

Lepton-nucleus scattering: from the quasi-elastic to the DIS region

Noemi Rocco

Neutrino Seminar series—Virtual Seminar

Fermilab

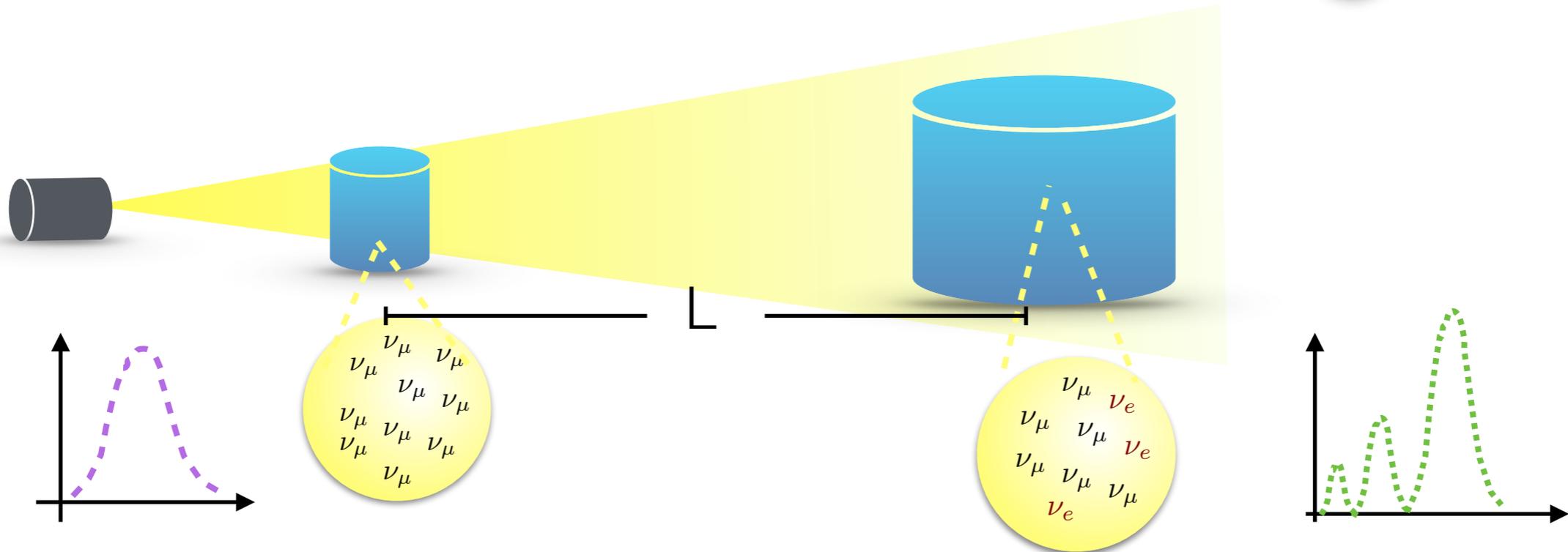
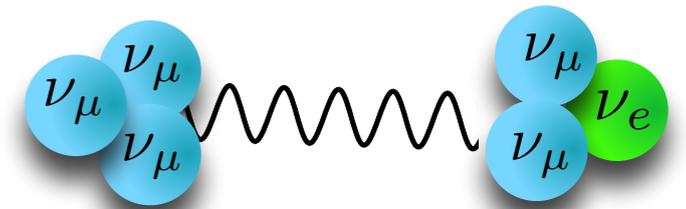
September 10, 2020



In Collaboration with: O. Benhar, J. Carlson, S. Gandolfi, J. Isaacson, W. Jay, T.S.H. Lee, A. Lovato, P. Machado, S.X. Nakamura, T. Sato, R. Schiavilla

Addressing Neutrino-Oscillation Physics

$$P_{\nu_\mu \rightarrow \nu_e}(E, L) \sim \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \rightarrow \Phi_e(E, L) / \Phi_\mu(E, 0)$$



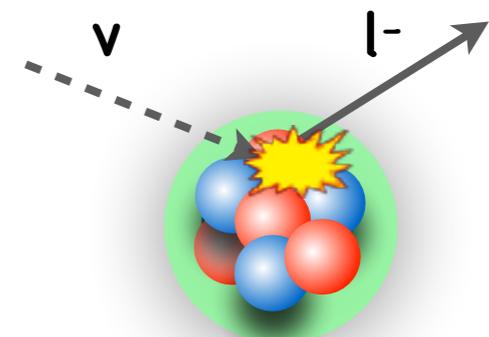
Detectors measure the neutrino interaction rate:

$$N_e(E_{\text{rec}}, L) \propto \sum_i \Phi_e(E, L) \sigma_i(E) f_{\sigma_i}(E, E_{\text{rec}}) dE$$

Reconstructed
ν energy

Cross Section

Smearing
matrix

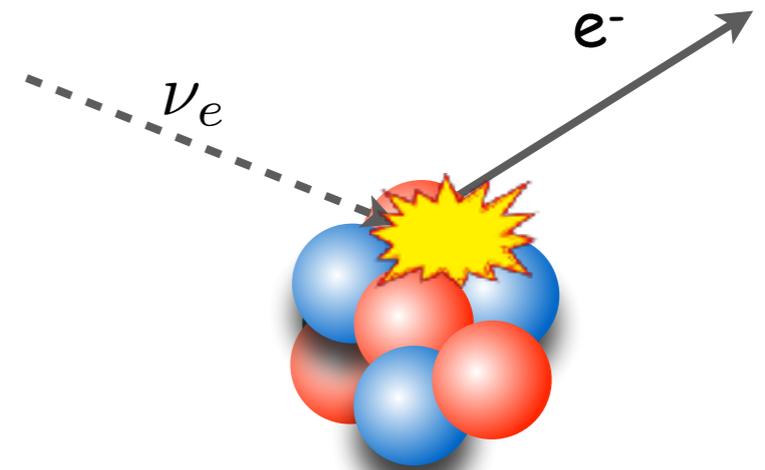


A quantitative knowledge of $\sigma(E)$ and $f_\sigma(E)$ is crucial to precisely extract ν oscillation parameters

To study neutrinos we need nuclei

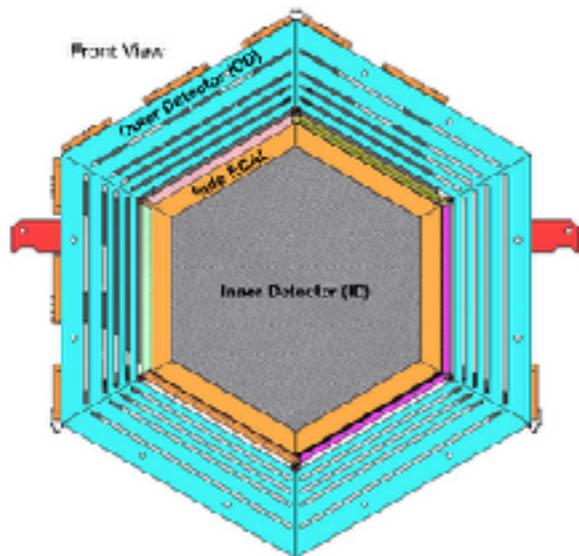
? Where does Nuclear Physics come into play

$$\text{Number of Interactions} = \underbrace{\sigma}_{\text{Cross Section}} \times \underbrace{\Phi}_{\text{Neutrino Flux}} \times \underbrace{N}_{\text{\# Targets}}$$



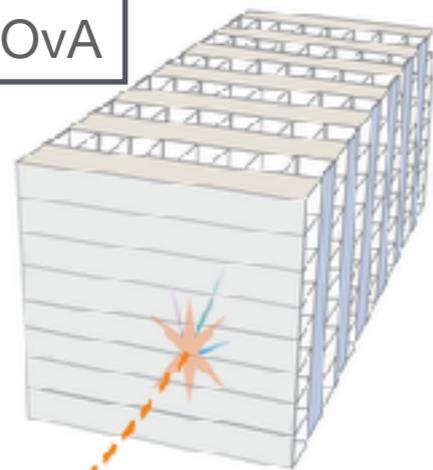
Utilize heavy target in neutrino detectors to maximize interactions \rightarrow understand nuclear structure

Carbon

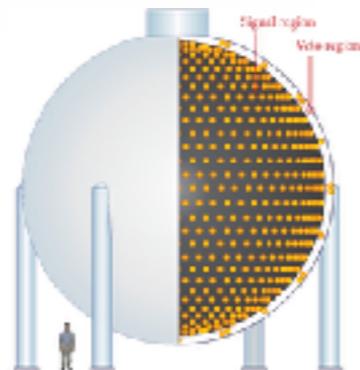


MINERvA

NOvA

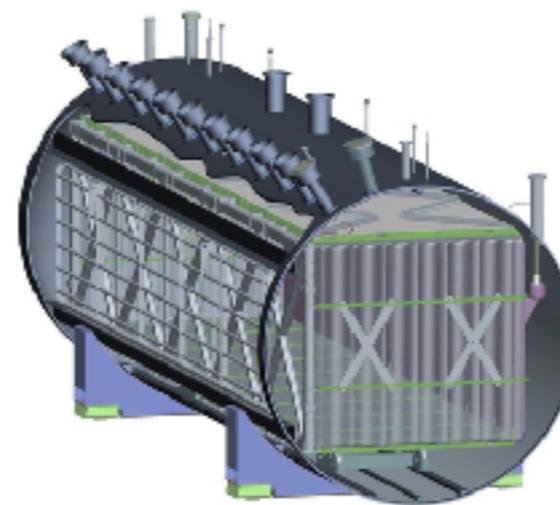


Neutrino from Fermilab



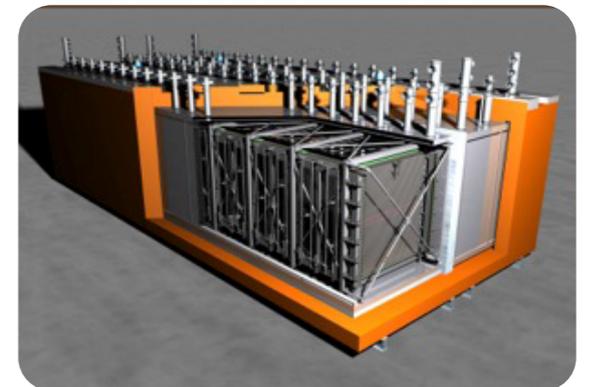
MiniBooNE

Liquid Argon

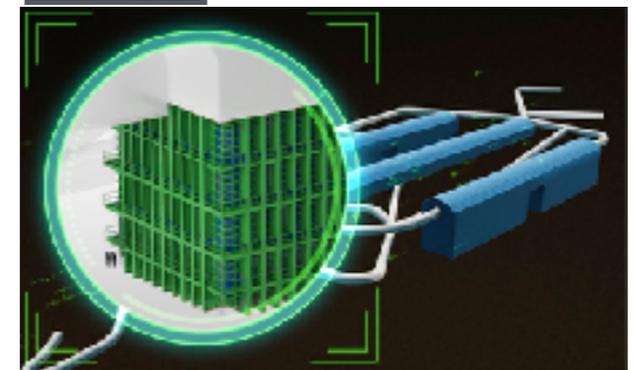


MicroBooNE

Icarus

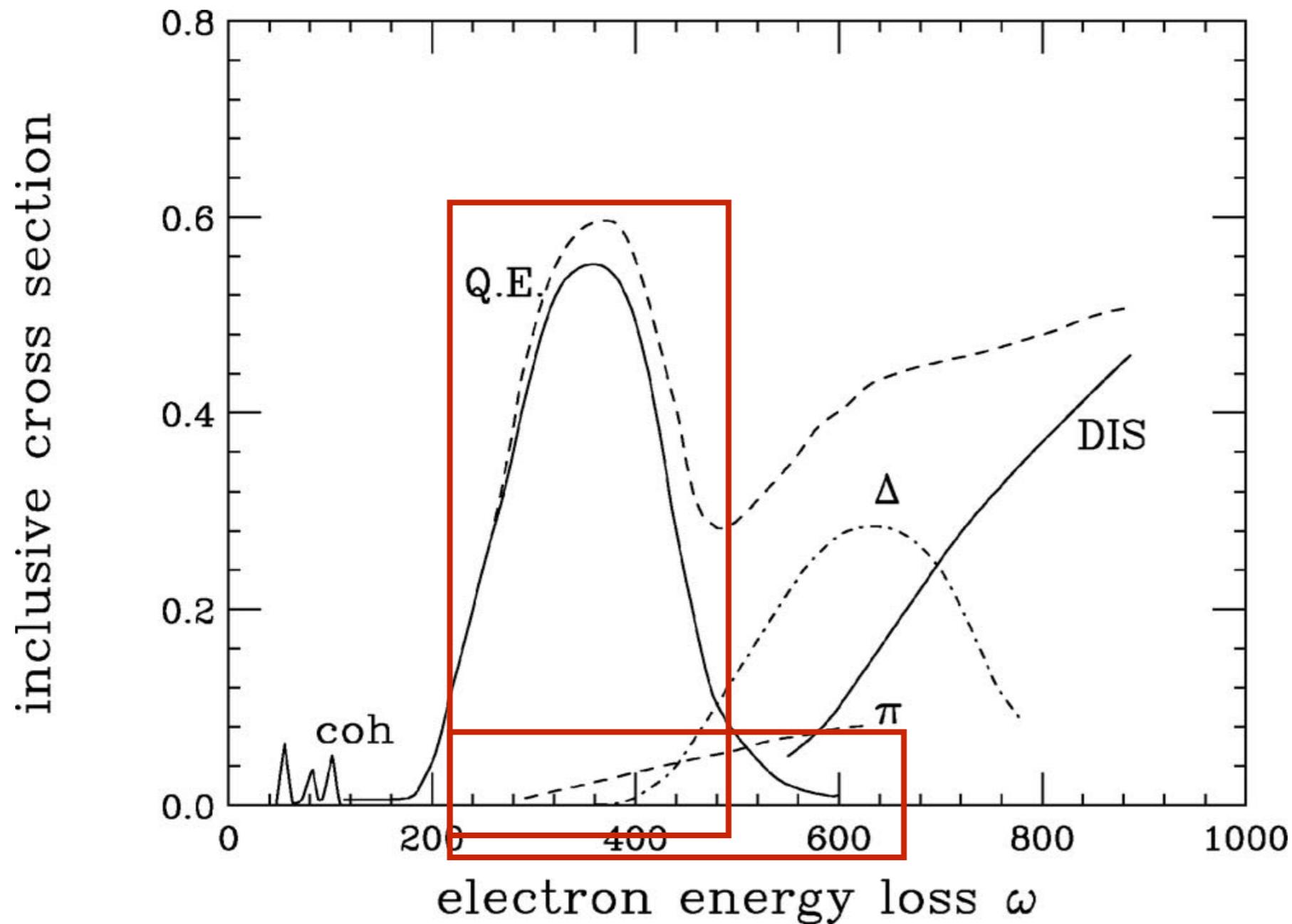


DUNE



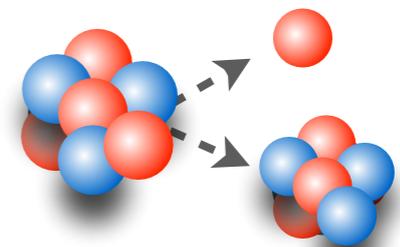
Outline of the talk

1st Part of the Presentation



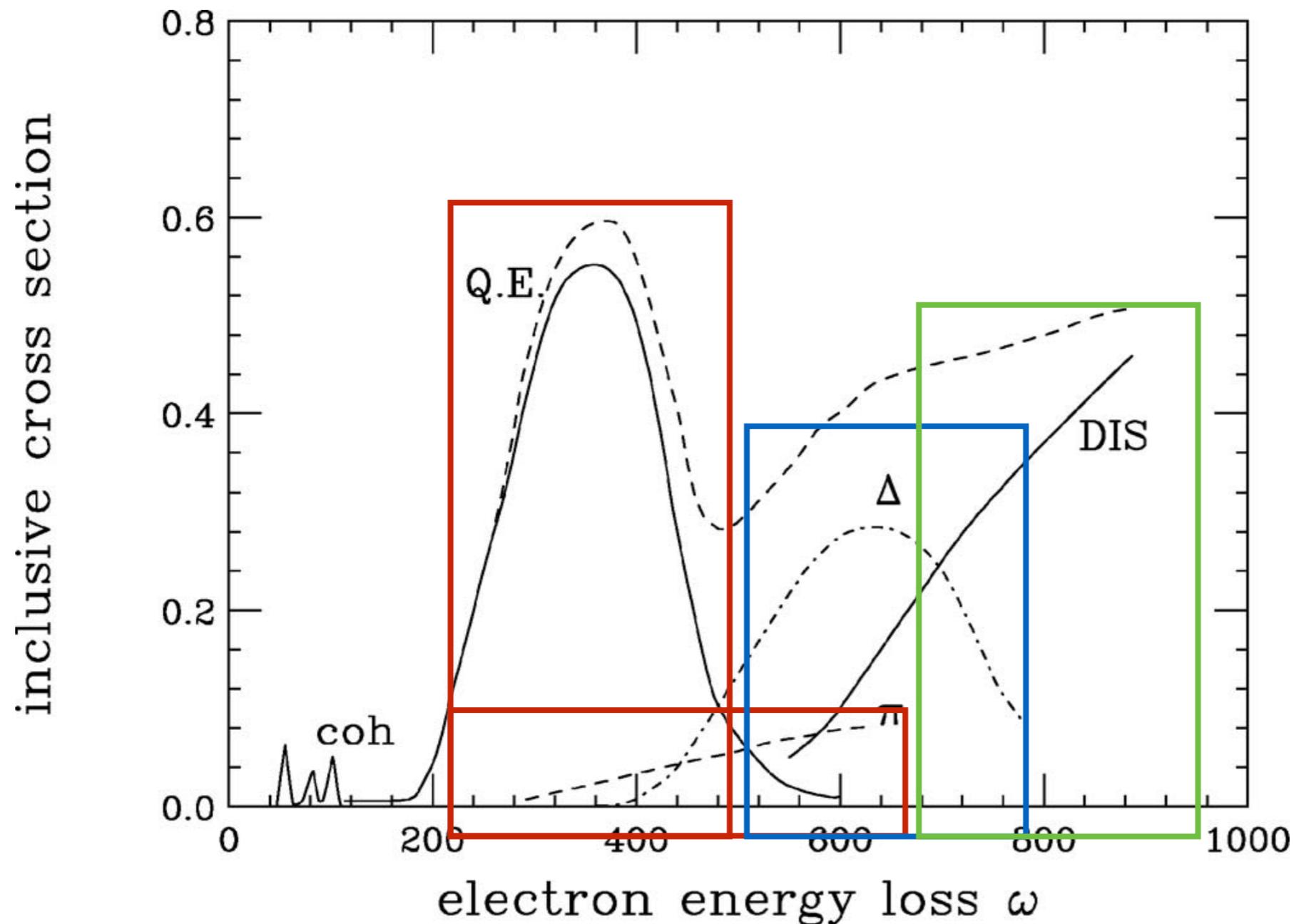
- Ab-initio calculations (GFMC) — able to describe how nuclei emerge starting from neutron and proton interactions — provide an accurate predictions of the QE region including one- and two-body currents

Quasielastic scattering on a nucleus:

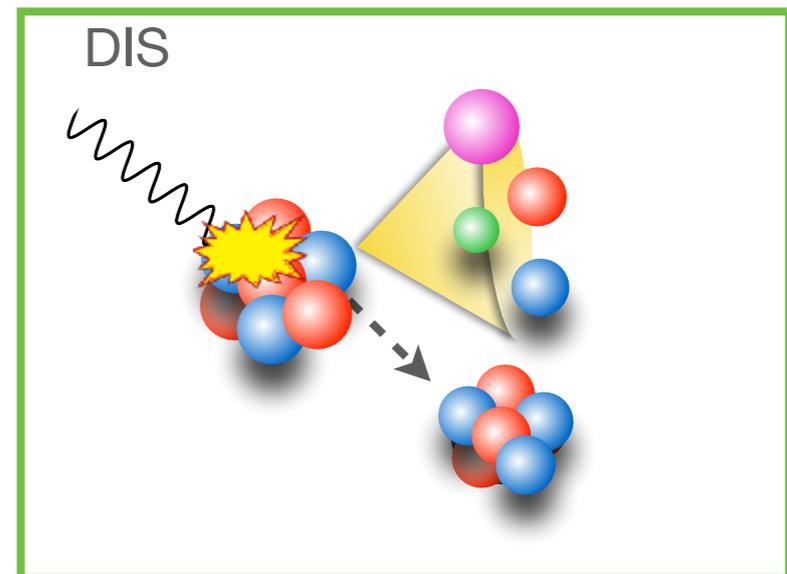
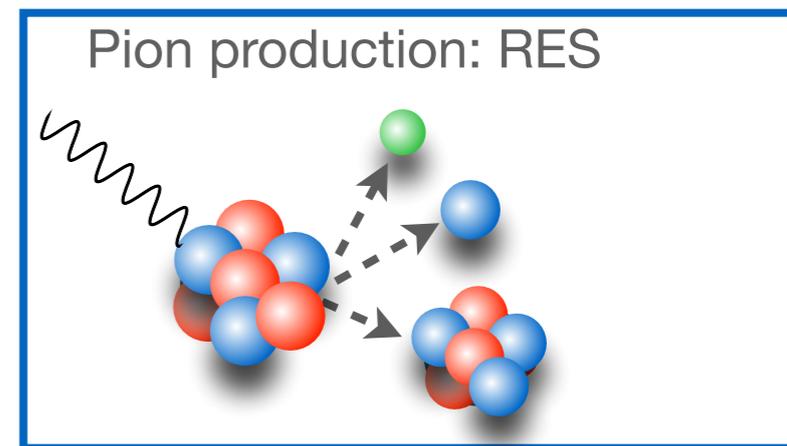


Outline of the talk

2nd Part of the Presentation

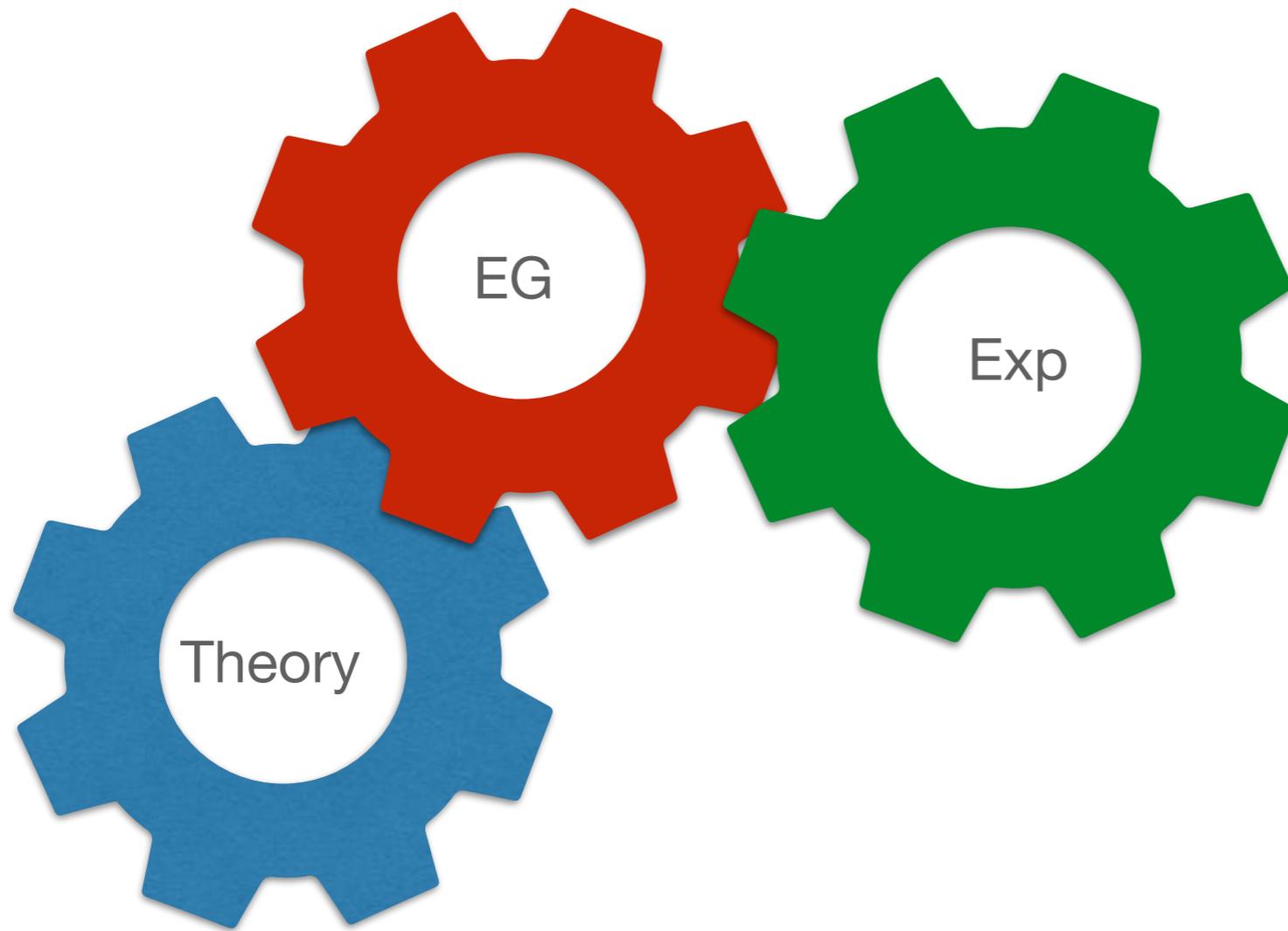


- More approximate approach: Extended Factorization scheme + Semi-phenomenological SF have been introduced to tackle QE, dip and π -production regions.



Outline of the talk

3rd Part of the Presentation



Synergistic effort among these three components:

what we are doing / plan to do for the event-generator component

Theory of lepton-nucleus scattering

The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$

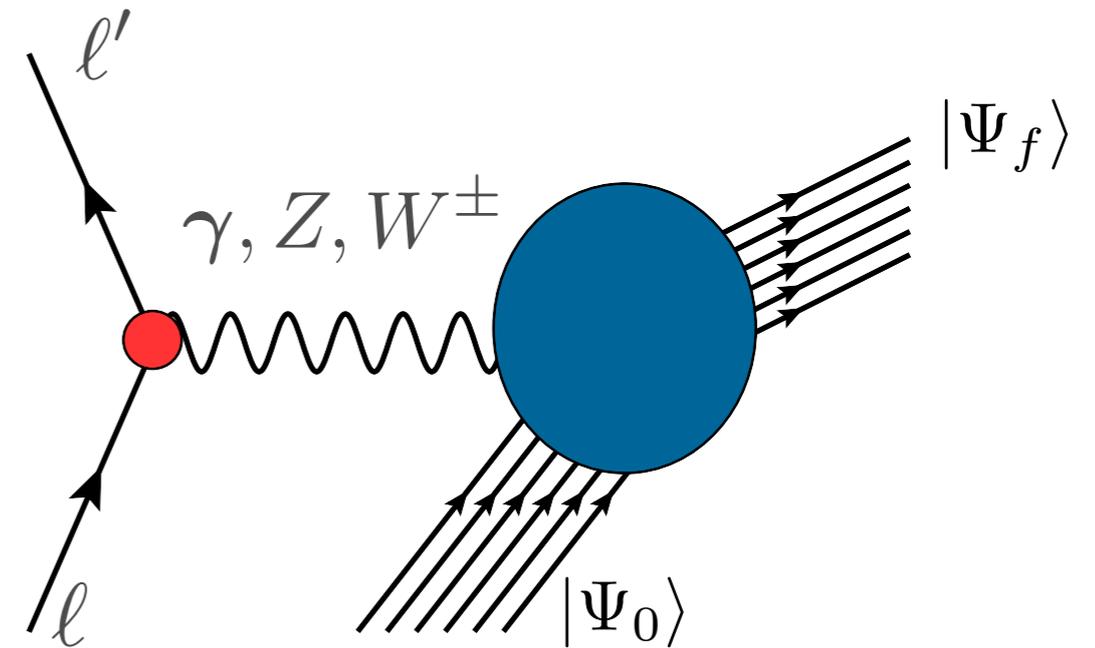
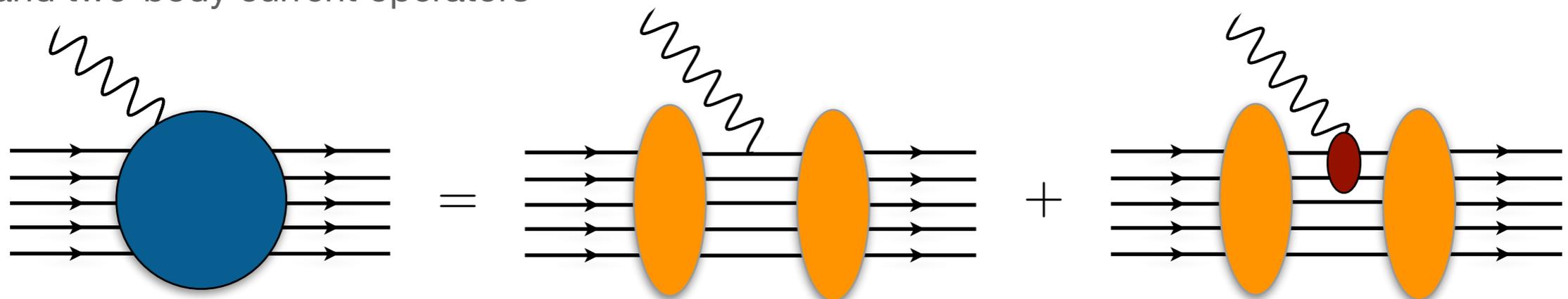
Nuclear response to the electroweak probe:

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0 | J_\alpha^\dagger(\mathbf{q}) | f \rangle \langle f | J_\beta(\mathbf{q}) | 0 \rangle \delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

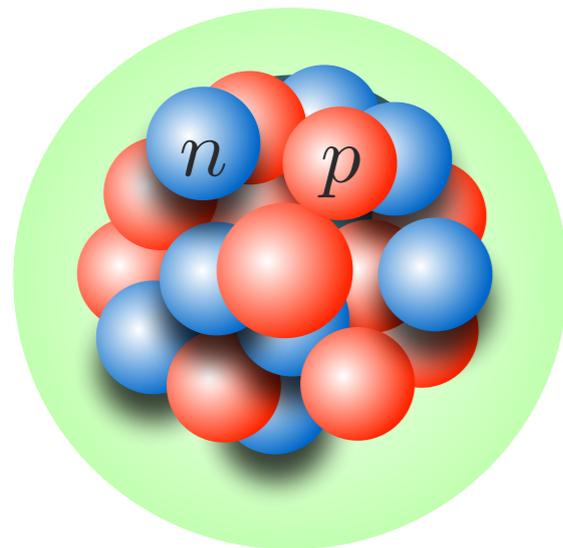
$$|0\rangle = |\Psi_0^A\rangle, |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \dots$$

One and two-body current operators



The basic model of nuclear theory

At low energy, the effective degrees of freedom are pions and nucleons:

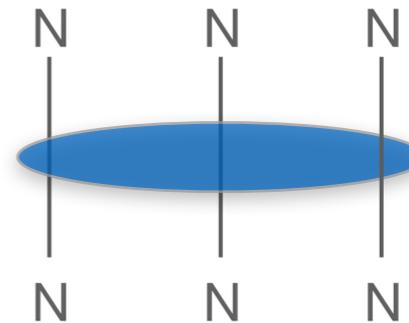
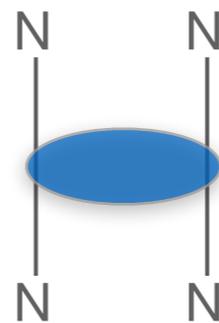


$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

1-body

2-body

3-body



The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\nabla \cdot \mathbf{J}_{\text{EM}} + i[H, J_{\text{EM}}^0] = 0 \quad [v_{ij}, j_i^0] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j_i^\mu + \sum_{i < j} j_{ij}^\mu + \dots$$

Quantum Monte Carlo approach

We want to solve the Schrödinger equation

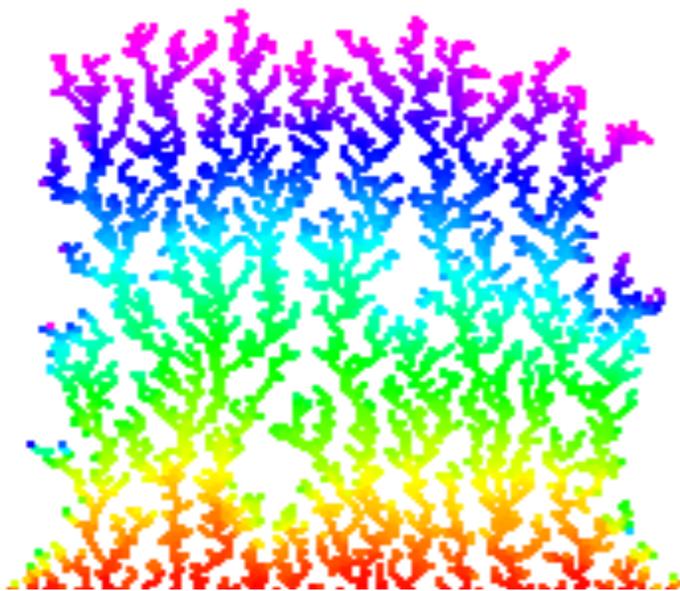
$$H\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A) = E\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A)$$

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

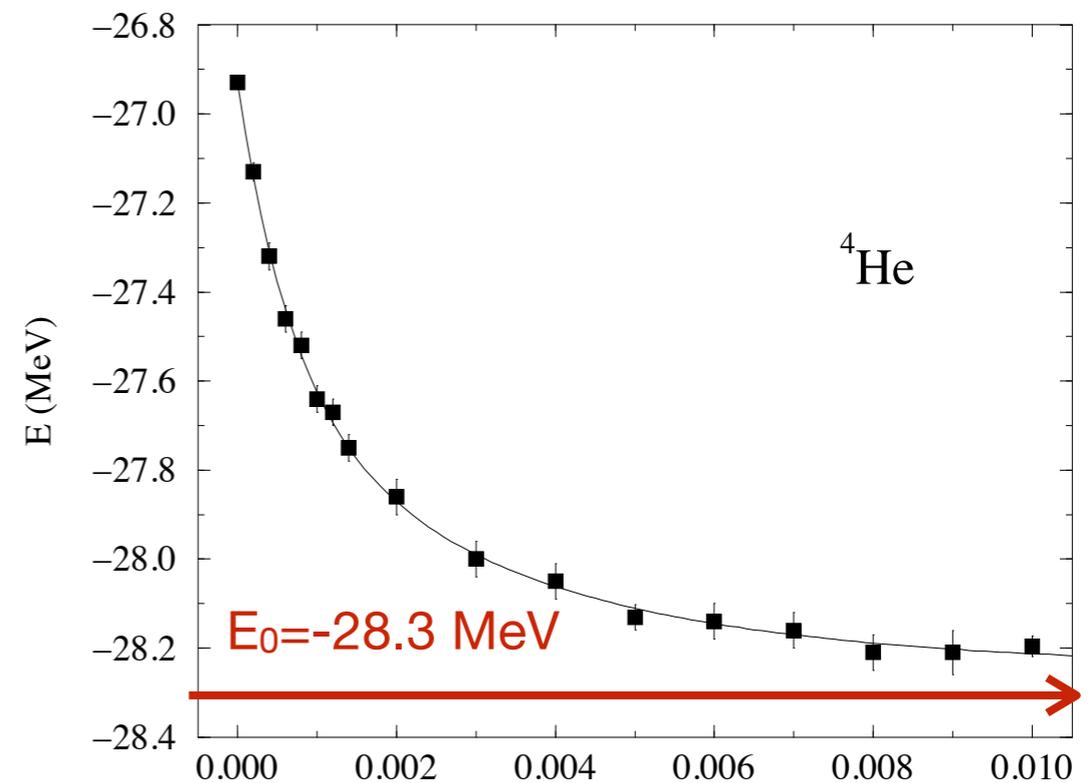
$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle$$

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

QMC techniques **projects out the exact lowest-energy state:** $e^{-(H-E_0)\tau} |\Psi_T\rangle \rightarrow |\Psi_0\rangle$



The system is cooled down by evolving it in time



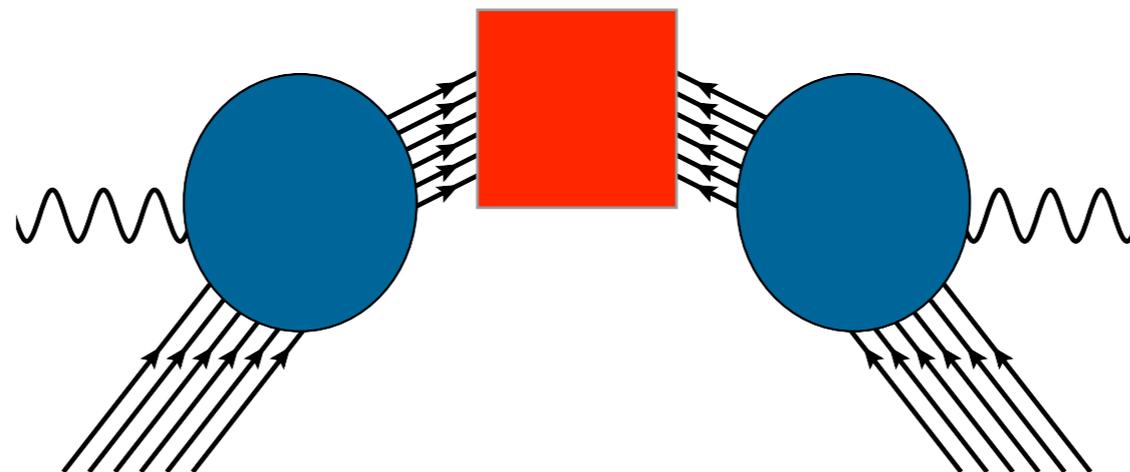
Integral Transform Techniques

► Nuclear responses obtained with QMC techniques (more in detail Greens' Function Monte Carlo)

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0 | J_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | J_{\beta}(\mathbf{q}) | 0 \rangle \delta(\omega - E_f + E_0)$$

Valuable information can be obtained from the integral transform of the response function

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q}) = \langle \psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) K(\sigma, H - E_0) J_{\beta}(\mathbf{q}) | \psi_0 \rangle$$



Integral Transform Techniques

$$E(\sigma, \mathbf{q}) \quad ? \quad R(\omega, \mathbf{q})$$

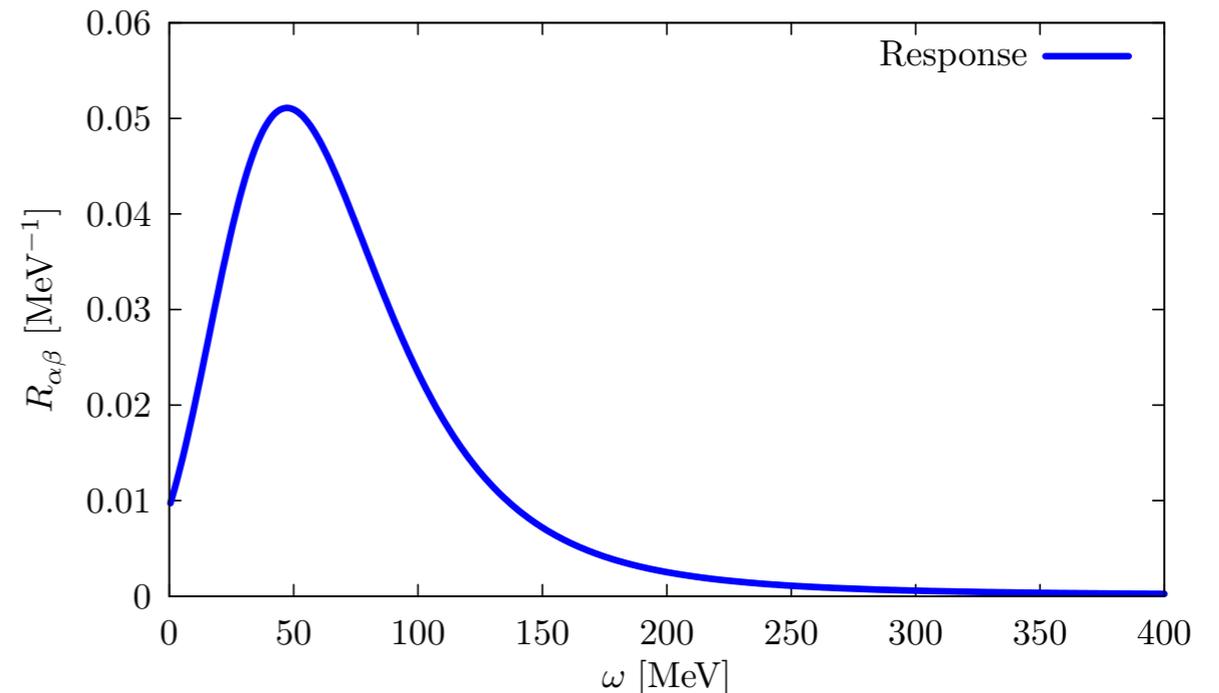
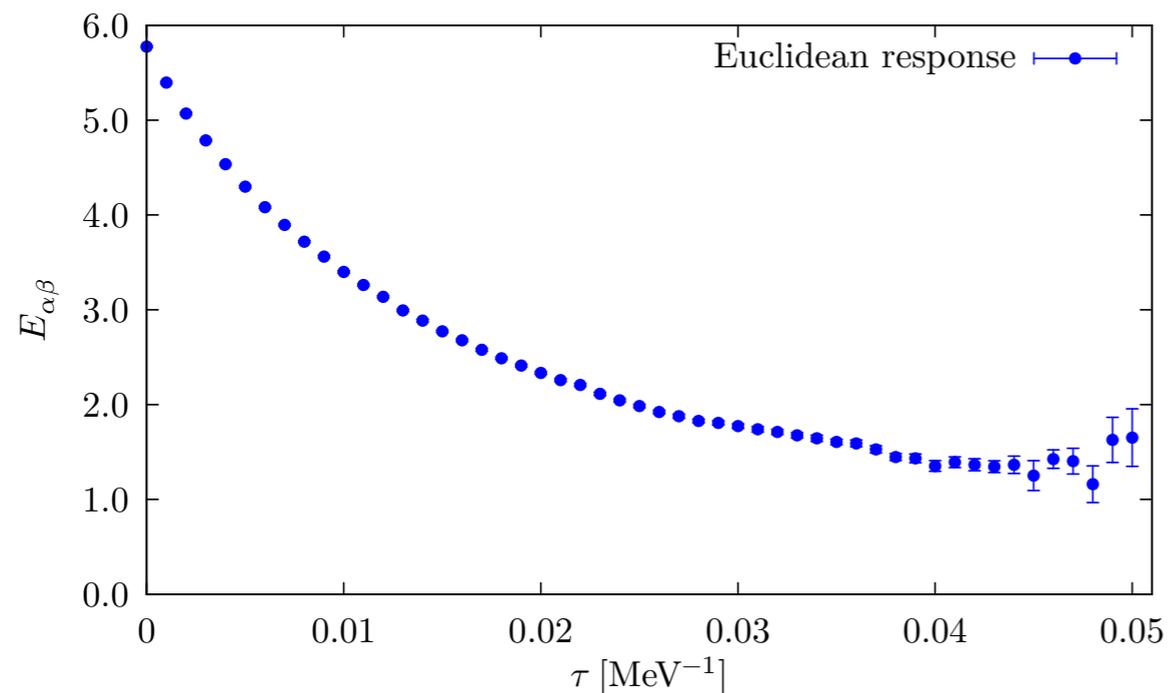



Inverting the integral transform is a complicated problem

$$E_{\alpha\beta}(\tau, \mathbf{q})$$



$$R_{\alpha\beta}(\omega, \mathbf{q})$$



Current solution for the quasielastic region: Maximum Entropy Techniques

A. Lovato et al, Phys.Rev.Lett. 117 (2016), 082501, Phys.Rev. C97 (2018), 022502



We are now exploring new strategies, based on **machine learning techniques**, to improve the accuracy of the inversion and to better estimate the associated uncertainties

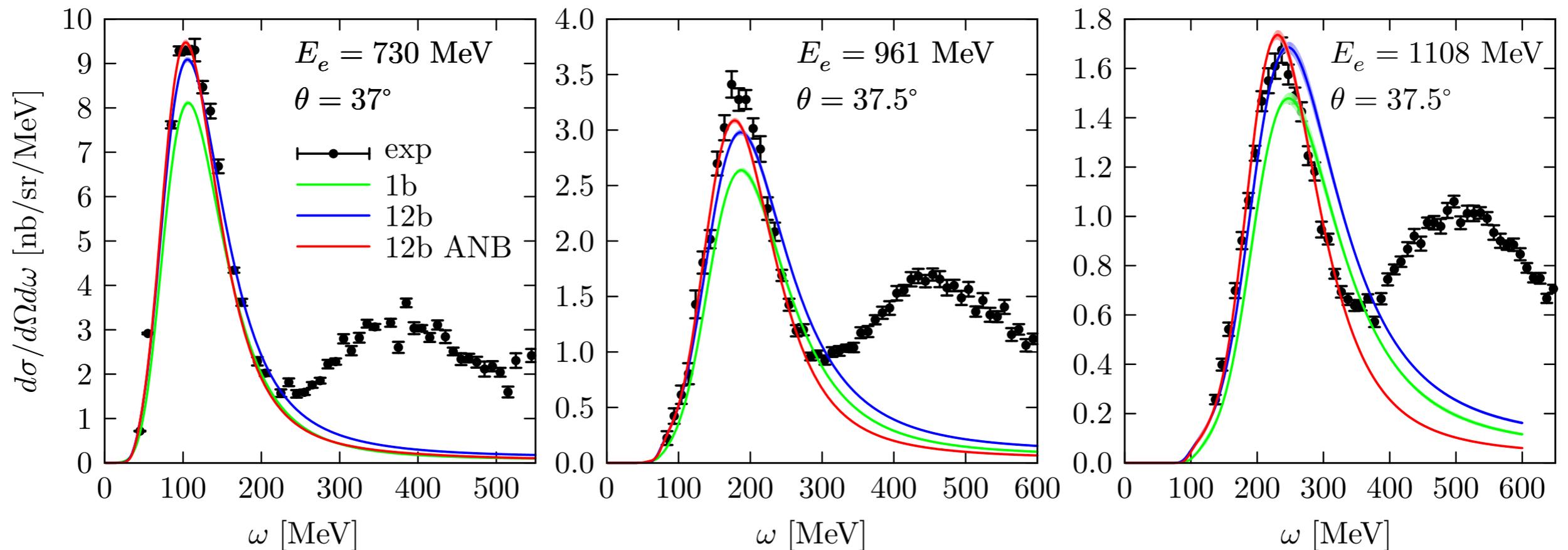
GFMC electron ^4He -cross sections



Virtually exact results for nuclear electroweak responses in the **quasi-elastic region** up to moderate values of q .
Initial and final state interactions fully accounted for.

Computational cost grows exponentially with the number of particles: currently limited to ^{12}C

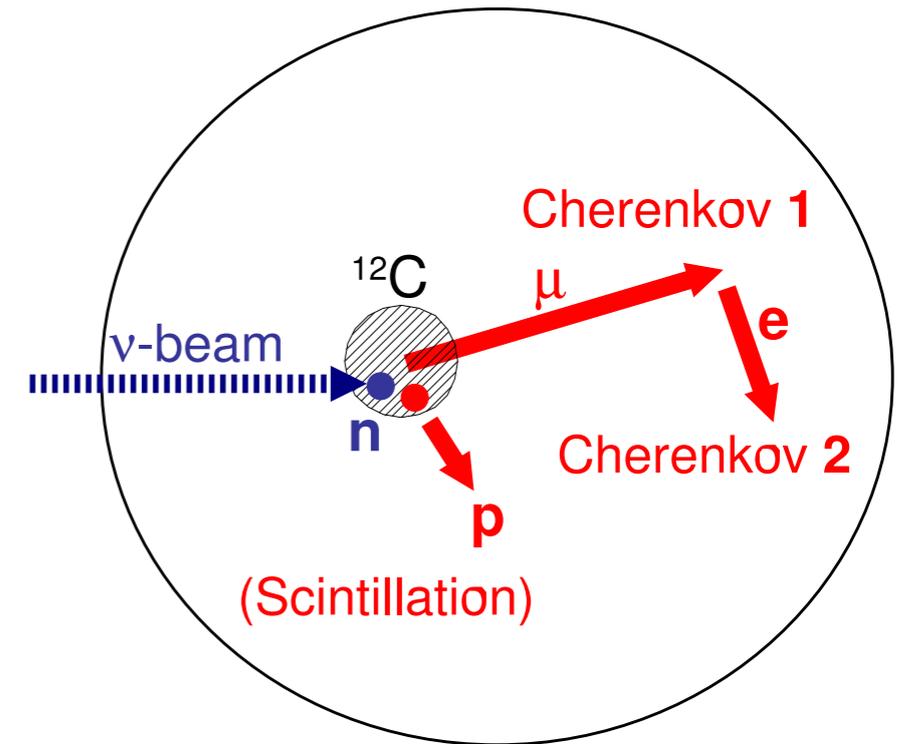
 N.R, W. Leidemann, et al PRC 97 (2018) no.5, 055501



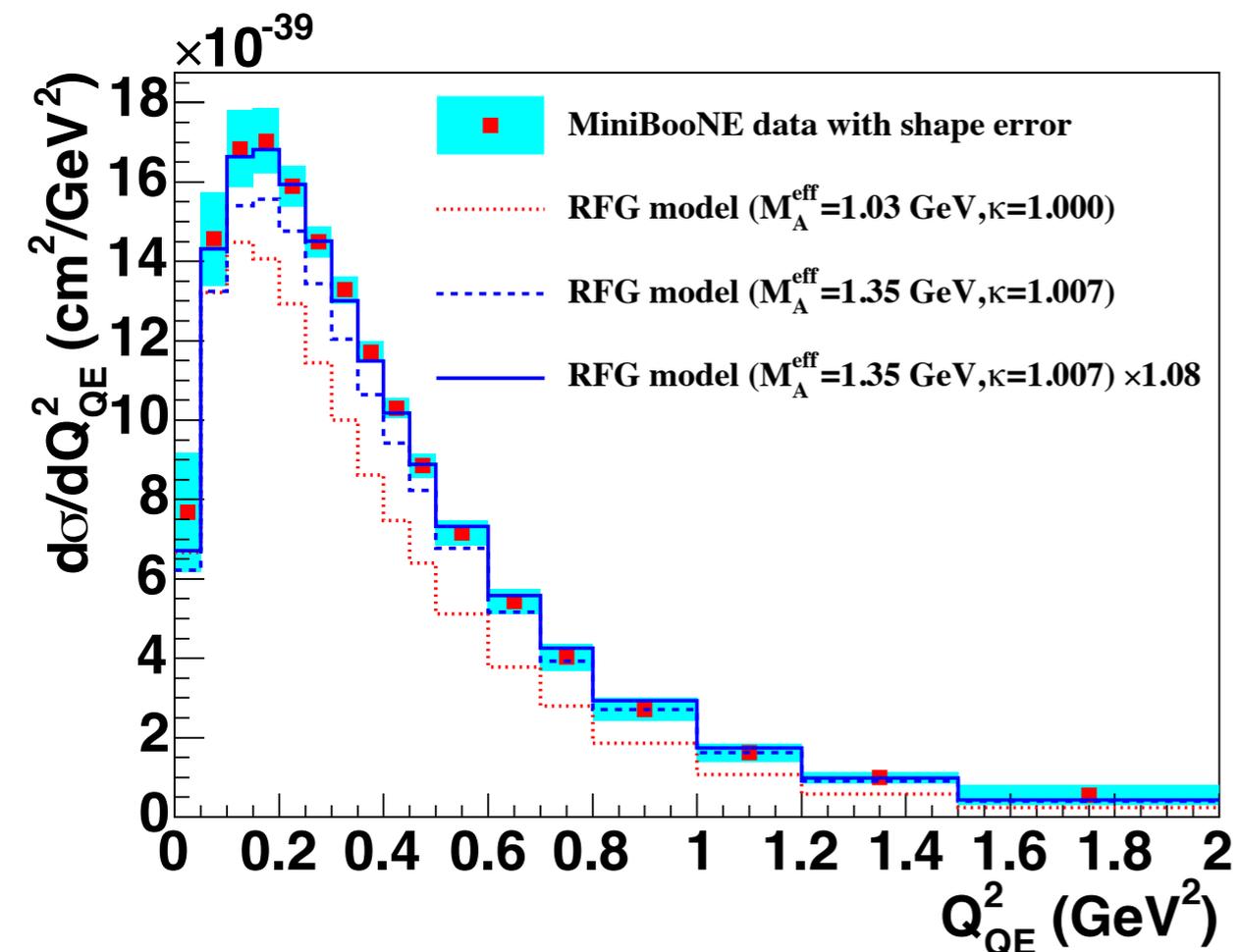
- Very good agreement in the quasielastic region when: one- and two-body currents are included
- Peak on the right: π production can not be described within this approach

MiniBooNE: the axial mass puzzle

- In MiniBooNE data analysis, an event is labeled as CCQE if **no final state pions** are detected in **addition to the outgoing muon**.
- To reproduce the data: $M_A \sim 1.35$ GeV is incompatible with former measurements: $M_A \sim 1.03$ GeV



MiniBooNE collaboration, PRDD81 (2010) 092005



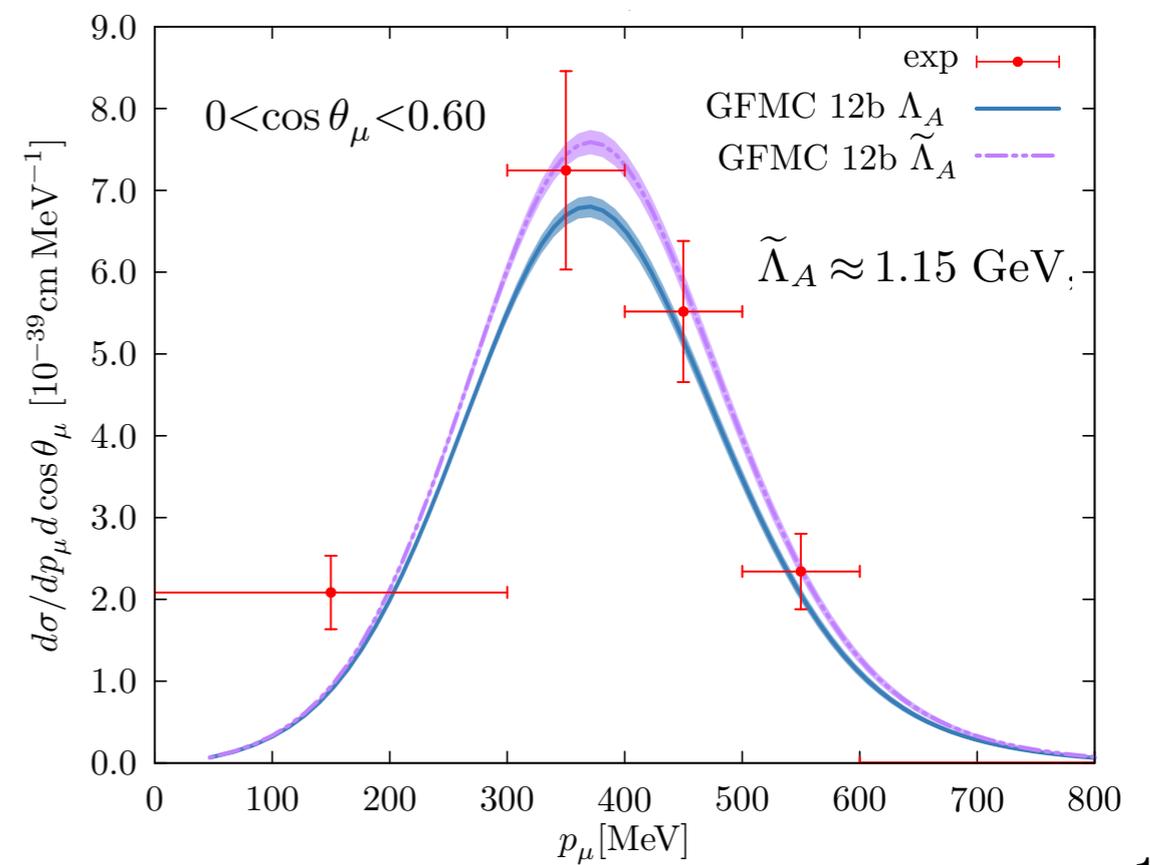
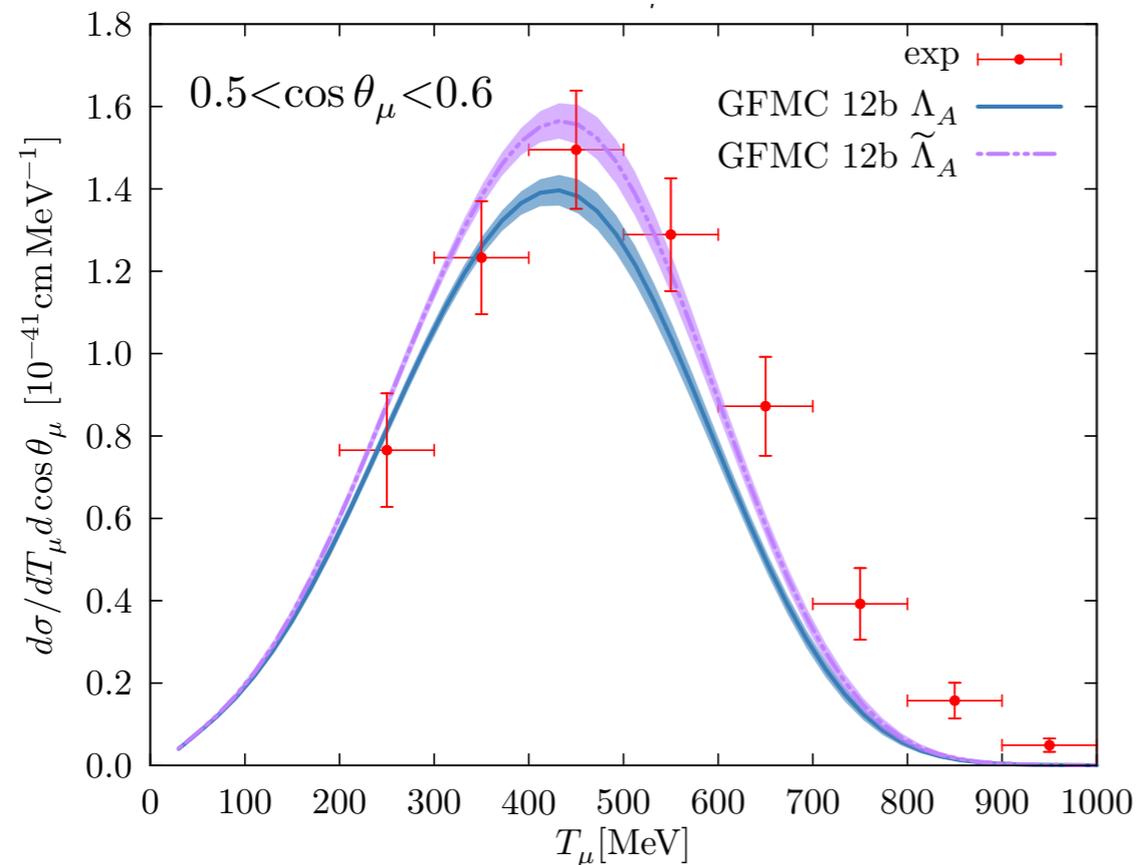
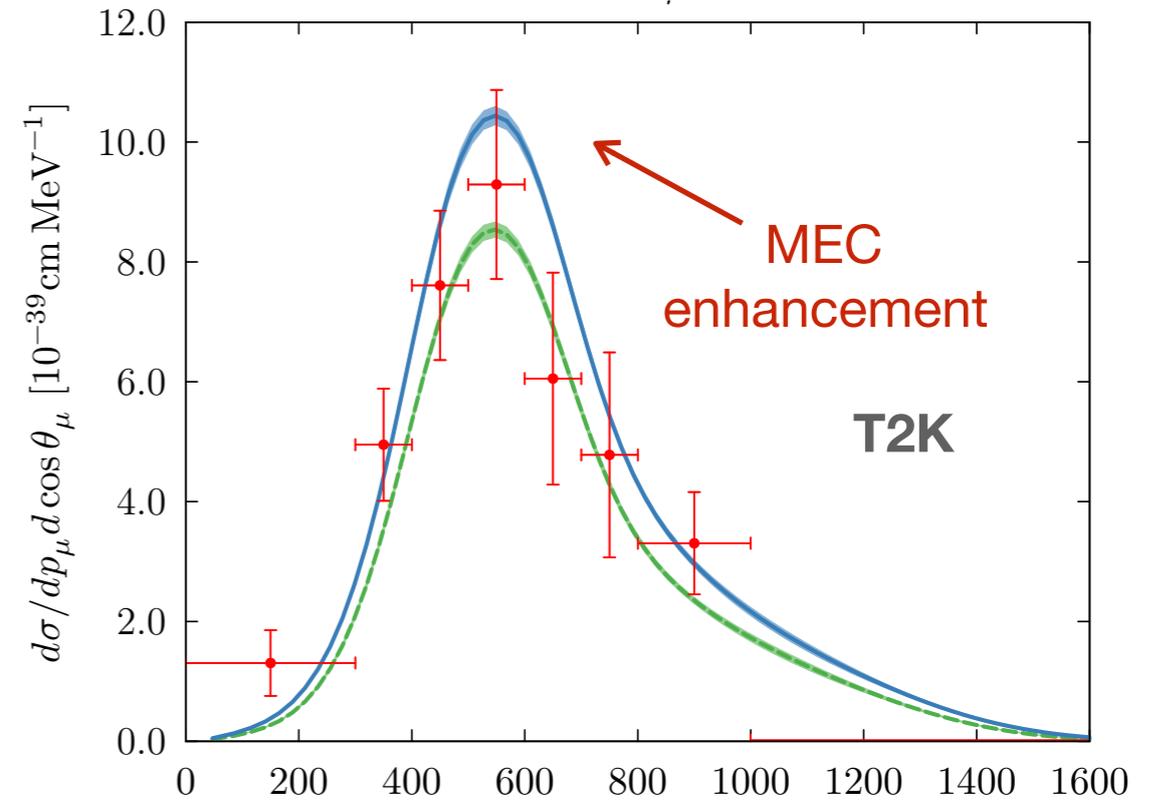
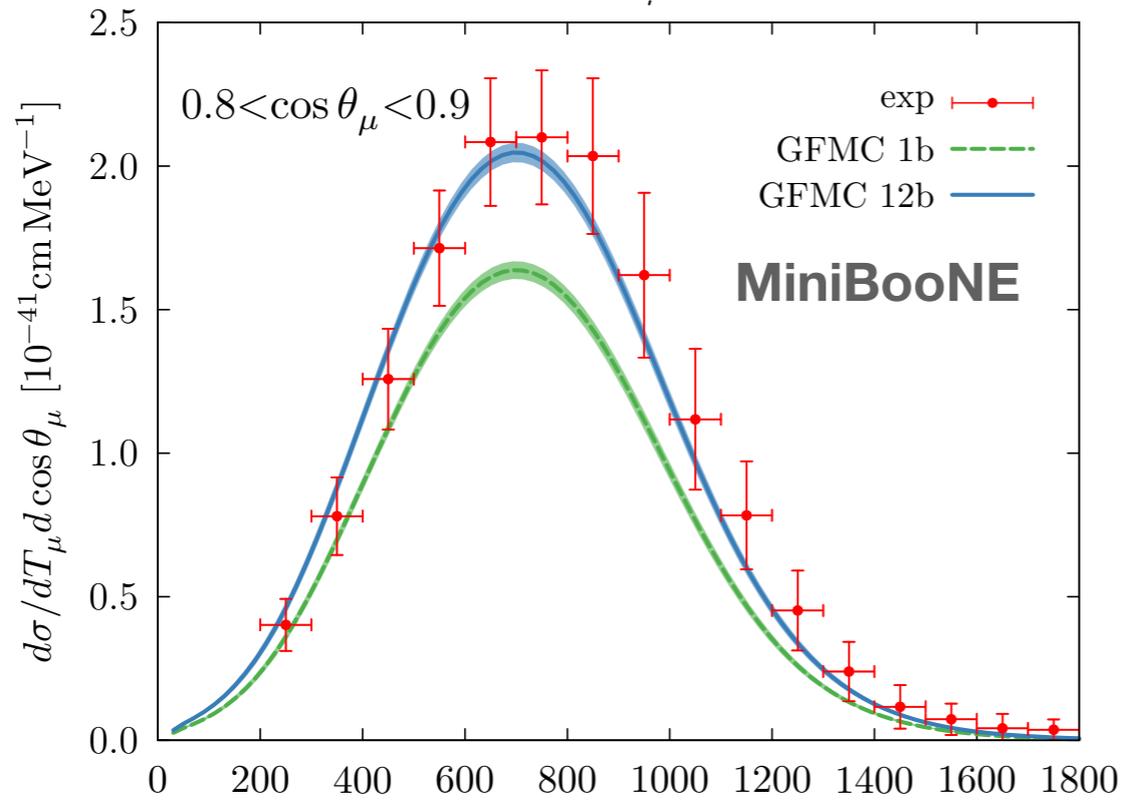
Nuclear effects might explain the axial mass puzzle



We have to include **meson-exchange currents**
—two nucleon emission: CCQE-like

GFMC CC ν_μ ^{12}C -cross sections

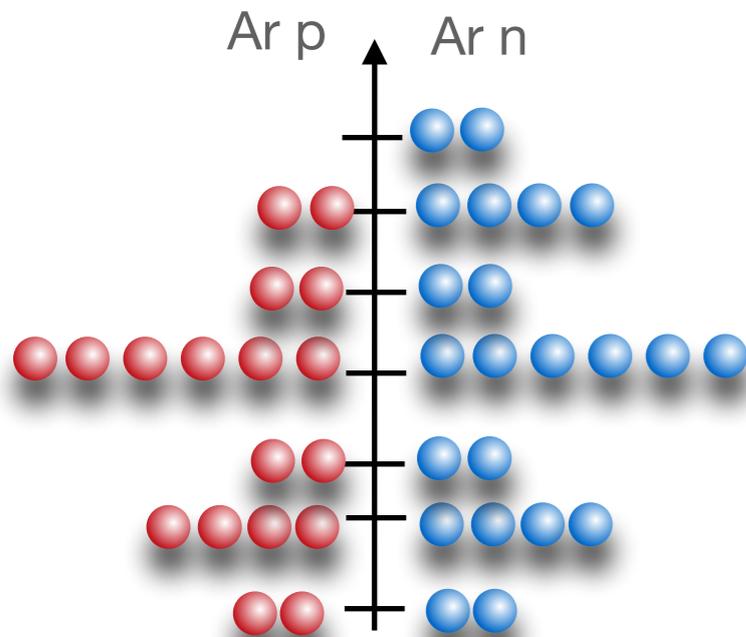
A.Lovato, NR et al, [arXiv:2003.07710](https://arxiv.org/abs/2003.07710), PRX in press



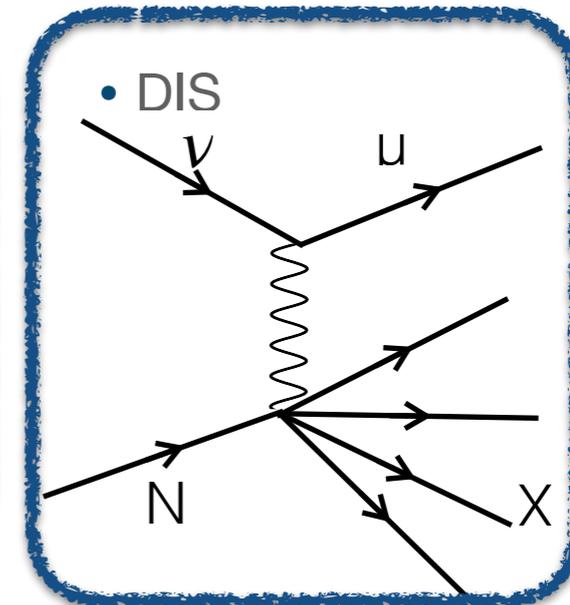
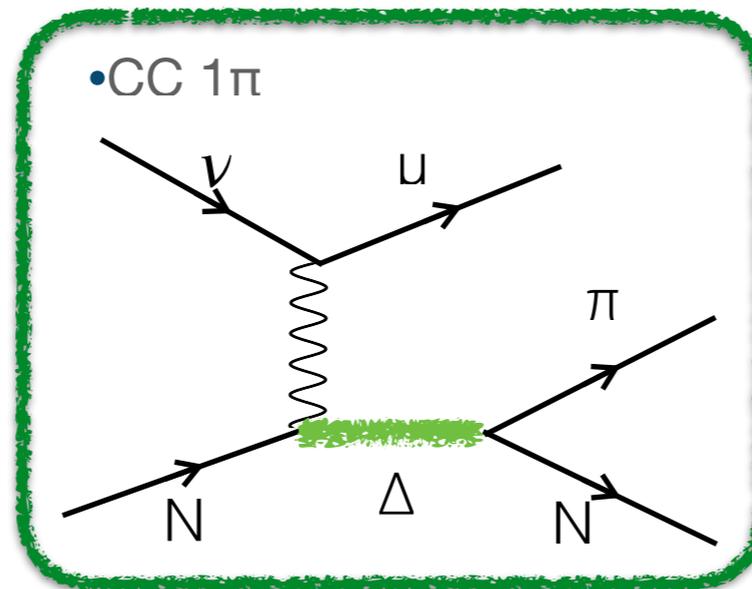
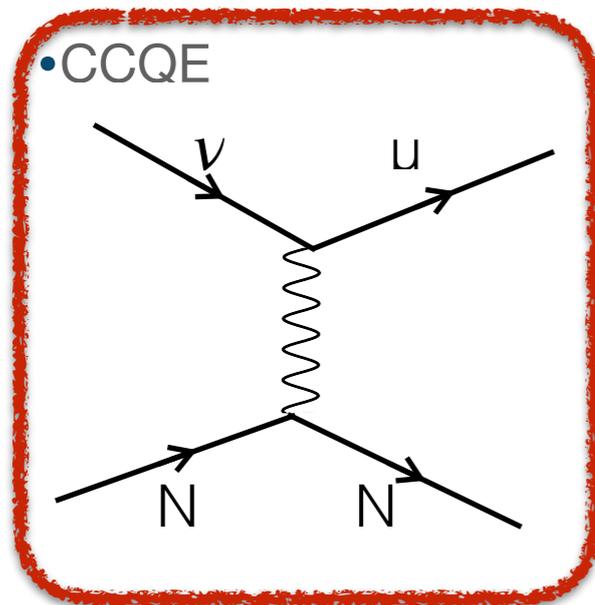
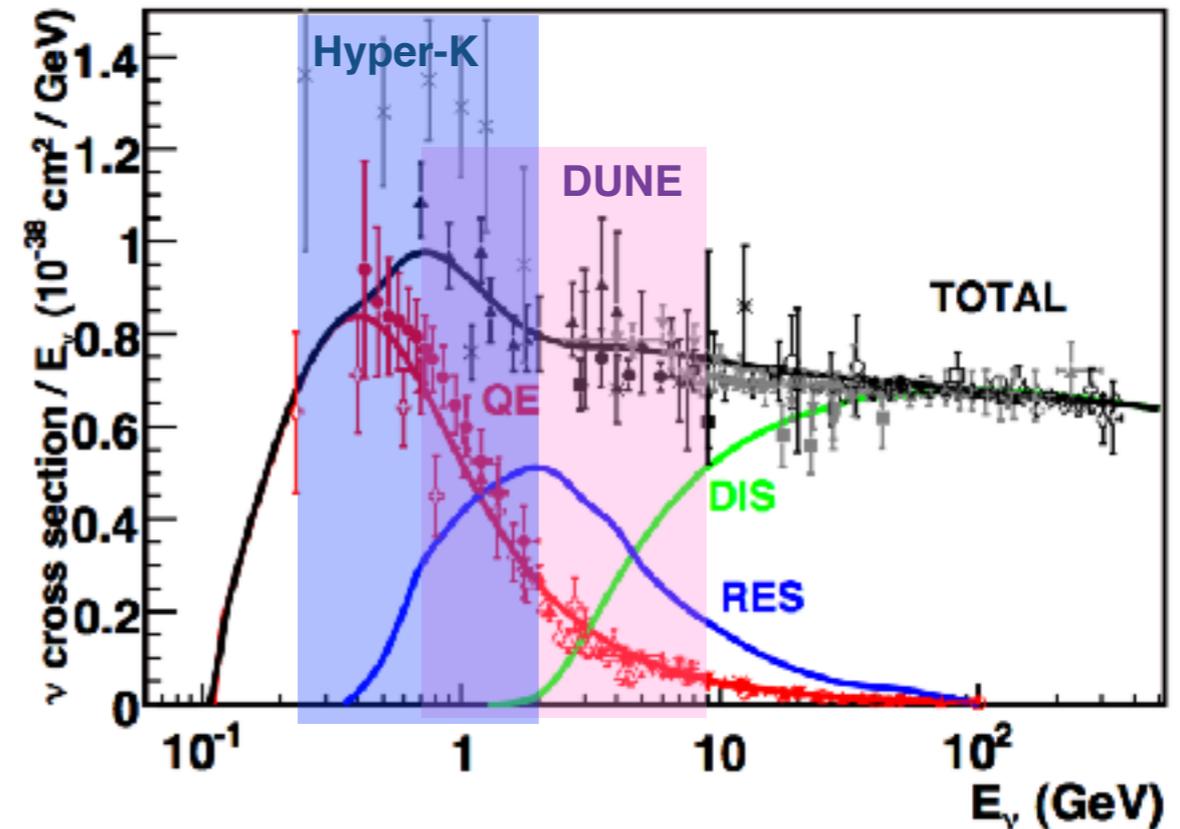
Addressing future precision experiments

- Liquid Argon TPC Technology

J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)



Ar has a complicated structure, out of the reach of most of the ab initio methods



The dominant reaction mechanism changes dramatically over the region of interest to oscillation experiment

Factorization Scheme and Spectral Function

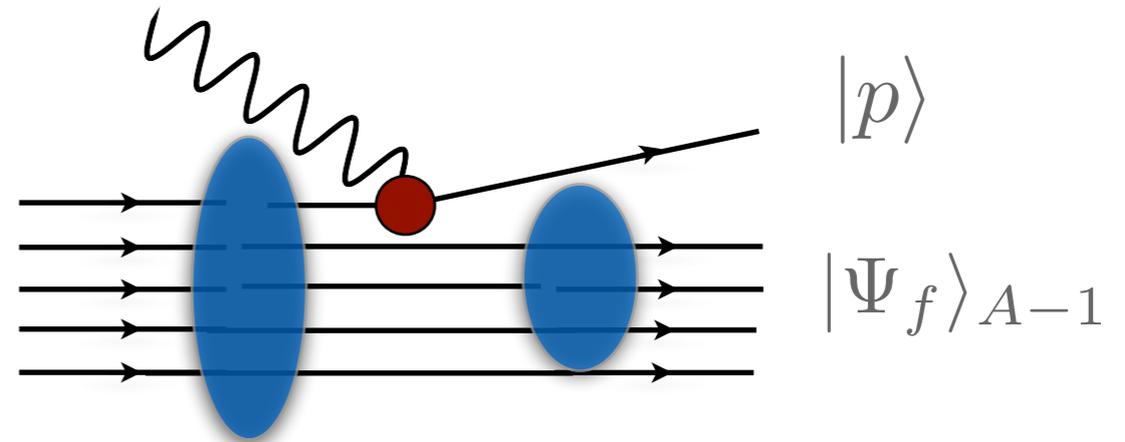
For sufficiently large values of $|\mathbf{q}|$, the **factorization scheme** can be applied under the assumptions

$$|\Psi_f\rangle \rightarrow |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$J_\alpha = \sum_i j_\alpha^i$$



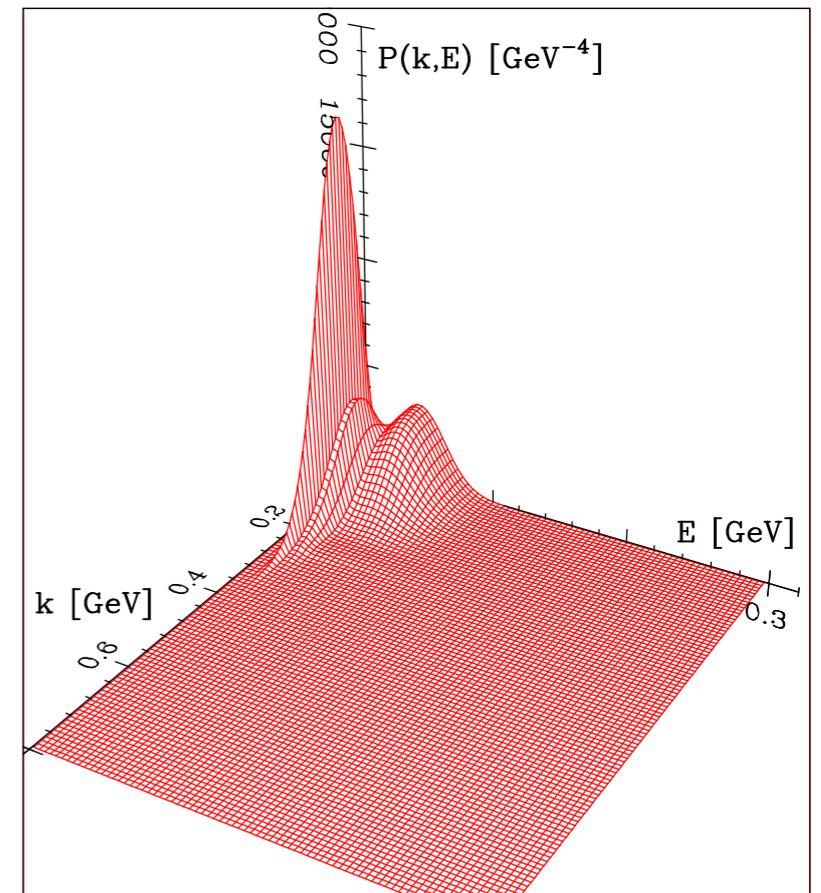
$$|\Psi_0\rangle$$



The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

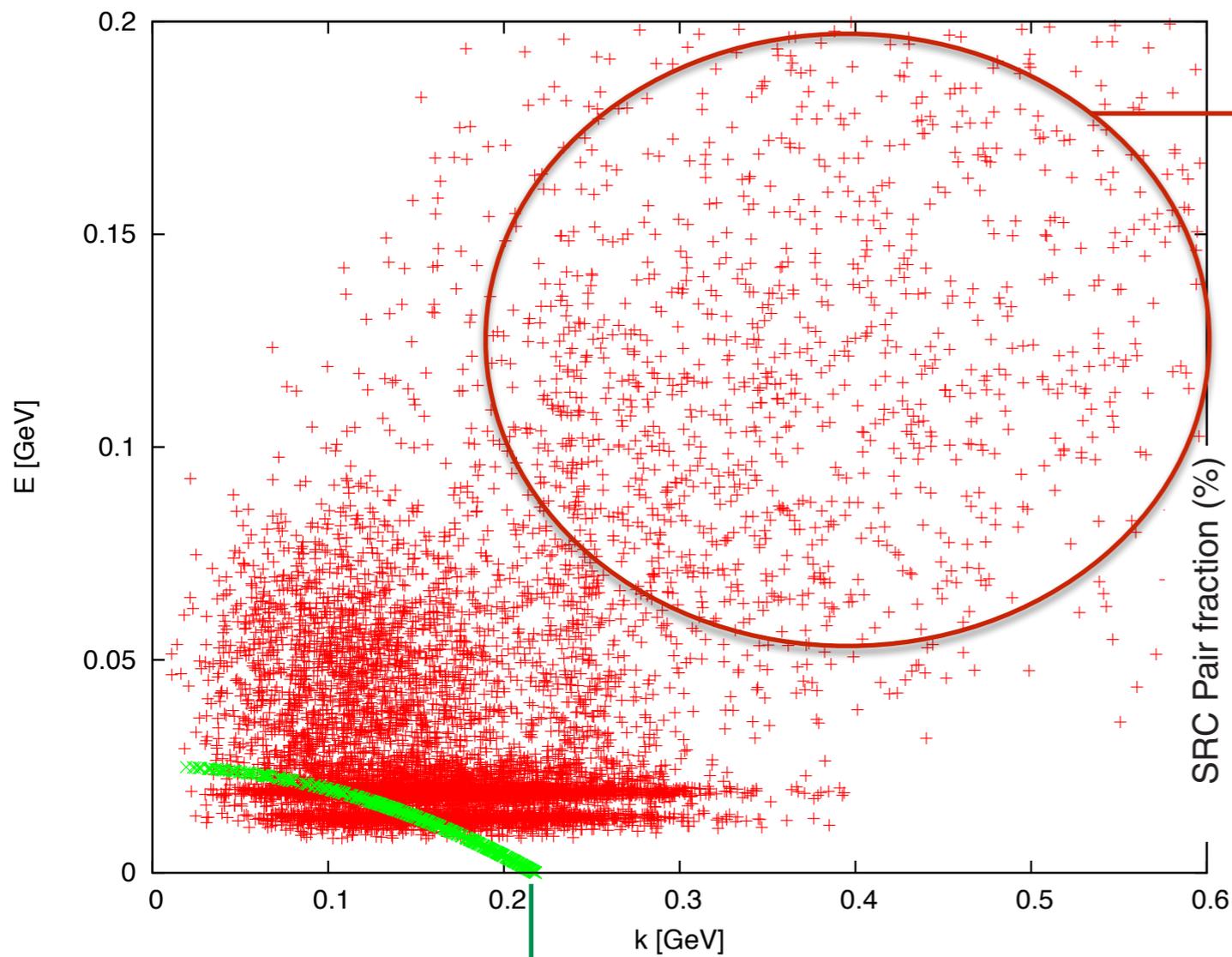
$$d\sigma_A = \int dE d^3k d\sigma_N P(\mathbf{k}, E)$$

The intrinsic properties of the nucleus are described by the **Spectral Function** → EFT and nuclear many-body methods



The one-nucleon Spectral Function

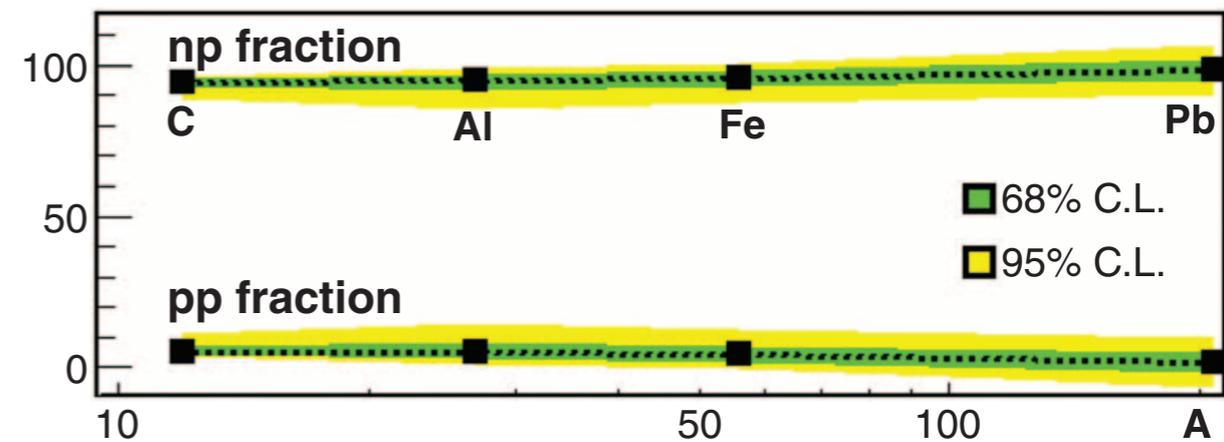
$P(\mathbf{k}, E)$ provides the **probability distribution** of removing a nucleon with momentum \mathbf{k} from the target nucleus, leaving the residual $(A - 1)$ -nucleon system with an **excitation energy E** .



SF
FG

High energy and momentum correlated pairs

Observed **dominance of np-over-pp** pairs for a variety of nuclei



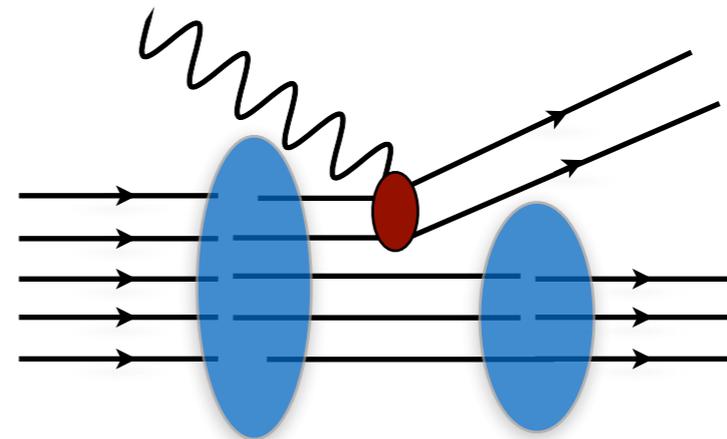
Consequence of the **tensor component** of the nucleon-nucleon interaction

Fermi gas contribution : $P_{FG}(\mathbf{k}, E) = \delta(E - \epsilon_B)\theta(k_F - |\mathbf{k}|)$

Extended Factorization Scheme

- Two-body currents are included rewriting the hadronic final state as

$$|f\rangle \rightarrow |pp'\rangle_a \otimes |f_{A-2}\rangle$$



The hadronic tensor for two-body current processes reads

$$W_{2b}^{\mu\nu}(\mathbf{q}, \omega) \propto \int dE \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} P_h(\mathbf{k}, \mathbf{k}', E) 2 \sum_{ij} \langle k k' | j_{ij}^{\mu\dagger} | p p' \rangle_a$$

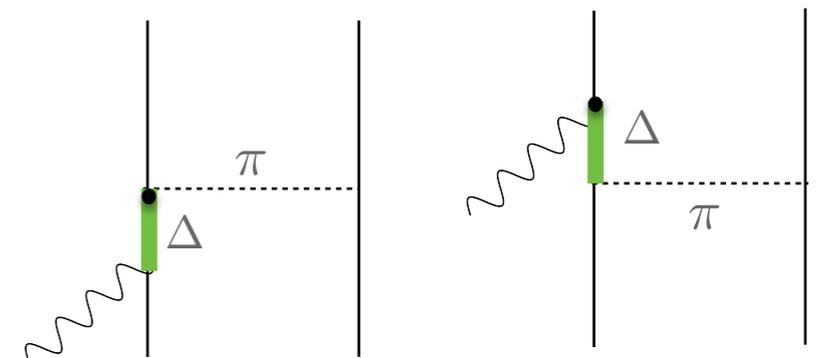
$$\times \langle p p' | j_{ij}^{\nu} | k k' \rangle \delta(\omega - E + 2m_N - e(\mathbf{p}) - e(\mathbf{p}')) .$$

[NR et al, Phys.Rev. C99 \(2019\) no.2, 025502](#)

[NR et al, Phys. Rev. Lett. 116, 192501 \(2016\)](#)

Relativistic two-body currents

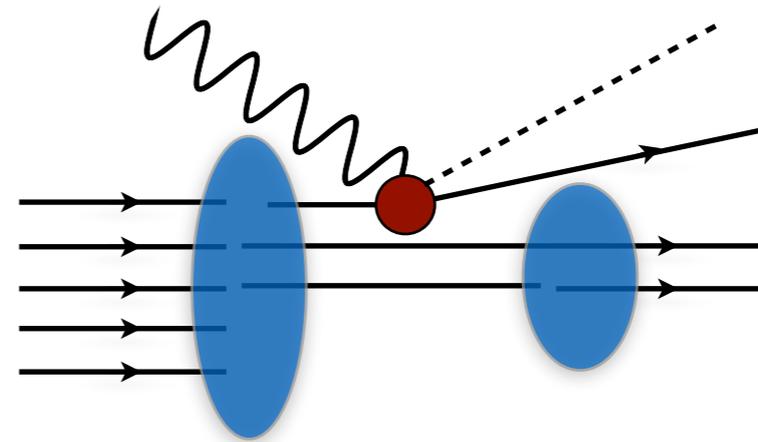
Dedicated code that **automatically** carries out the calculation of the **MEC spin-isospin matrix elements**, performing the integration using the Metropolis MC algorithm



Extended Factorization Scheme

- Production of real π in the final state

$$|f\rangle \rightarrow |p_\pi p\rangle \otimes |f_{A-1}\rangle$$



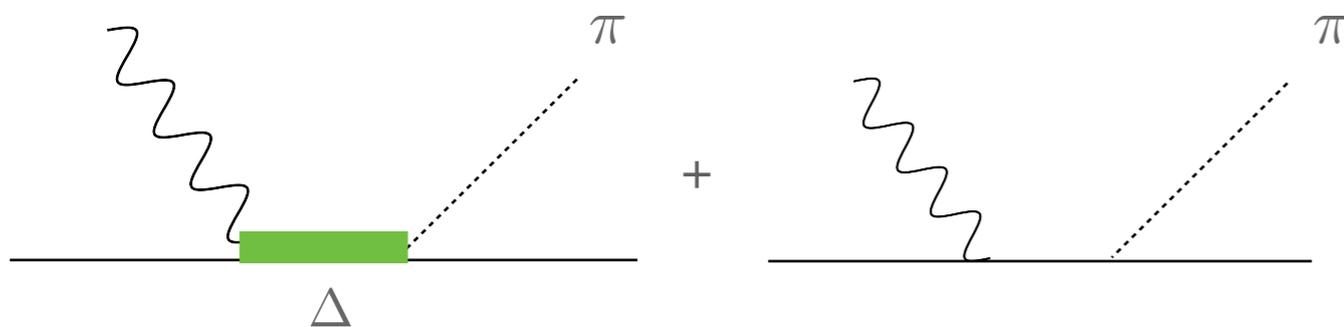
The hadronic tensor for two-body current processes reads

$$W_{1b1\pi}^{\mu\nu}(\mathbf{q}, \omega) \propto \int \frac{d^3 k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \frac{d^3 p_\pi}{(2\pi)^3} \sum_i \langle k | j_i^{\mu\dagger} | p_\pi p \rangle \langle p_\pi p | j_i^\nu | k \rangle$$

$$\times \delta(\omega - E + m_N - e(\mathbf{p}) - e_\pi(\mathbf{p}_\pi))$$

Pion production elementary amplitudes derived within the extremely sophisticated **Dynamic Couple Channel approach**; includes meson baryon channel and nucleon resonances up to $W=2$ GeV

- The diagrams considered resonant and non resonant π production



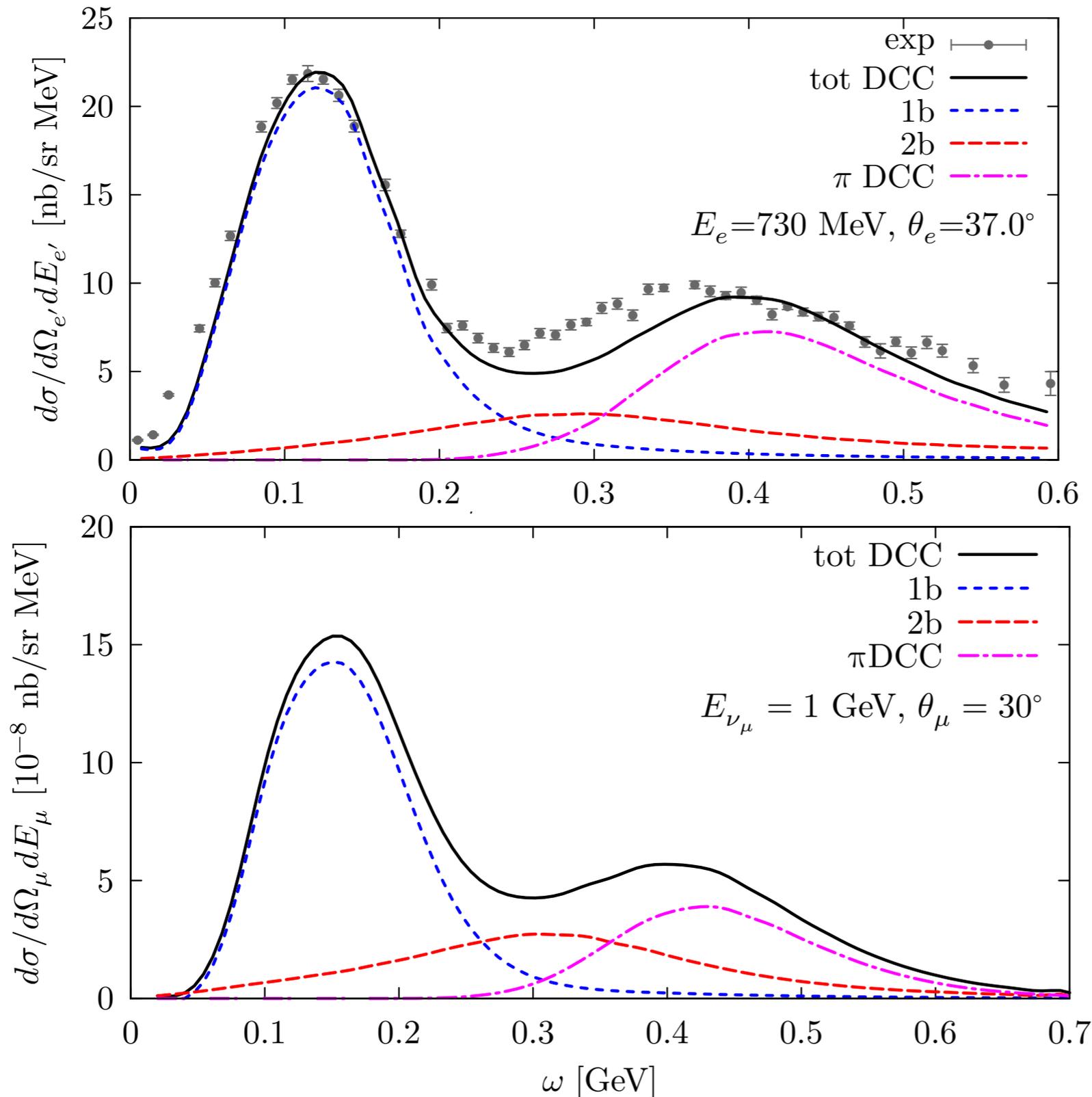
NR, et al, PRC100 (2019) no.4, 045503

H. Kamano et al, PRC 88, 035209 (2013)

S.X.Nakamura et al, PRD 92, 074024 (2015)

Electron and neutrino -¹²C cross sections-SF

NR, S. Nakamura, T.S.H. Lee, A. Lovato, PRC100 (2019) no.4, 045503



- We included in the Extended Factorization Scheme the **one-** and **two-body current** contributions and the **pion production** amplitudes.
- Good agreement with electron scattering data when all reaction mechanisms are included
- Ongoing calculation of flux folded cross sections

A QMC based approach to intranuclear cascade

The propagation of **nucleons** through the nuclear medium is crucial in the analysis of electron-nucleus scattering and neutrino oscillation experiments.

Describing nucleons' propagation in the nuclear medium would in principle require a fully quantum-mechanical description of the hadronic final state.

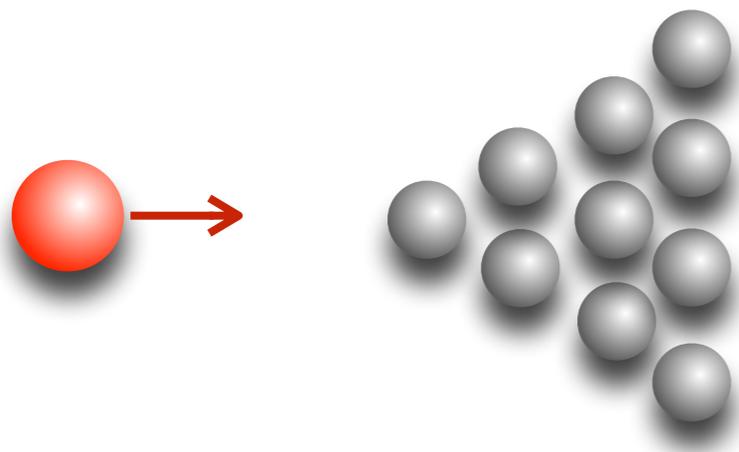
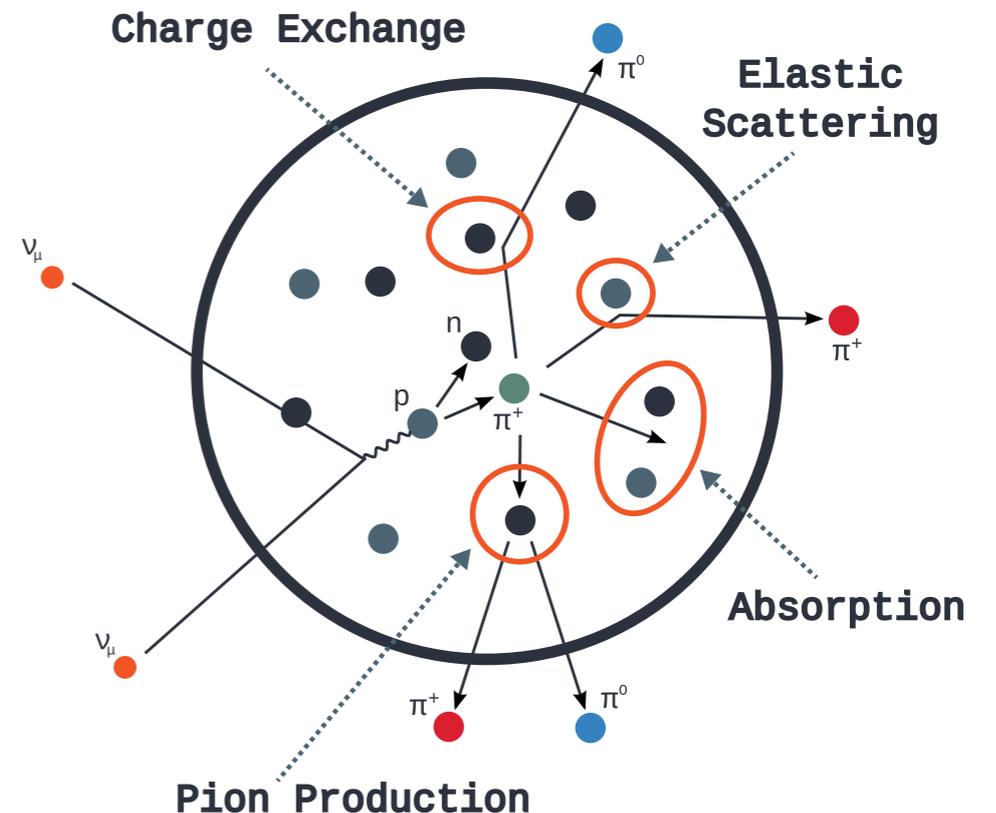
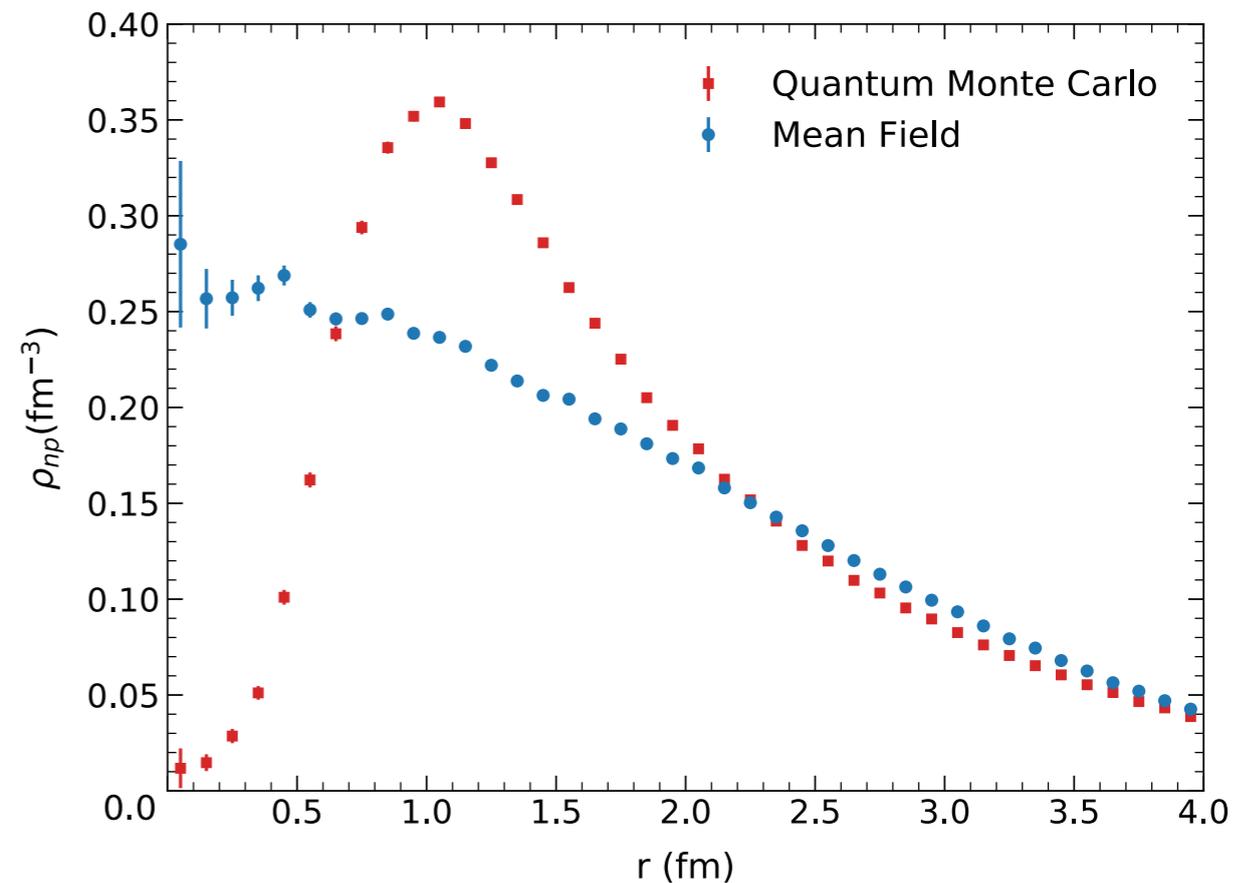
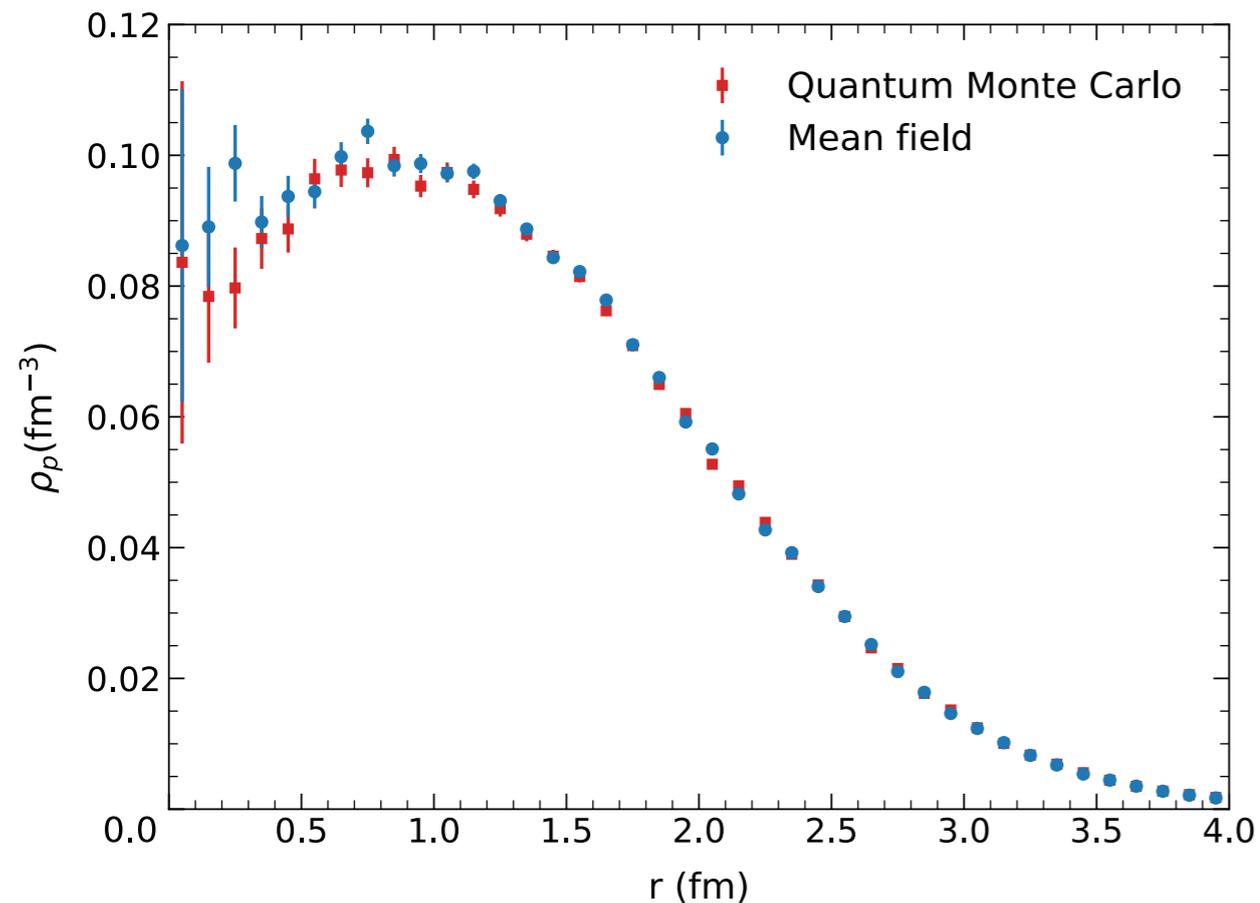


Figure by T. Golan



Due to its tremendous difficulty we follow a seminal work of Metropolis and develop a **semi-classical intranuclear cascade** (INC) that assume classical propagation between consecutive scatterings

Sampling nucleon configurations



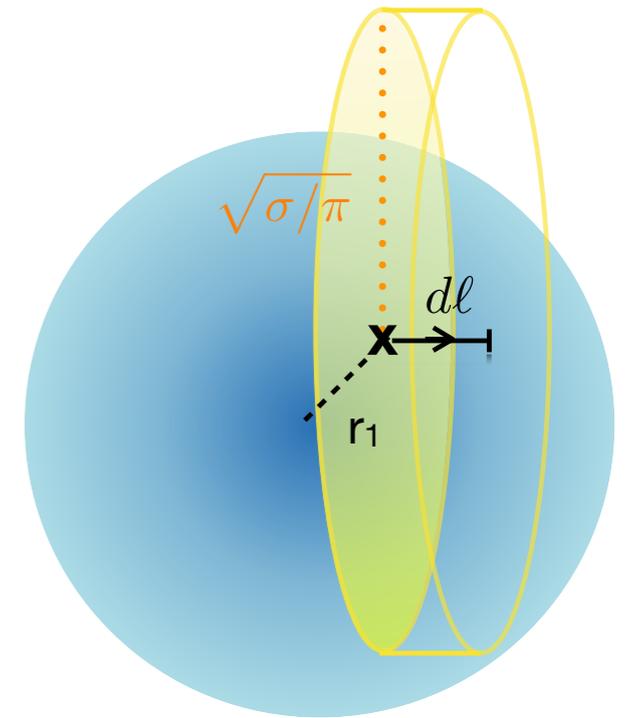
The nucleons' positions utilized in the INC are sampled from **36000 GFMC configurations**. For benchmark purposes we also sampled **36000 mean-field (MF) configurations** from the single-proton distribution.

The differences between GFMC and MF configurations are apparent when comparing the **two-body density distributions**: repulsive nature of two-body interactions reduced the probability of finding two particles close to each other

Probability of interaction

To check if an interaction between nucleons occurs an accept-reject test is performed on the closest nucleon according to a probability distribution.

We use a **cylinder probability distribution**, this mimics a more classical billiard ball like system where each billiard ball has a radius
In addition we consider a **gaussian probability distribution**



For benchmark purposes, we also implemented the **mean free path approach**, routinely used in event generators

$$P = \sigma \bar{\rho} dl \quad \text{where a constant density is assumed} \quad \rho(r_1) \sim \rho(r_1 + dl) \sim \bar{\rho}$$

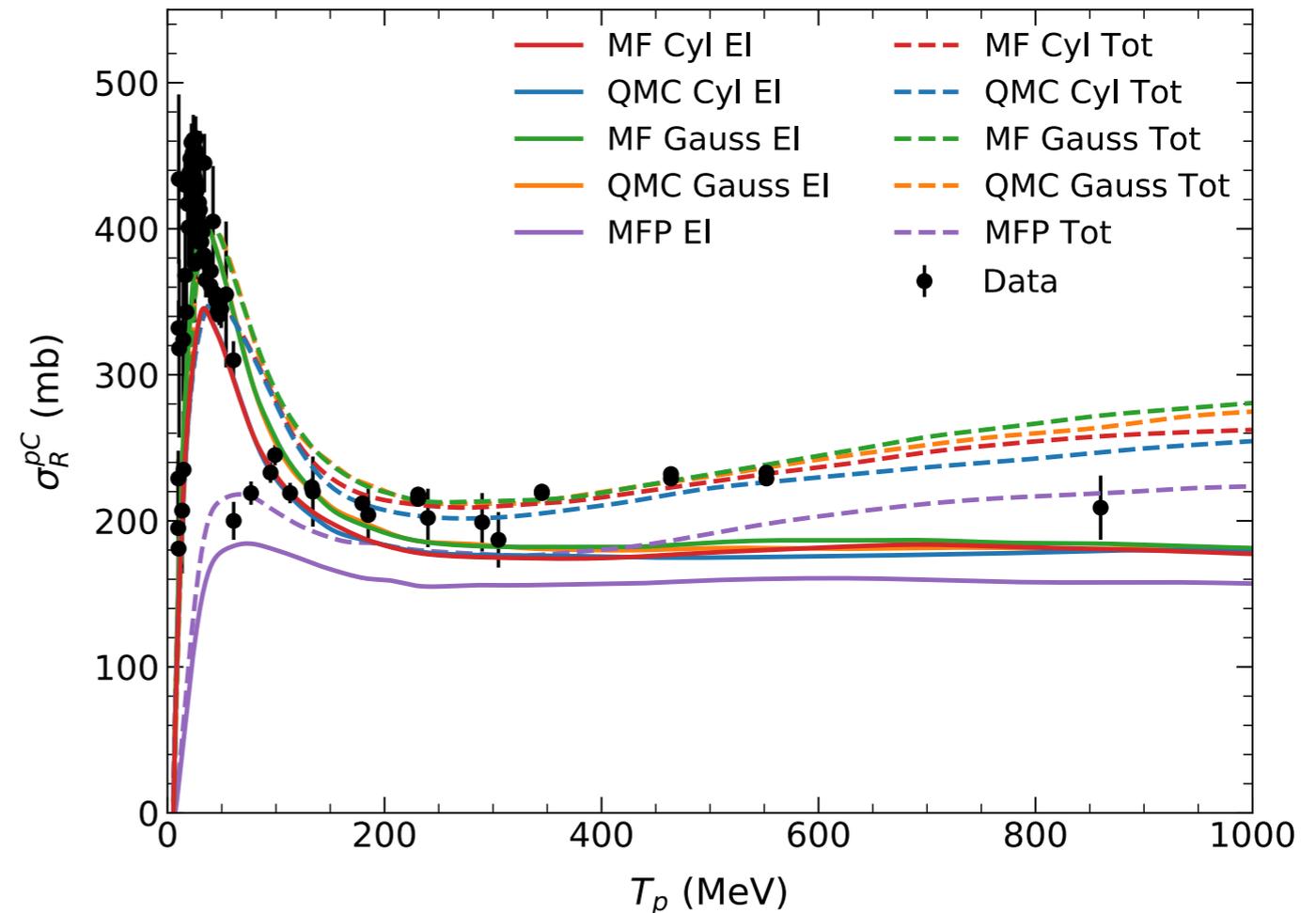
we sample a number $0 \leq x \leq 1$ $\left\{ \begin{array}{ll} x < P & \checkmark \text{ the interaction occurred, check Pauli blocking} \\ x > P & \times \text{ the interaction DID NOT occur} \end{array} \right.$

Results: proton-Carbon cross section

Reproducing proton-nucleus cross section measurements is an important test of the accuracy of the INC model.

- We define a beam of protons with energy E , uniformly distributed over an area A .
- We propagate each proton in time and check for scattering at each step.
- The Monte Carlo cross section is defined as:

$$\sigma_{\text{MC}} = A \frac{N_{\text{scat}}}{N_{\text{tot}}}$$



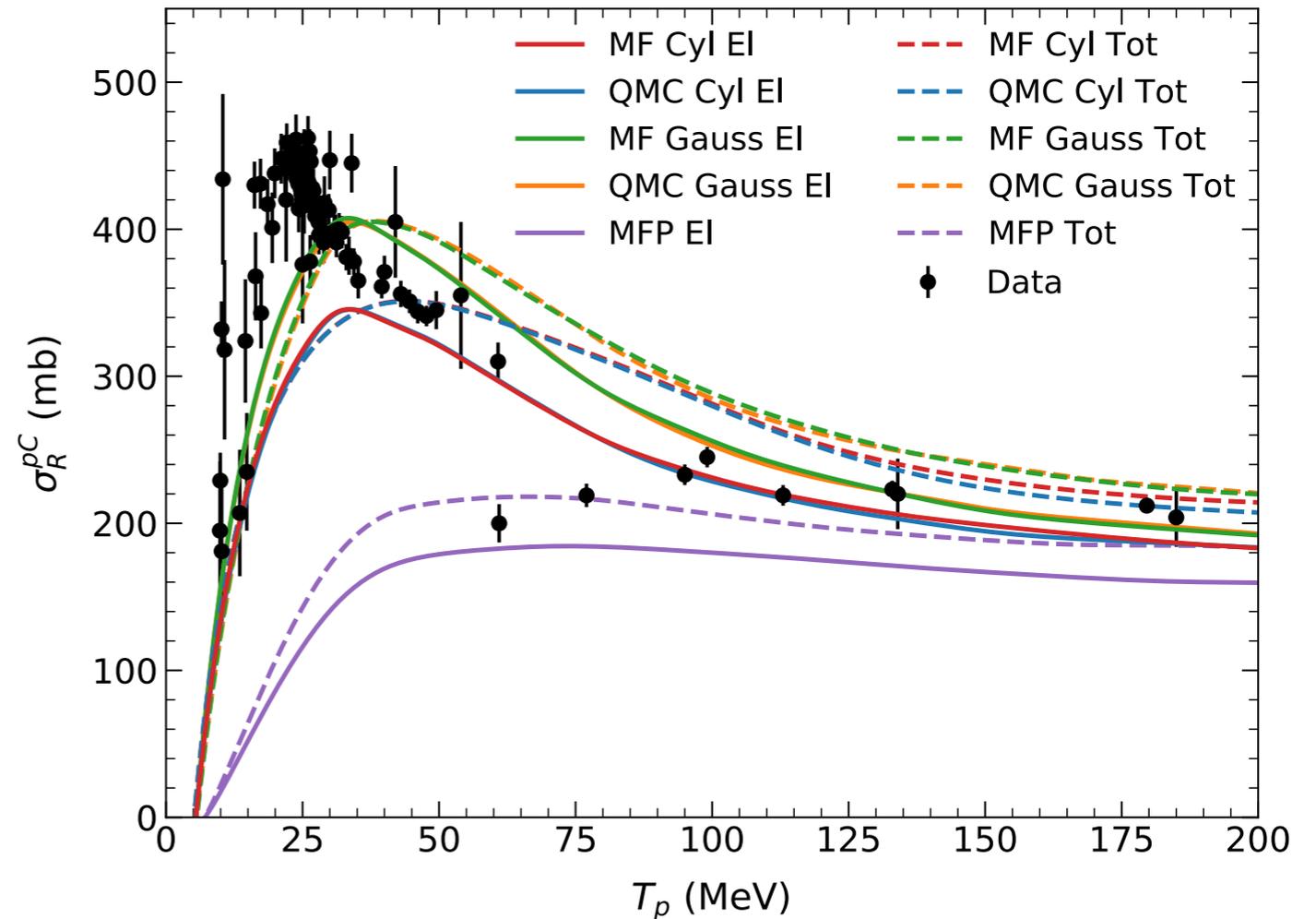
The **solid lines** have been obtained using the nucleon-nucleon cross sections from the **SAID database** in which only the **elastic contribution** is retained. The **dashed lines** used the **NASA parameterization**, which includes **inelasticities**.

Results: proton-Carbon cross section

The **Gauss** and **cylinder probability** distribution yield **similar results**

Large difference with the mean-free-path implementation: conceptual differences with respect to the previous cases

QMC and MF distribution lead to almost identical results: this observable does not depend strongly on correlations among the nucleons



The **solid lines** have been obtained using the nucleon- nucleon cross sections from the SAID database in which only the **elastic contribution** is retained. The **dashed lines** used the NASA parameterization , which includes **inelasticities**.

Results: nuclear transparency

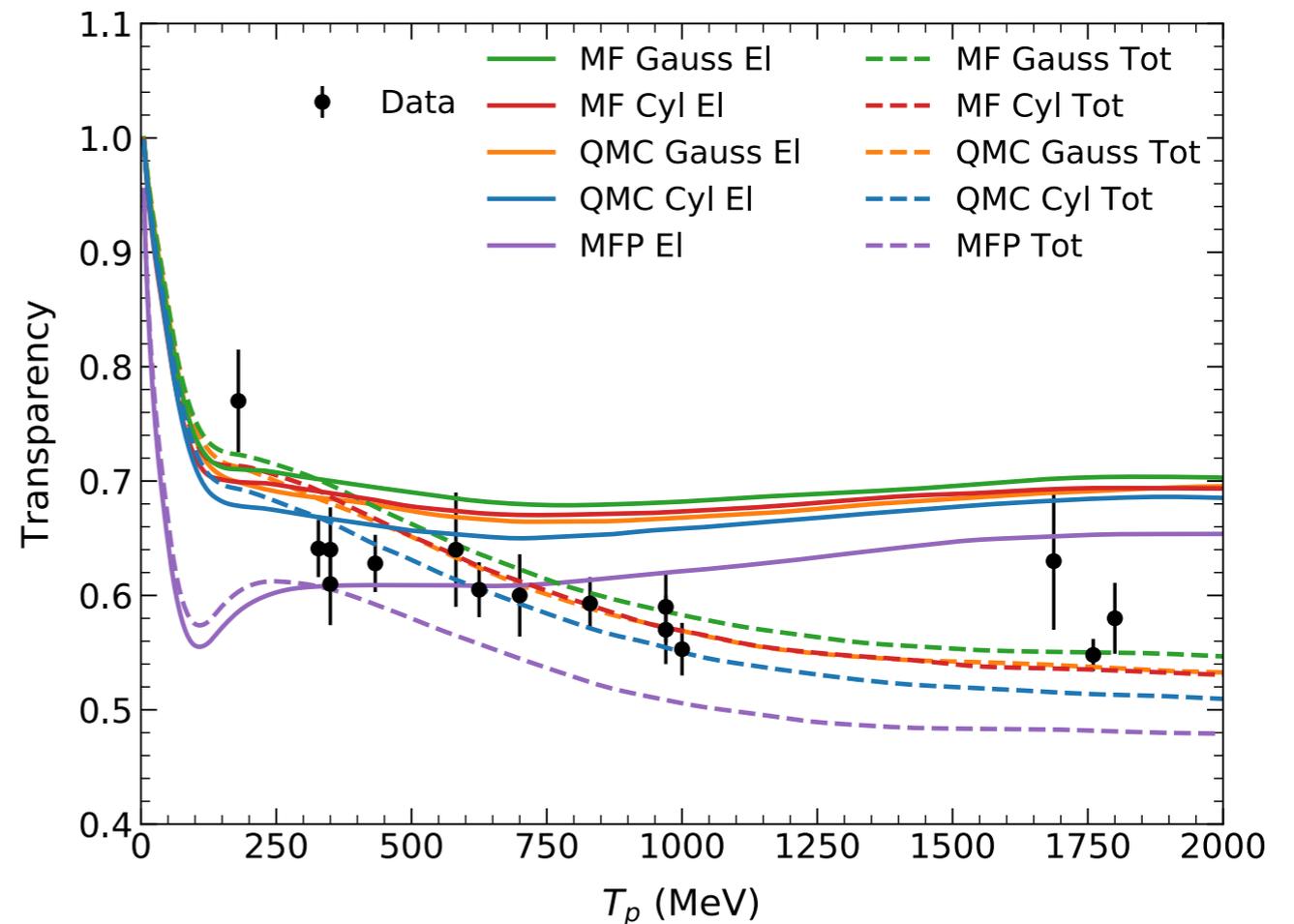
The **nuclear transparency** yields the average probability that a struck nucleon leaves the nucleus without interacting with the spectator particles

Nuclear transparency is **measured in (e,e'p) scattering** experiments

Simulation: we randomly sample a nucleon with kinetic energy T_p and propagate it through the nuclear medium

$$T_{MC} = 1 - \frac{N_{\text{hits}}}{N_{\text{tot}}}$$

Gaussian and cylinder curves are consistent and correctly reproduces the data. Correlations do not seem to play a big role.



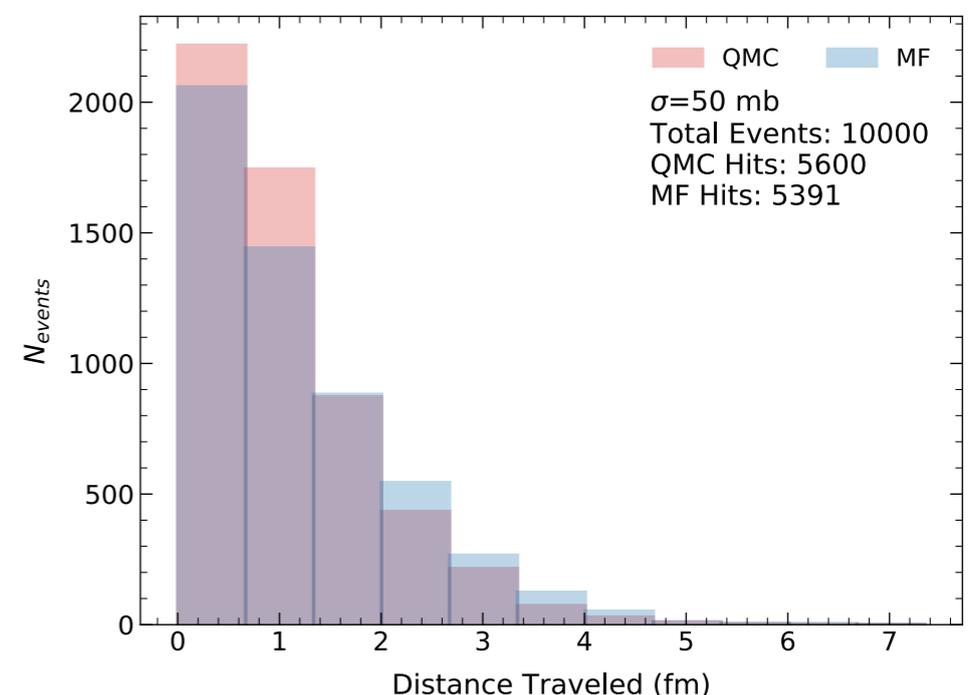
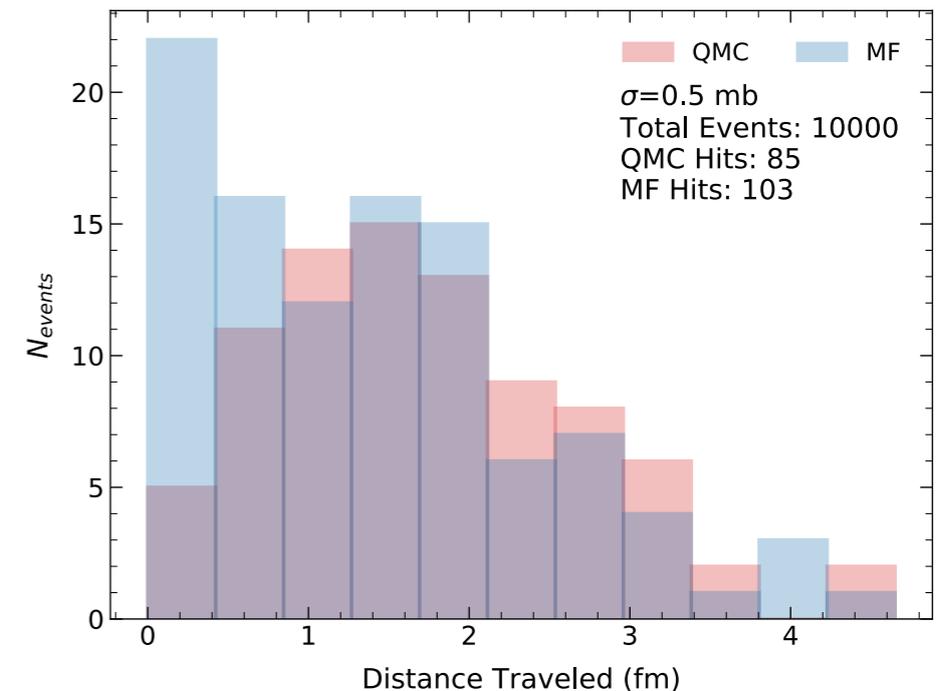
Results: correlation effects

Histograms of the **distance traveled** by a struck particle **before the first interaction** takes place for different values of the interaction cross section

When using **QMC configurations**, the hit nucleon is surrounded by a short-distance **correlation hole**: expected to propagate freely for ~ 1 fm before interacting

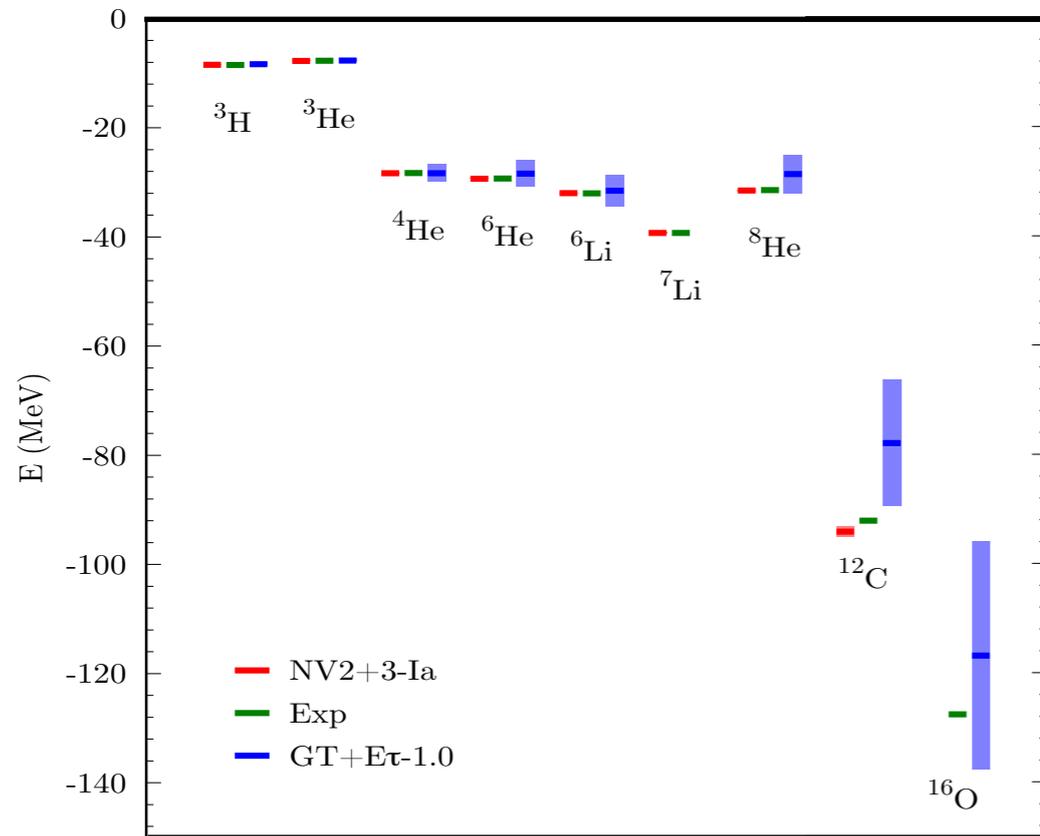
For $\sigma=0.5$ mb the **MF distribution peaks toward smaller distances than the QMC one**: originates from the repulsive nature of the nucleon-nucleon potential

For $\sigma=50$ mb large cylinder, **MF and QMC distributions become similar**. The propagating particle is less sensitive to the local distribution of nucleons and more sensitive to the integrated density over a larger volume, reducing the effect of correlations



Future theory efforts

 S.Gandolfi, D.Lonardonì, et al, *Front.Phys.* 8 (2020) 117

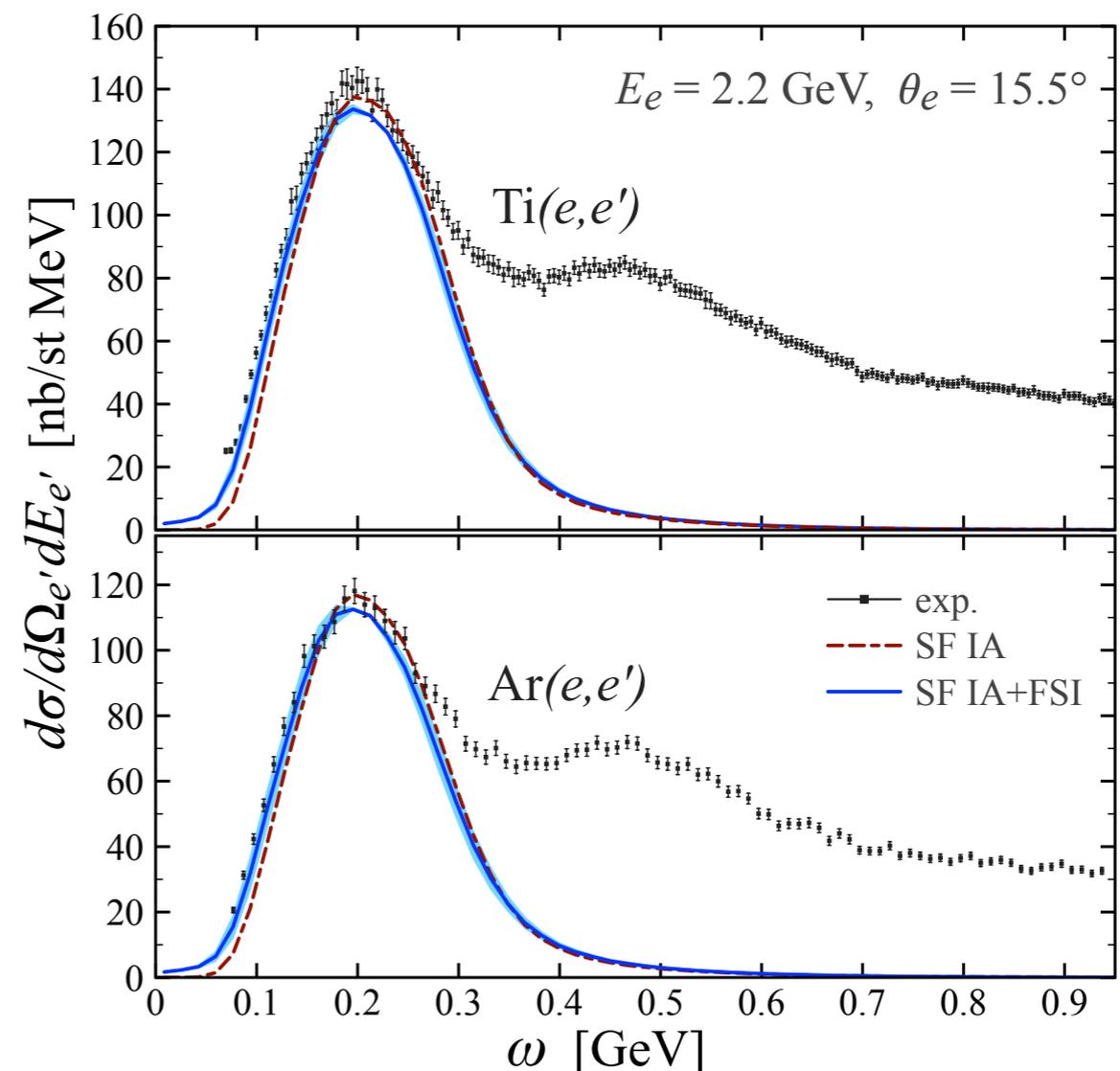


Using more approximate methods,
calculation of lepton-Ar cross sections.
Extend the factorization scheme to the DIS

Intranuclear cascade: include π degrees of freedom: π production, absorption and elastic scattering as well as **in medium corrections**

Theoretical uncertainty estimate: truncation of the chiral expansion and statistical uncertainty of the ab-initio method

Devise an **hybrid QMC** approach able to describe larger nuclei such as ${}^{16}\text{O}$ and use machine learning algorithms to obtain cross sections



 C.Barbieri, NR, V.Somà, *PRC* 100 (2019) 6, 062501

Thank you for your attention!