_2^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_2 b}\lambda_{RV_0}^{\ell_1 b}}{M_{V_0}^2}$
$\frac{C_3^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_2 u}\lambda_{LS_0}^{\ell_1 u}}{2M_{S_0}^2}$	$\frac{C_3^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_1 b}\lambda_{RV_0}^{\ell_2 b}}{M_{V_0}^2}$
$\frac{C_4^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_2 u}\lambda_{LS_{1/2}}^{\ell_1 u}}{2M_{S_{1/2}}^2}$	$\frac{C_4^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_1 b}\lambda_{RV_{1/2}}^{\ell_2 b}}{M_{V_{1/2}}^2}$

Leptoquarks as an example

★ Leptoquark interaction parameters for tau-mu transitions

$$\frac{|\lambda^{\mu t}_{RS_0}\lambda^{\tau t}_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^{\mu t}_{RS_{1/2}}\lambda^{\tau t}_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.3\times 10^{-4}\ \mathrm{GeV}^{-2},$$

$$\frac{|\lambda^{\tau b}_{LV_0}\lambda^{\mu b}_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^{\tau b}_{LV_{1/2}}\lambda^{\mu b}_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 4.4\times 10^{-6}\ \mathrm{GeV}^{-2}$$

★ … and the same for tau-e

$$\frac{|\lambda^{et}_{RS_0}\lambda^{\tau t}_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^{et}_{RS_{1/2}}\lambda^{\tau t}_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.2\times 10^{-4}\ \mathrm{GeV}^{-2}$$

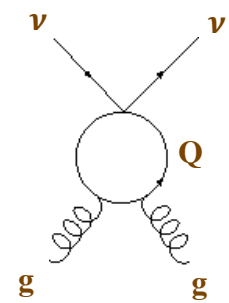
$$\frac{|\lambda^{\tau b}_{LV_0}\lambda^{eb}_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^{\tau b}_{LV_{1/2}}\lambda^{eb}_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 4.2\times 10^{-6}\ \mathrm{GeV}^{-2}$$

AAP and D. Zhuridov
PRD89 (2014) 033005

4. What about neutrinos?

★ Similar effective Lagrangian, but different assumptions (and less operators)

$$\mathcal{L} = \frac{c_1^{(\nu)}}{\Lambda^2}(\overline{\nu}_R \nu_L)\; G^a_{\mu\nu} G^{a\mu\nu} + \frac{c_2^{(\nu)}}{\Lambda^2}(\overline{\nu}_R \nu_L)\; G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$$



...with Wilson coefficients

$$c_1^{(\nu)} = -\frac{2}{9}\frac{\beta_L}{4\alpha_s}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_1^{q\nu} + C_2^{q\nu}\right),$$

$$c_2^{(\nu)} = -\frac{2i}{9}\frac{\beta_L}{4\alpha_s}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_1^{q\nu} - C_2^{q\nu}\right)$$

AAP, to be published

Can constrain both flavor-changing and flavor-conserving interactions

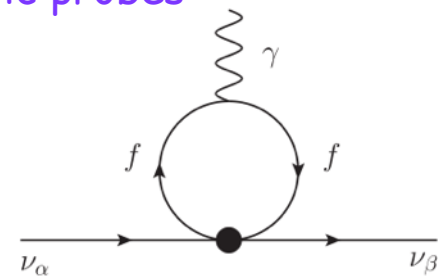
- can constrain NSI of neutrinos with heavy quarks
- ...but need to know absolute flux of neutrinos, expt?

★ Can other (but related) quantities constrain NSI with heavy quarks?

NSIs and neutrino's magnetic moment

★ Same effects can be constrained with electromagnetic probes

$$-\mathcal{L}_{\rm eff} \subset \sum_a \frac{\tilde{\epsilon}^{fa}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma_a f)(\bar{f}\Gamma_a \nu_\alpha) + {\rm H.c.}$$



…which leads to neutrino magnetic moment contribution

$$\mu_{\alpha\beta} = \mu^0_{\alpha\beta} - \sum_q \epsilon^q_{\alpha\beta} \frac{N_c Q_q}{\pi^2} \frac{m_e m_q}{M^2} \ln\left(\frac{M^2}{m_q^2}\right)\mu_B$$

Healy, AAP, Zhuridov PRD87 (2013) 117301

Experimental constrains on magnetic moment (best form astro): $\mu_\nu < 3 \times 10^{-12}\ \mu_B$

- … with (almost) no background from SM: $\mu^{\rm SM}_{ii} \approx 3.2 \times 10^{-20}\left(\frac{m_i}{0.1\ {\rm eV}}\right)\mu_B$

$$\mu^{\rm SM}_{ij} \approx -4 \times 10^{-24}\left(\frac{m_i + m_j}{0.1\ {\rm eV}}\right) \sum_{\ell=e,\mu,\tau} \left(\frac{m_\ell}{m_\tau}\right)^2 U^*_{\ell i} U_{\ell j} \mu_B$$

… can therefore constrain NSIs:

TABLE I. Upper bounds on the couplings $\epsilon^f_{\alpha\beta}$.

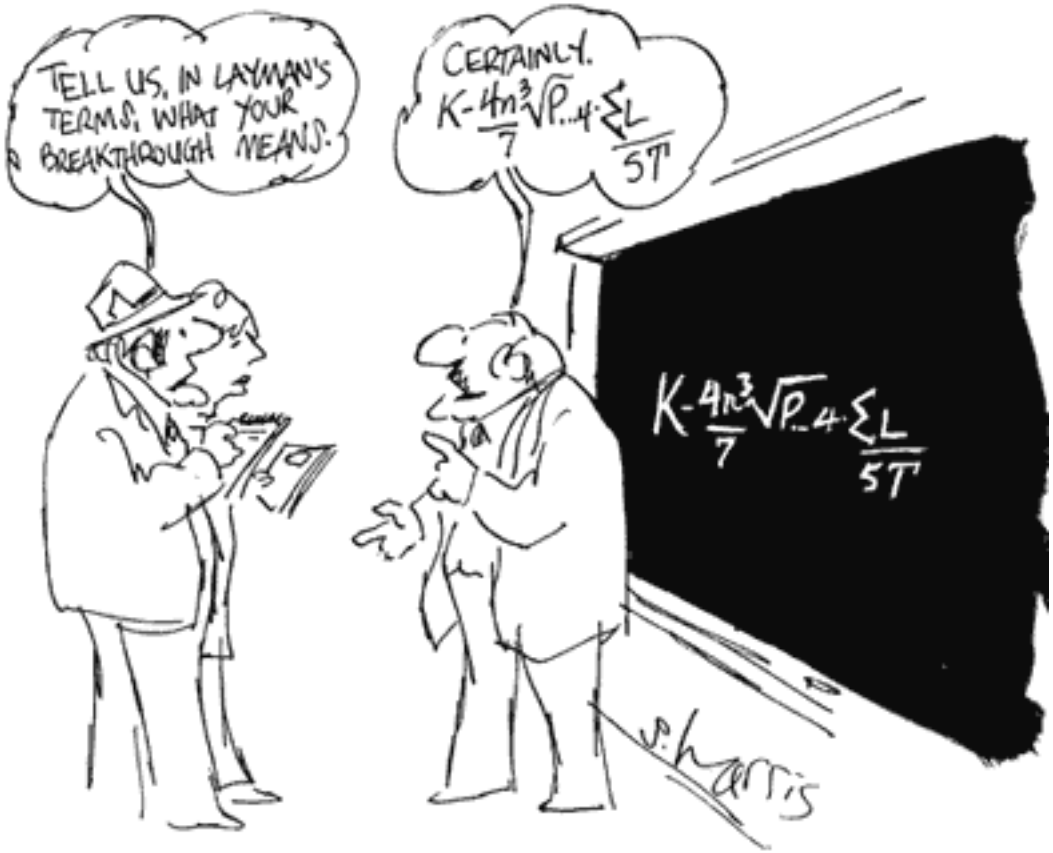
$\lvert\epsilon^e_{\alpha\beta}\rvert$	3.9	$\lvert\epsilon^d_{\alpha\beta}\rvert$	0.49	$\lvert\epsilon^u_{\alpha\beta}\rvert$	0.49
$\lvert\epsilon^\mu_{\alpha\beta}\rvert$	3.0×10^{-2}	$\lvert\epsilon^s_{\alpha\beta}\rvert$	3.3×10^{-2}	$\lvert\epsilon^c_{\alpha\beta}\rvert$	1.7×10^{-3}
$\lvert\epsilon^\tau_{\alpha\beta}\rvert$	2.6×10^{-3}	$\lvert\epsilon^b_{\alpha\beta}\rvert$	1.2×10^{-3}	$\lvert\epsilon^t_{\alpha\beta}\rvert$	4.8×10^{-5}

5. Conclusions

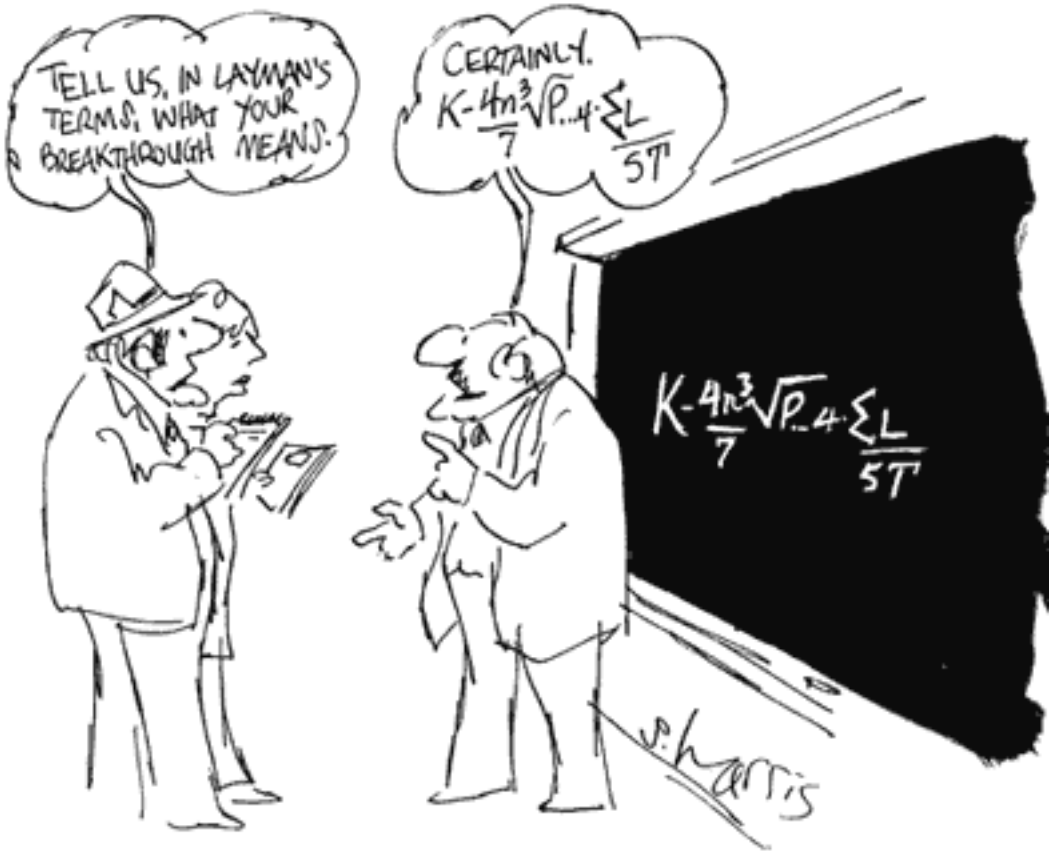
- Flavor puzzle is still a big problem for particle physics
  - Standard Models simply parameterizes the solution
  - New Physics models use flavor as input, not output
- Flavor-changing neutral current transitions provide great opportunities for studies of flavor in the SM and BSM
  - charge lepton transitions offer practically SM-background-free playground
    - large contributions from New Physics are possible, but not seen
    - EFT approach can be useful in studies of muon conversion
    - ... as current methods rarely go beyond dim-6 operators
    - ... and thus do not constrain NP-heavy fermion couplings very well
- New data from Belle-II, but also from LHCb on tau decays!
- Similar physics contributes to NSI and neutrino's magnetic moments

- Maybe flavor physics will be the first place to see glimpses of New Physics
- ...but then again, maybe not.

Thank you for your attention!



Thank you for your attention!

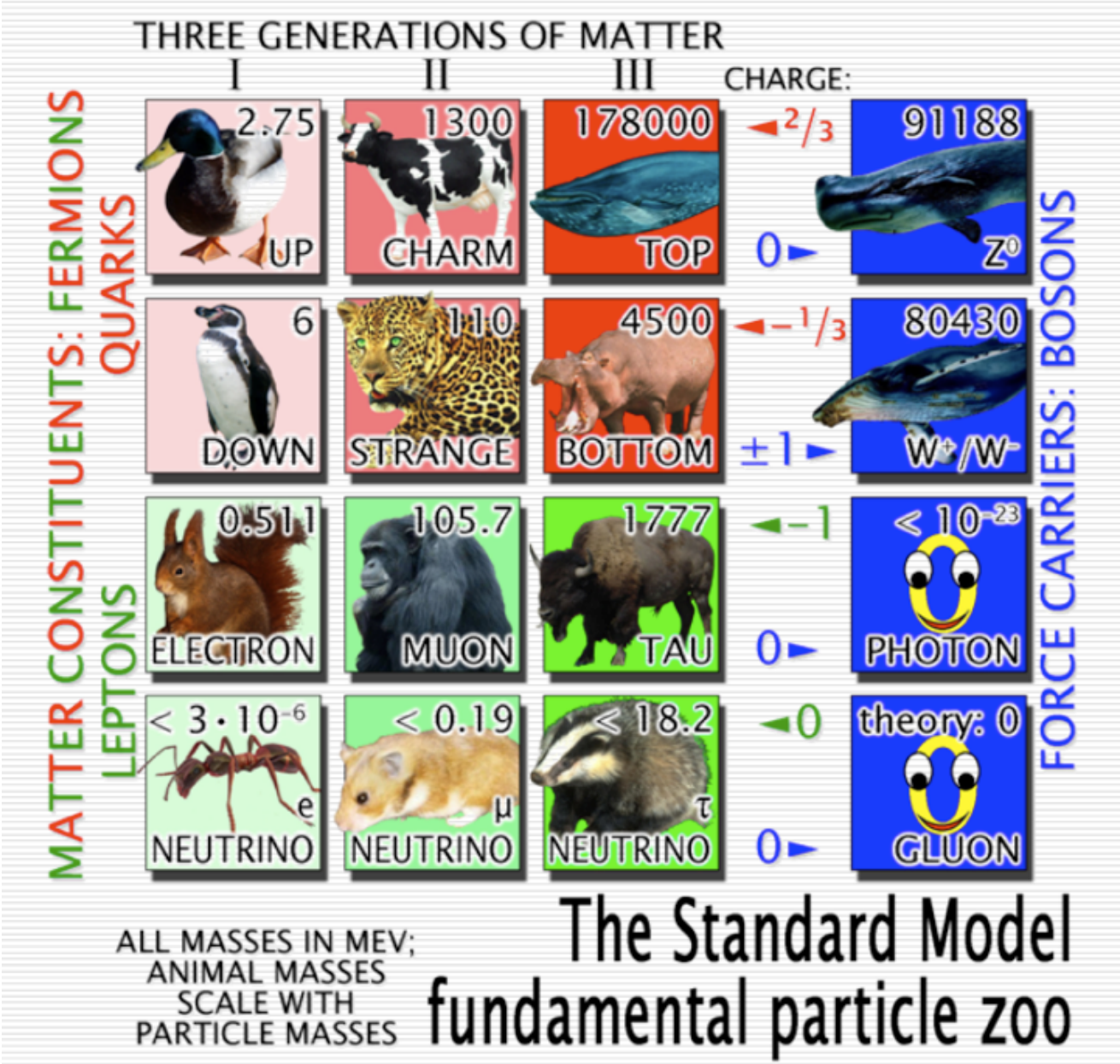


Hopefully, I did better than him…

Additional slides

"Frankly, I even find it hard to believe some of the things I've been coming up with."

S. Harris


THREE GENERATIONS OF MATTER
I
II
III
CHARGE:
MATTER CONSTITUENTS: FERMIONS
QUARKS
LEPTONS
FORCE CARRIERS: BOSONS
2.75
UP
1300
CHARM
178000
TOP
2/3
91188
Z0
0
6
DOWN
110
STRANGE
4500
BOTTOM
−1/3
80430
W+/W−
±1
0.511
ELECTRON
105.7
MUON
1777
TAU
−1
< 10−23
PHOTON
0
< 3 · 10−6
e
NEUTRINO
< 0.19
μ
NEUTRINO
< 18.2
τ
NEUTRINO
0
theory: 0
GLUON
0
ALL MASSES IN MEV;
ANIMAL MASSES
SCALE WITH
PARTICLE MASSES
The Standard Model
fundamental particle zoo