Status and future prospects of the 3ν mixing

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Neutrino Seminar Series

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Neutrino evolution

In the 3ν scenario, neutrino evolution is described by the Schrödinger equation

$$i\frac{d\vec{\nu}}{dt} = \frac{1}{2E}\left[U^{\dagger} Diag(0, \Delta m^2_{21}, \Delta m^2_{31}) U \pm V_{mat}\right]\vec{\nu}$$

$$\vec{\nu} = (\nu_e\, \nu_\mu\, \nu_\tau)^T \qquad V_{mat} = \sqrt{2} G_F N_e E\, \mathrm{diag}(1, 0, 0)$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-\delta_{cp}} \\ 0 & 1 & 0 \\ -s_{13}e^{\delta_{cp}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

- N_e electron density along the neutrino path.
- In the case of Majorana neutrinos, $P = \mathrm{diag}(1, e^{\imath\delta_1}, e^{\imath\delta_2})$ encode two additional phases.
- P cannot be measured in oscillation experiments.

Neutrino evolution

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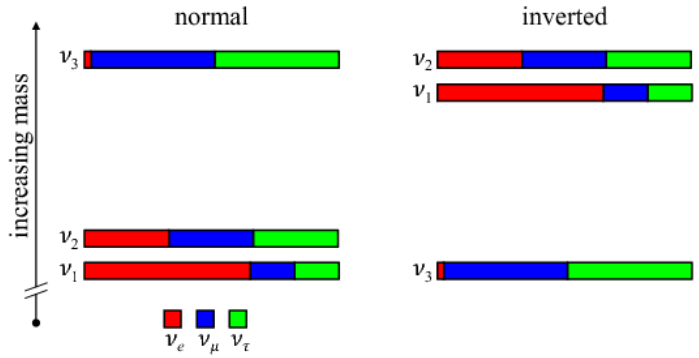
Two possible mass hierarchies



The global fit goal is the determination of the six parameters describing the evolution.

π^+
π^0
γ
γ
μ^+
e^+
e^-
γ
γ
e^+
ν_e
$\bar{\nu}_\mu$
ν_μ

Measuring oscillation parameters

Experiment	Dominant	Important
Solar	θ_{12}	$\Delta m^2_{21}, \theta_{13}$
Reactor LBL	Δm^2_{21}	θ_{12}, θ_{13}
Ractor MBL	θ_{13}	$\left\lvert \Delta m^2_{3l} \right\rvert$
Atmospheric	θ_{23}	$\left\lvert \Delta m^2_{3l} \right\rvert, \theta_{13}, \delta_{CP}$
Accelerator LBL ν_μ Disapp	$\left\lvert \Delta m^2_{3l} \right\rvert, \theta_{23}$	
Accelerator LBL ν_e App	δ_{CP}	$\theta_{13}, \theta_{23}, \mathrm{sign}\left(\Delta m^2_{3l}\right)$

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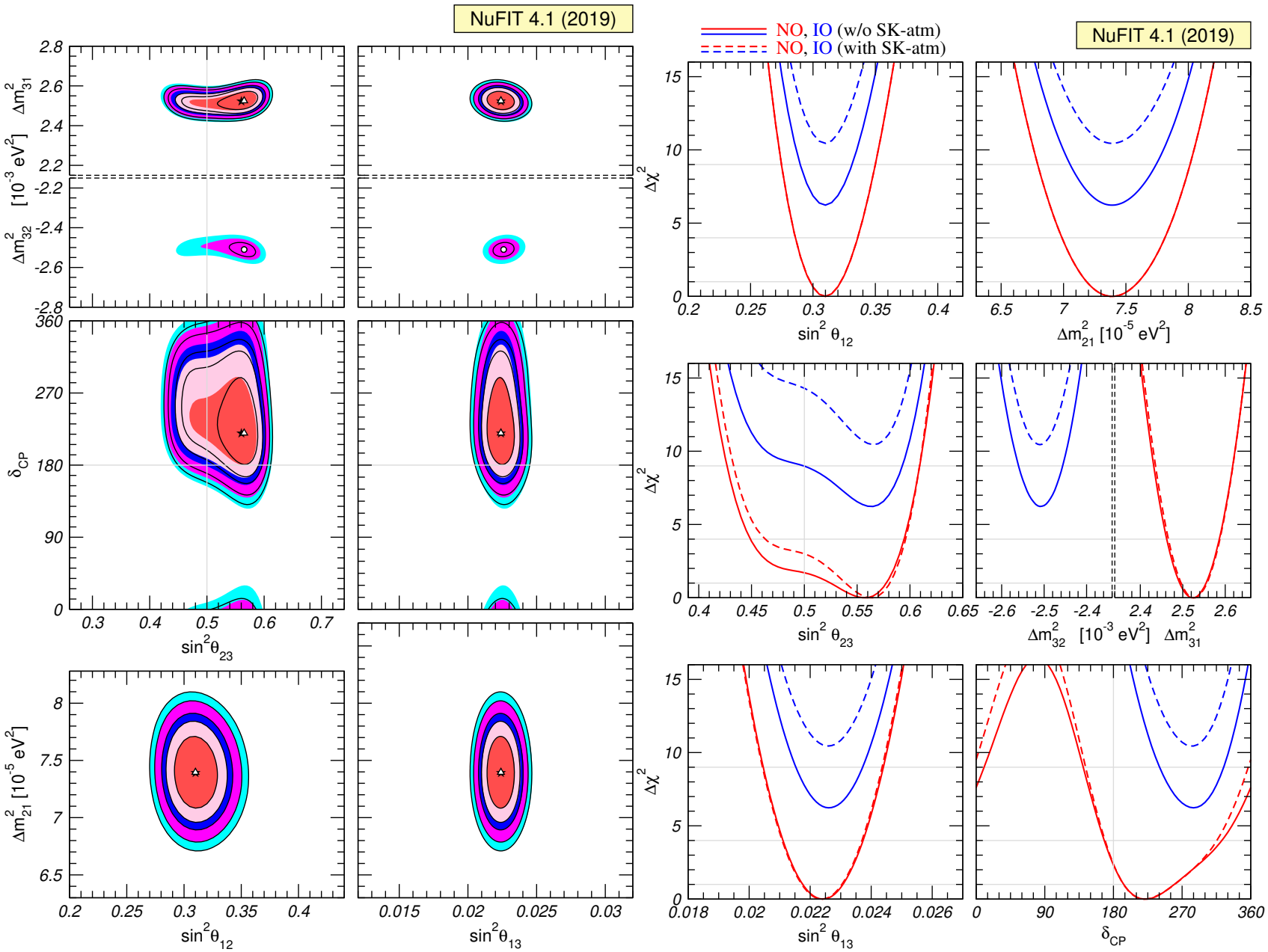
Solar	θ_{12}	$\Delta m^2_{21}, \theta_{13}$
Reactor LBL	Δm^2_{21}	θ_{12}, θ_{13}
Ractor MBL	θ_{13}	$\left\|\Delta m^2_{3l}\right\|$
Atmospheric	θ_{23}	$\left\|\Delta m^2_{3l}\right\|, \theta_{13}, \delta_{CP}$
Accelerator LBL ν_μ Disapp	$\left\|\Delta m^2_{3l}\right\|, \theta_{23}$	
Accelerator LBL ν_e App	δ_{CP}	$\theta_{13}, \theta_{23}, \text{sign}\left(\Delta m^2_{3l}\right)$

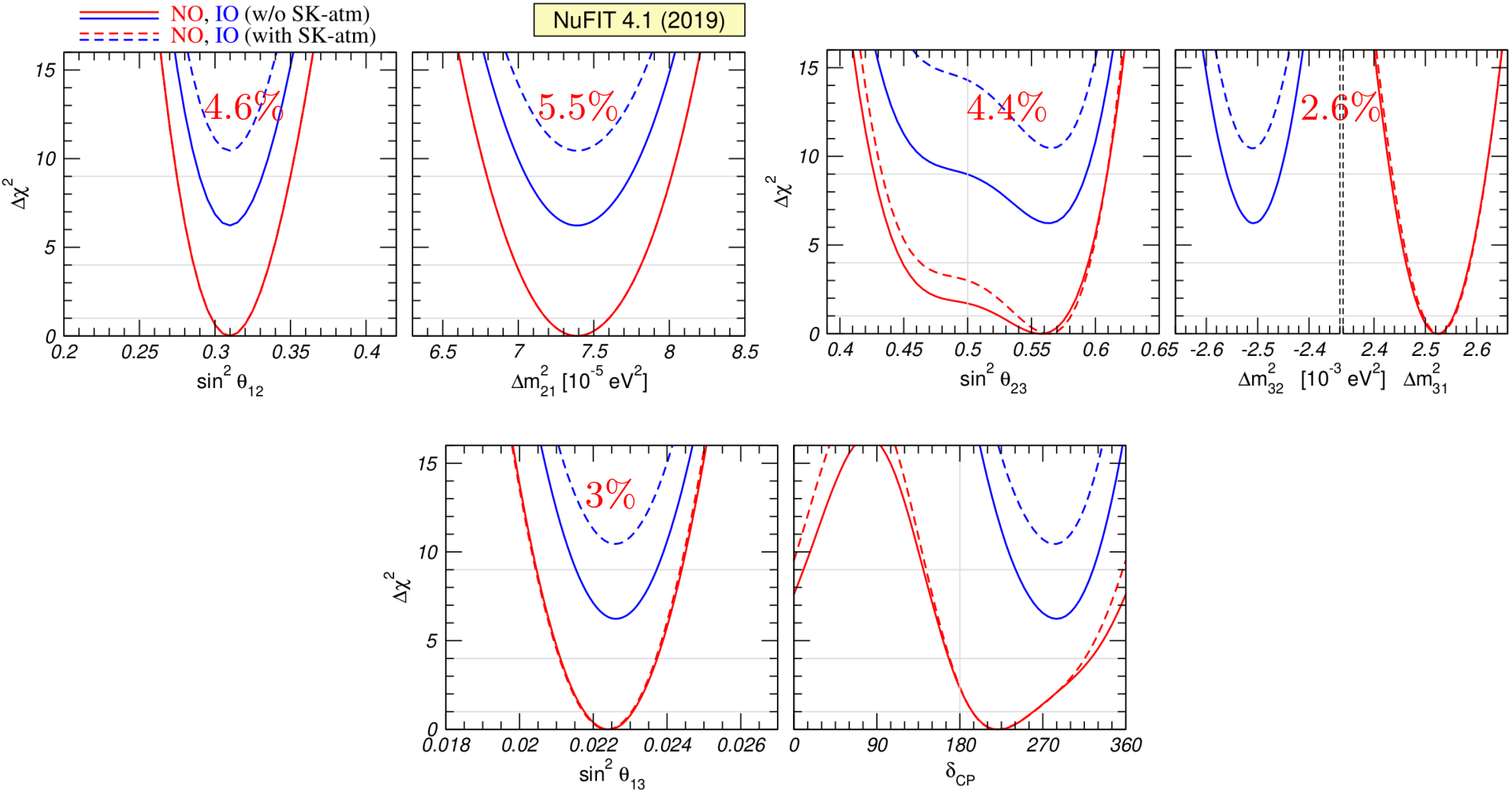
Summary of the data included in the analysis (NuFIT 4.1)

Solar:	SNO, SK and Borexino
Reactor LBL:	KamLAND
Reactor MBL:	Daya Bay, RENO and Double Chooz
Atmospheric:	IceCube/DeepCore and SK(χ^2 map)
Accelerator LBL:	MINOS, T2K and NOvA(ν_μ disapp. and ν_e app.)

Present precision



Present precision

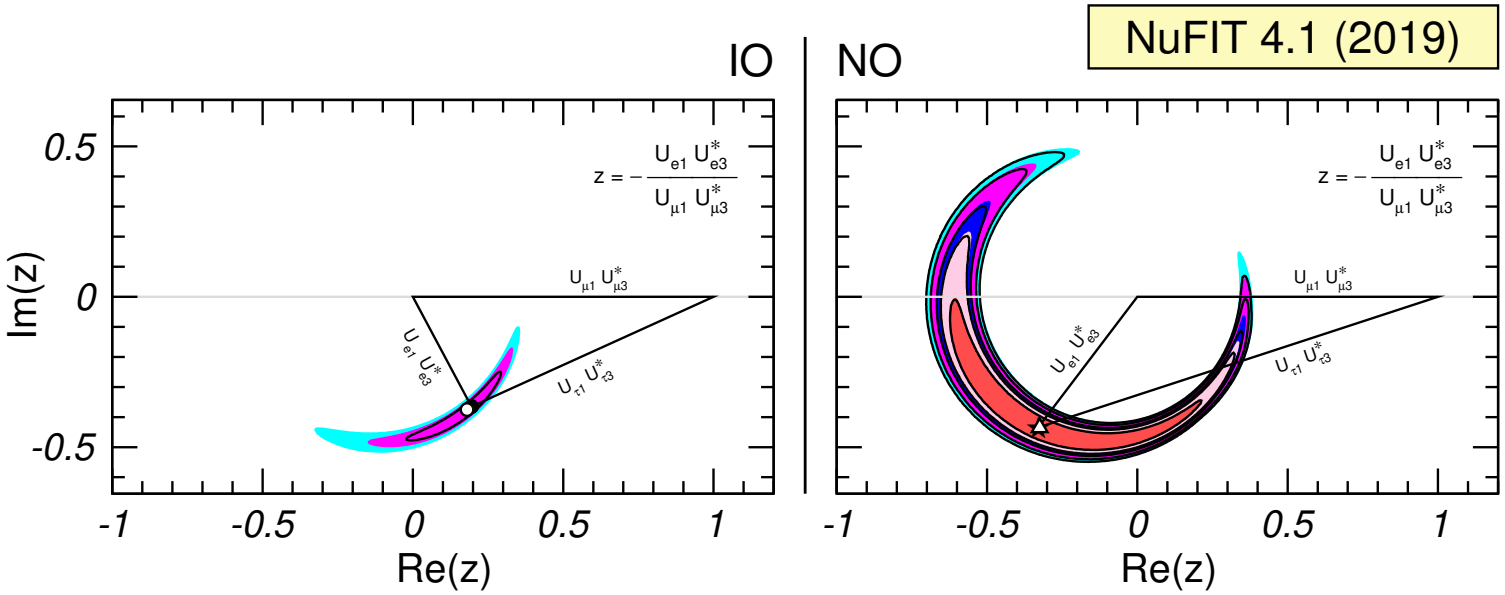


- CP-conserving excluded at 2σ
- $\delta_{cp} = \pi/2$ excluded at more than 3σ

Unitarity triangle

NuFIT 4.1 (2019)

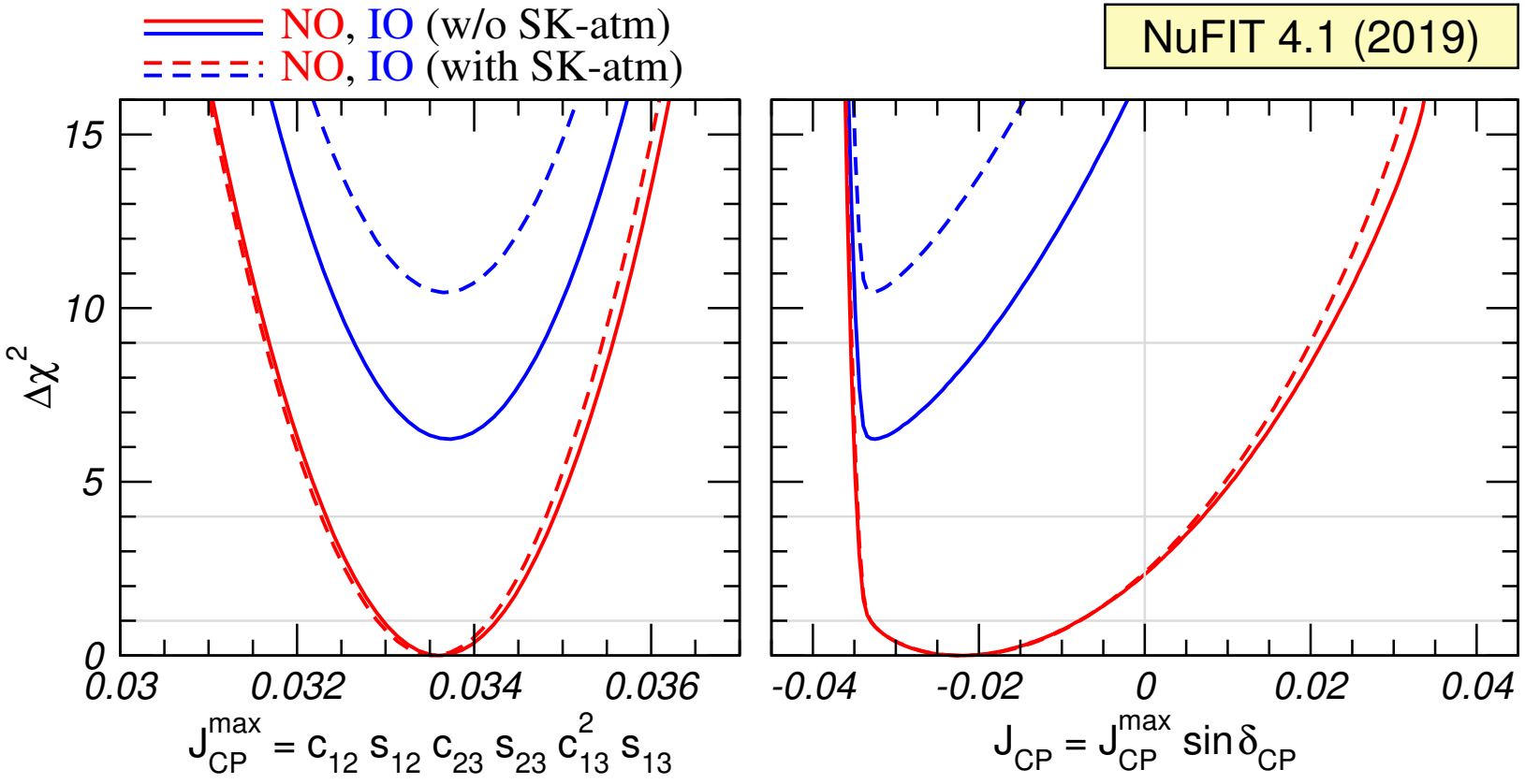
$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.244 \to 0.496 & 0.467 \to 0.678 & 0.646 \to 0.772 \\ 0.287 \to 0.525 & 0.488 \to 0.693 & 0.618 \to 0.749 \end{pmatrix}$$



$$J_{CP} = \text{Im}\left[U_{\alpha i} U_{\alpha j}^* U_{\beta i} U_{\beta j}^*\right]$$

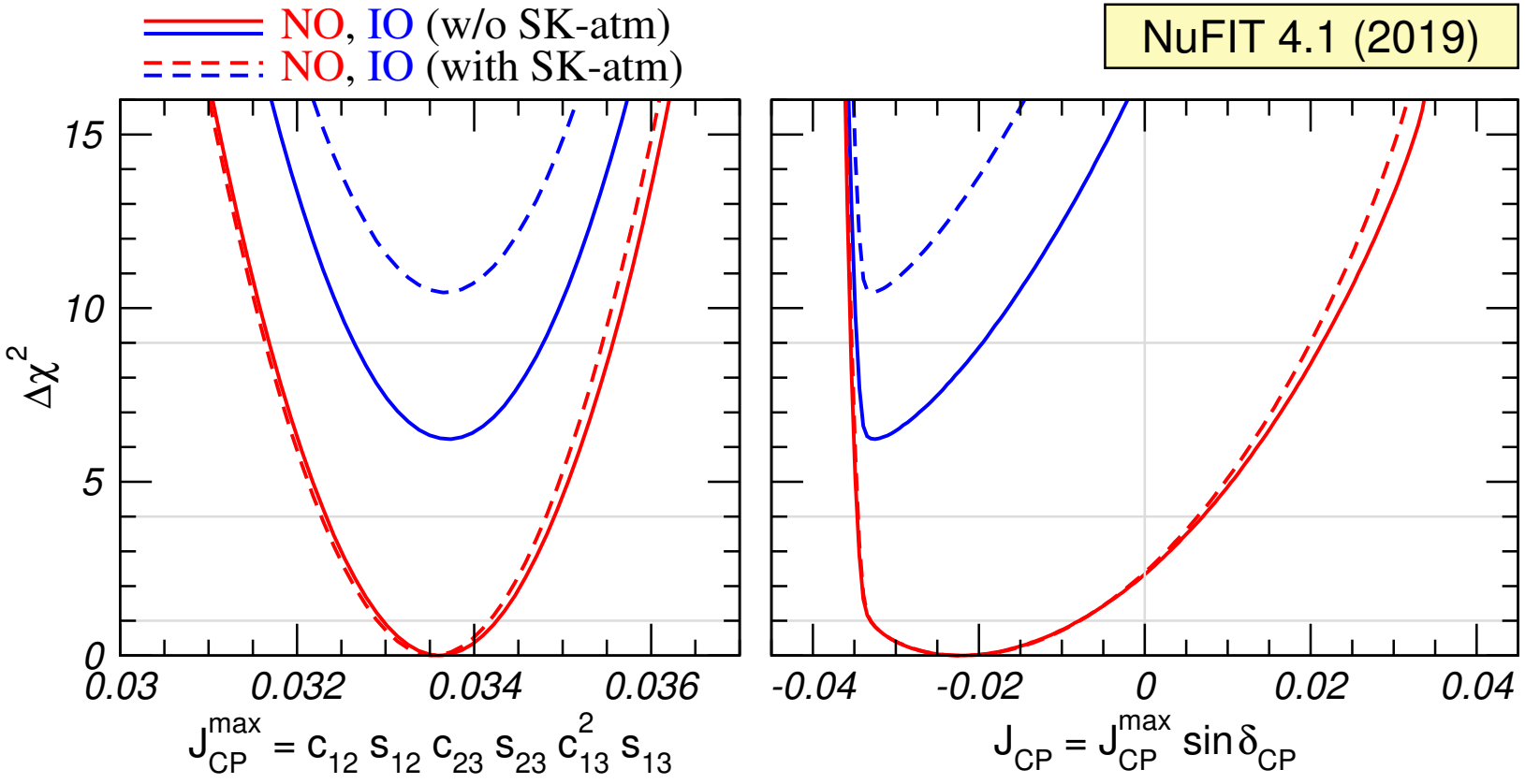
Parke and Ross-Lonergan, Phys.Rev.D 93 (2016) 11 [1508.05095]
Ellis, Kelly and Li [2004.13719]
IMS and Minakata [1806.10152], IMS and Minakata [1908.04855]
Blennow, Fernandez-Martinez, Gehrlein, Hernandez-Garcia, Salvado, Eur.Phys.J.C 78 (2018) 10 [1803.02362]...

Jarlskog invariant



$$J_{CP} = J_{CP}^{\max} \sin\delta_{CP} = \cos\theta_{12} \sin\theta_{12} \cos\theta_{23} \sin\theta_{23} \cos^2\theta_{13} \sin\theta_{13} \sin\delta_{CP}$$

Jarlskog invariant



Lepton sector

$J_{CP} = -0.019$

Quark sector[1]

$J_{CP} = (3.08^{+0.16}_{-0.18} \times 10^{-5})$

[1] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).

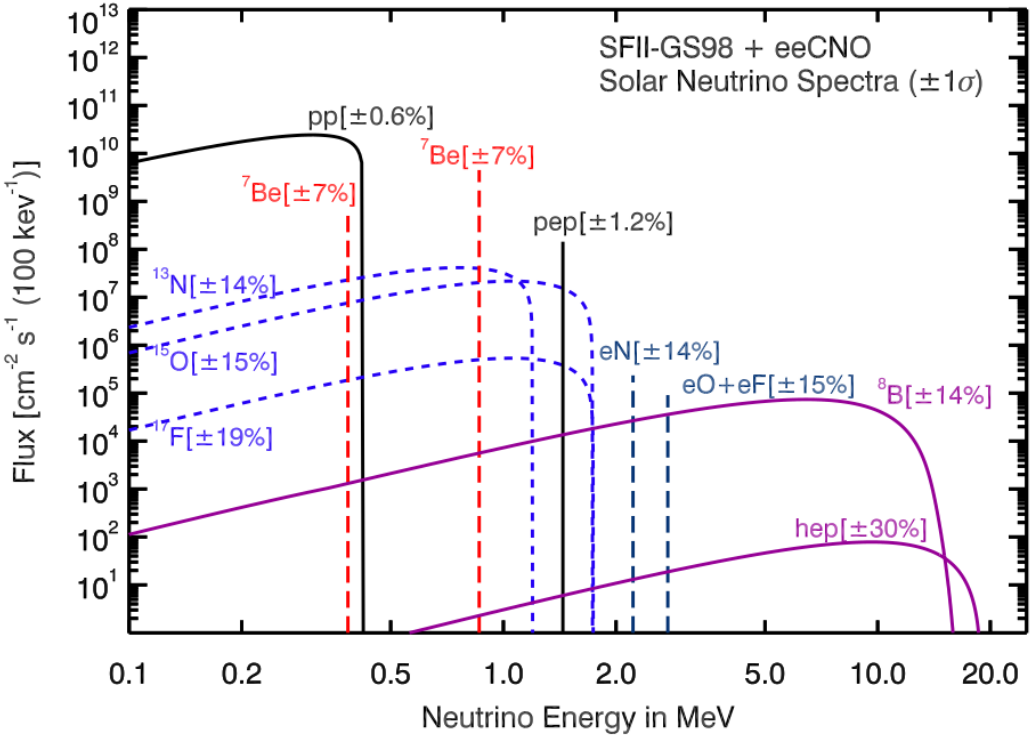
Solar neutrinos and Long-baseline reactor experiments

Solar neutrinos

- Solar neutrinos are produced by two different nuclear fusion reactions: pp chains and CNO cycles.

$$4p \rightarrow^{4} He + 2e^{+} + 2\nu_e + \gamma$$

- The flux is composed by ν_e with a characteristic energy (^{7}Be,pep) or spectrum (pp, CNO, ^{8}B, hep).



M. Tanabashi et al. (PDG) Phys. Rev. D 98, 030001 (2018)

Solar neutrinos

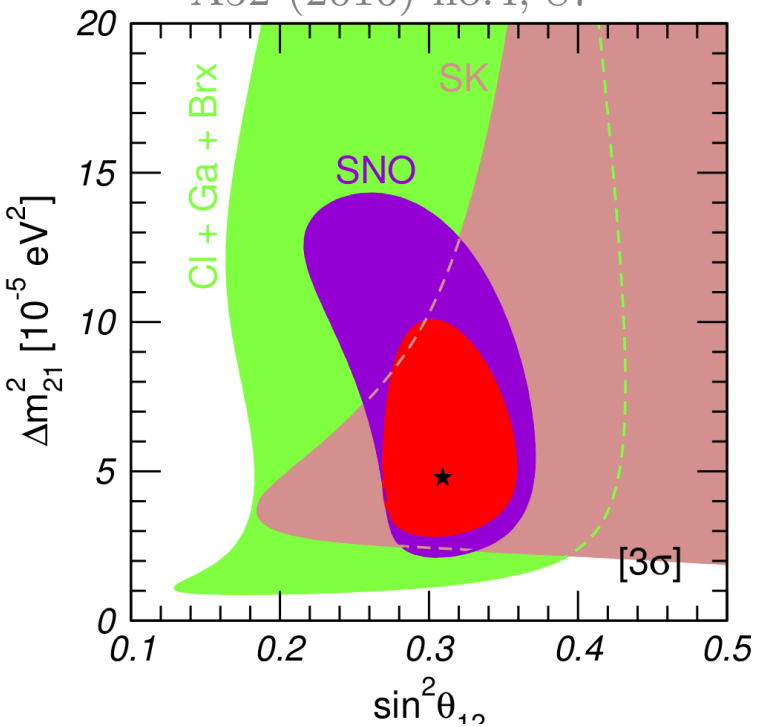
- $\Delta m^2_{31} >> E/L$

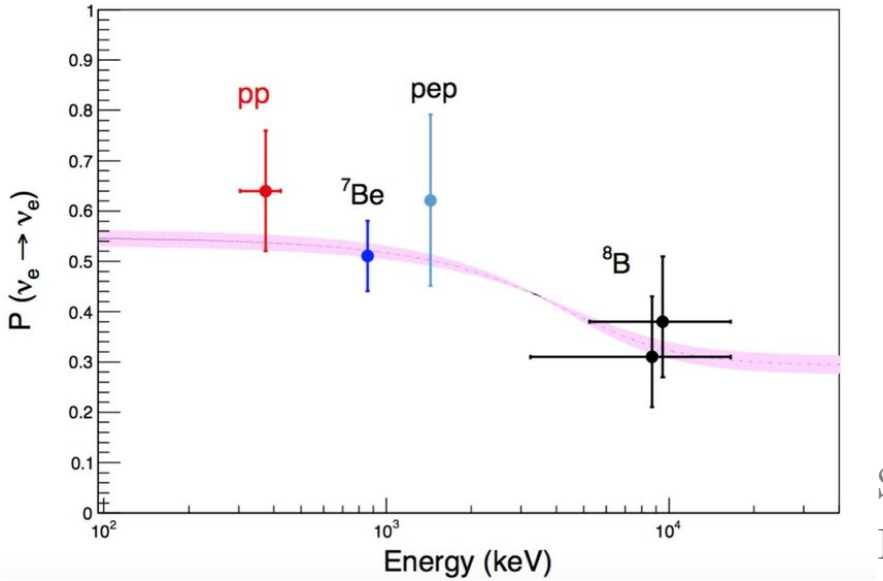
$$P^{3\nu}_{ee} \approx \cos^2\theta_{13}\cos^2\theta^m_{13} P^{2\nu}_{eff}(\Delta m^2_{21}, \theta_{12}) + \sin^2\theta^m_{13}\sin^2\theta_{13}$$

- For neutrinos created in high densities

$$P^{2\nu}_{eff}(\Delta m^2_{21}, \theta_{12}) = \frac{1}{2}(1 + \cos\theta^m_{12}\cos\theta_{12})$$

Maltoni, Smirnov Eur.Phys.J. A52 (2016) no.4, 87



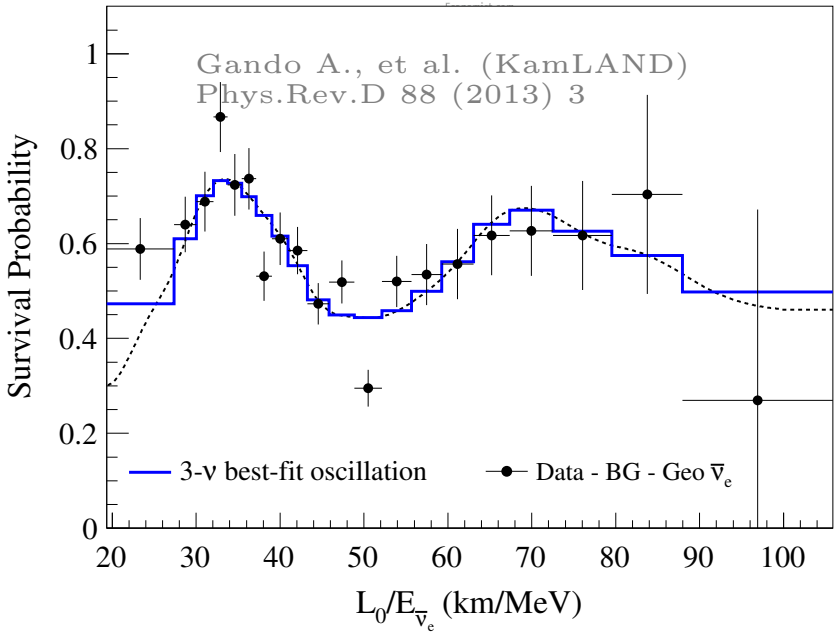


S. Marcocci, et al. (Borexino) Magellan Workshop 2016

KamLAND

- Long-baseline reactor experiment.
- $\overline{\nu}_e$ with $E_\nu \sim$ few MeV.
- Liquid scintillator detector

$$P_{ee}^{3\nu} \simeq c_{13}^4 \left(1 - \sin^2 2\theta_{12M} \sin^2 \left(\frac{\Delta m_{21M}^2 L}{4E}\right)\right) + s_{13}^4$$

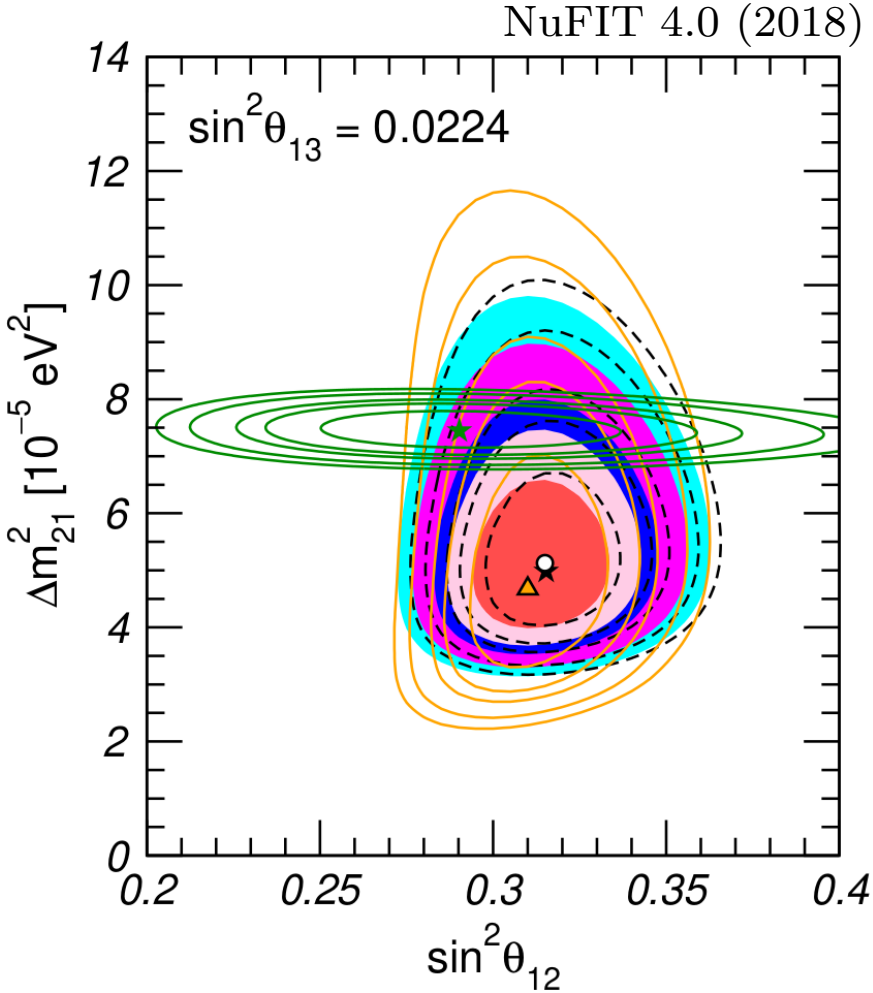


$$\imath \frac{d\vec{\nu}^{2\nu}}{dt} = \left[\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} \pm \sqrt{2} G_F N_e \begin{pmatrix} c_{13}^2 & 0 \\ 0 & 0 \end{pmatrix}\right] \vec{\nu}, \ \vec{\nu} = \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}$$

Determination of the solar parameters (Δm^2_{21}, θ_{21})

- Δm^2_{21} is determined by KamLAND
- θ_{12} is determined by the solar neutrinos

2σ tension in the determination of Δm^2_{21} between KamLAND and solar experiments

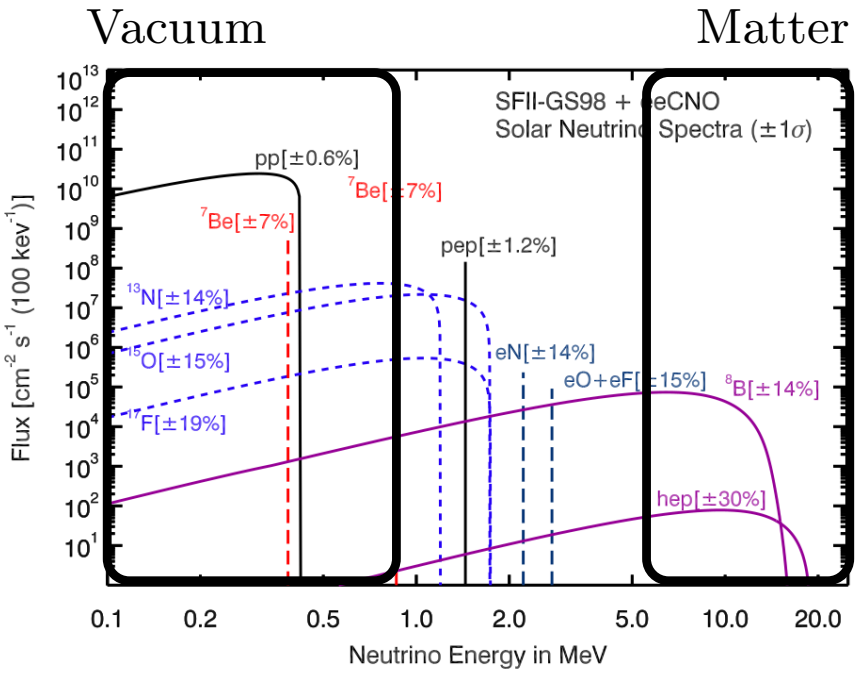


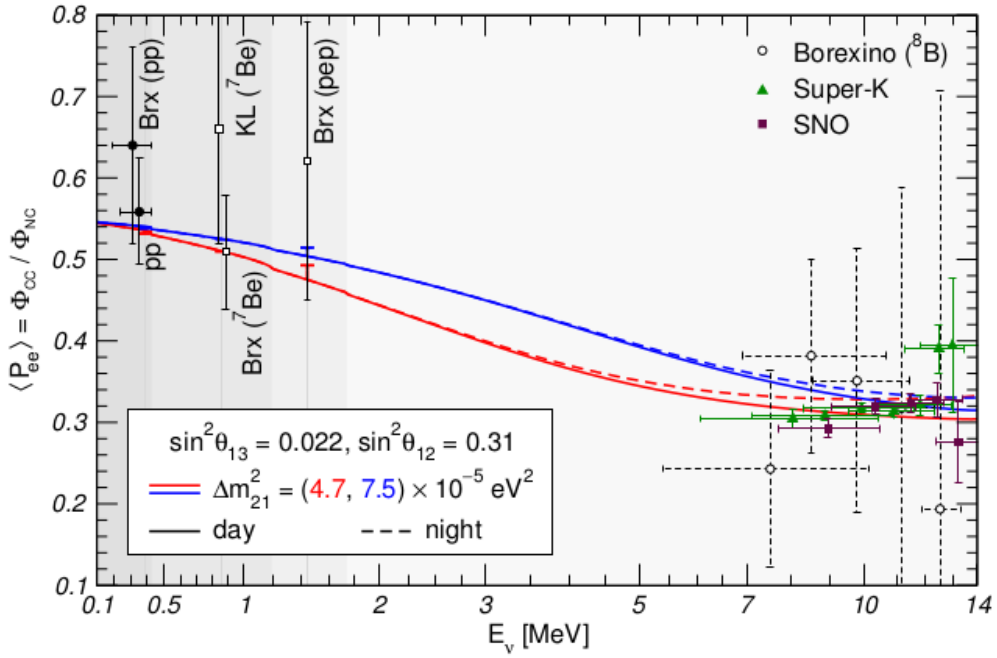
Tension in Δm^2_{21}

Non observation of low-energy turn up ^8B neutrino spectrum measured.

Observations indicates:

- $P_{ee} \sim 30\%$ at high energy (^8B, hep).
- $P_{ee} \sim 60\%$ at low energy (pp, ^7Be, CNO and low ^8B).

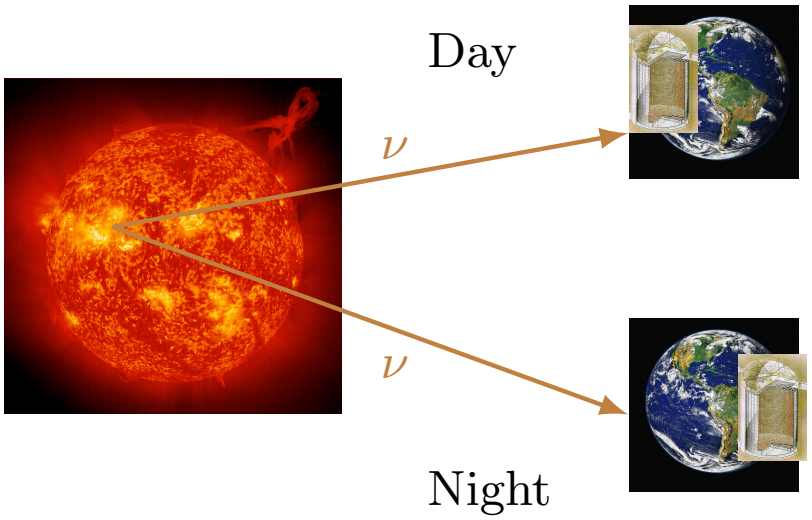


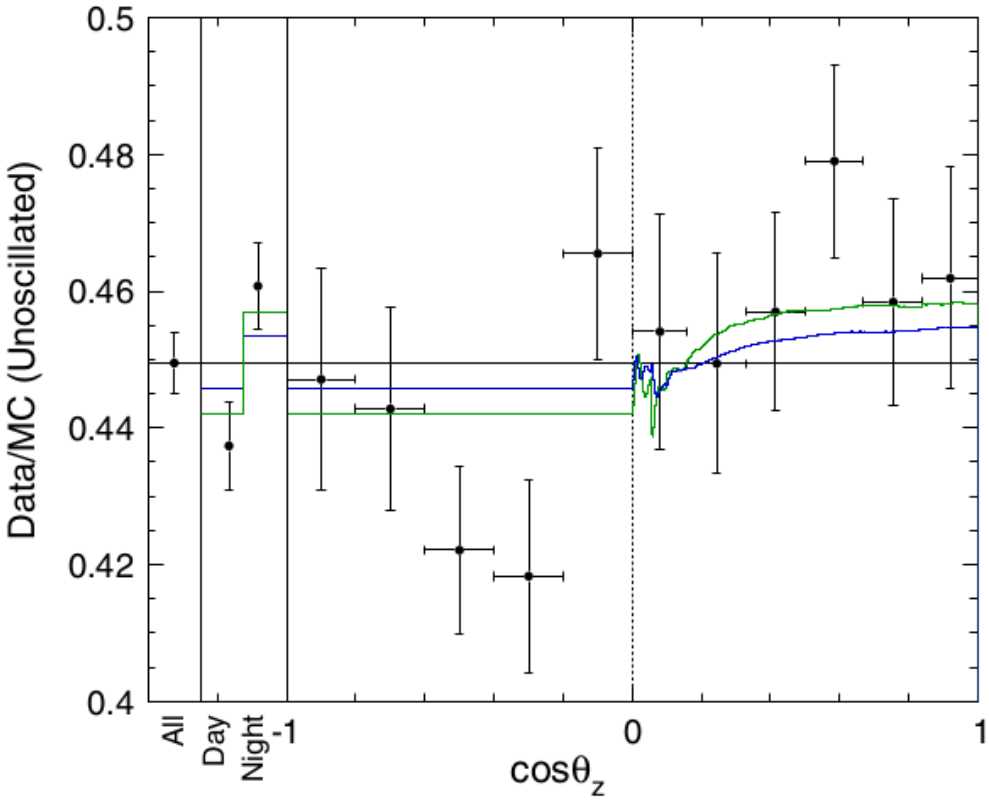


M. Tanabashi et al. (PDG) Phys. Rev. D 98, 030001 (2018) Maltoni, Smirnov Eur.Phys.J. A52 (2016)

Tension in Δm^2_{21}

Observation of a larger day/night asymmetry than predicted by KamLAND.



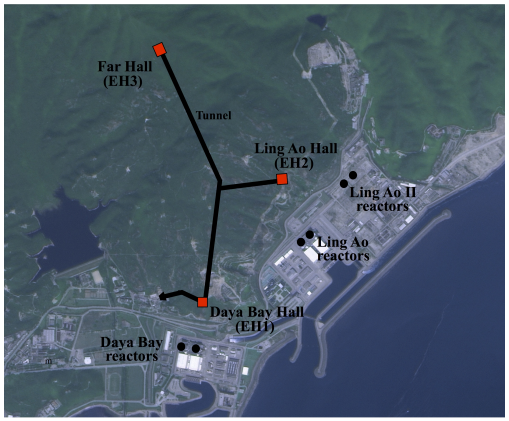


K. Abe et al. (SK) Phys. Rev. D94, 052010 (2016)

Medium baseline reactor experiments

Reactor neutrinos: determination of θ_{13} and Δm^2_{31}

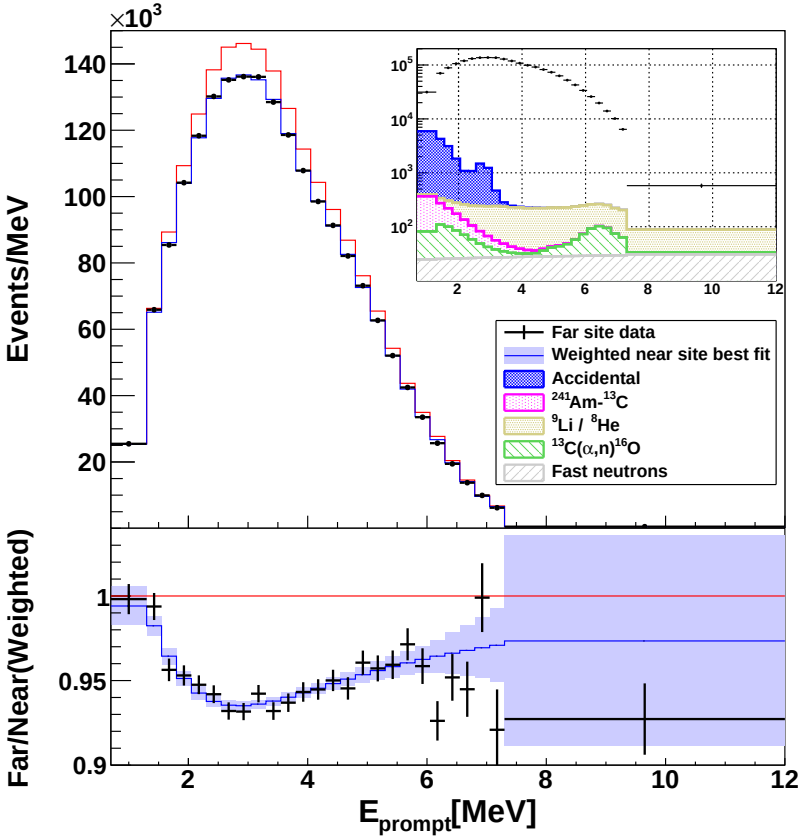
- Neutrino baseline ~ 1 km
- Reactor neutrinos are sensitive to θ_{13} and Δm^2_{31}.
- A rate-only analysis determines $\sin^2 2\theta_{13} = 0.0856 \pm 0.0029$ (Daya Bay)



$$P_{ee} \approx 1 - c_{13}^4 \sin^2 2\theta_{21} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$

$$\Delta m^2_{ee} = \cos^2 \theta_{12} |\Delta m^2_{31}| + \sin^2 \theta_{12} |\Delta m^2_{32}|$$

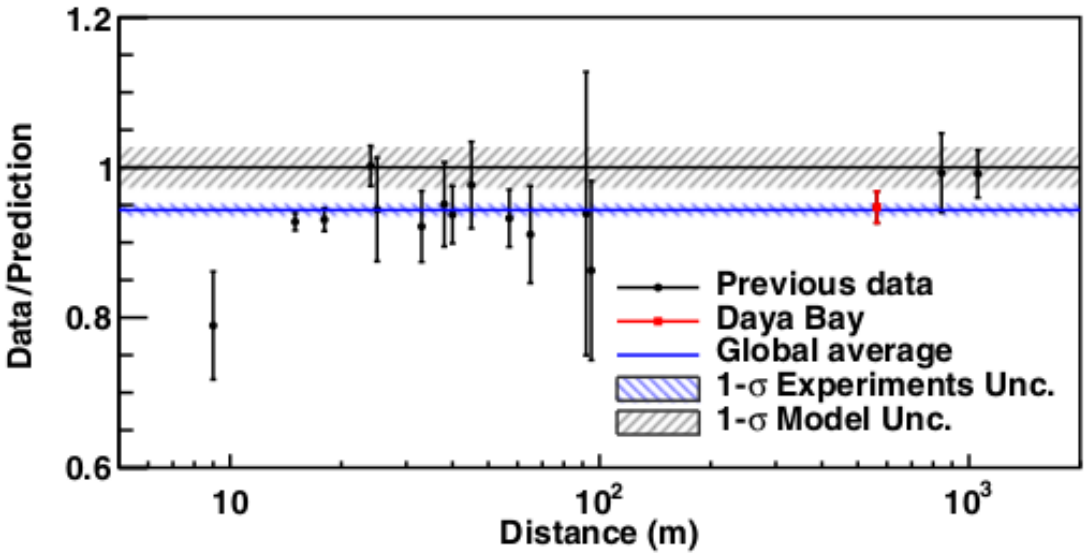
Nunokawa, Parke and Funchal, Phys. Rev. D72 (2005) 013009



D. Adey et al. (Daya Bay) Phys.Rev.Lett. 121 (2018) 24

Reactor anomaly

- The reevaluation of the $\overline{\nu}_e$ flux determined a deficit in the experimental data.
  - Data/Prediction $= 0.952 \pm 0.014 \pm 0.023$ (Daya Bay), 0.918 ± 0.018 (RENO)
- A large initial flux prefers larger values of θ_{13}.
- Small impact over θ_{13} since it is dominated by experiment with near detector.

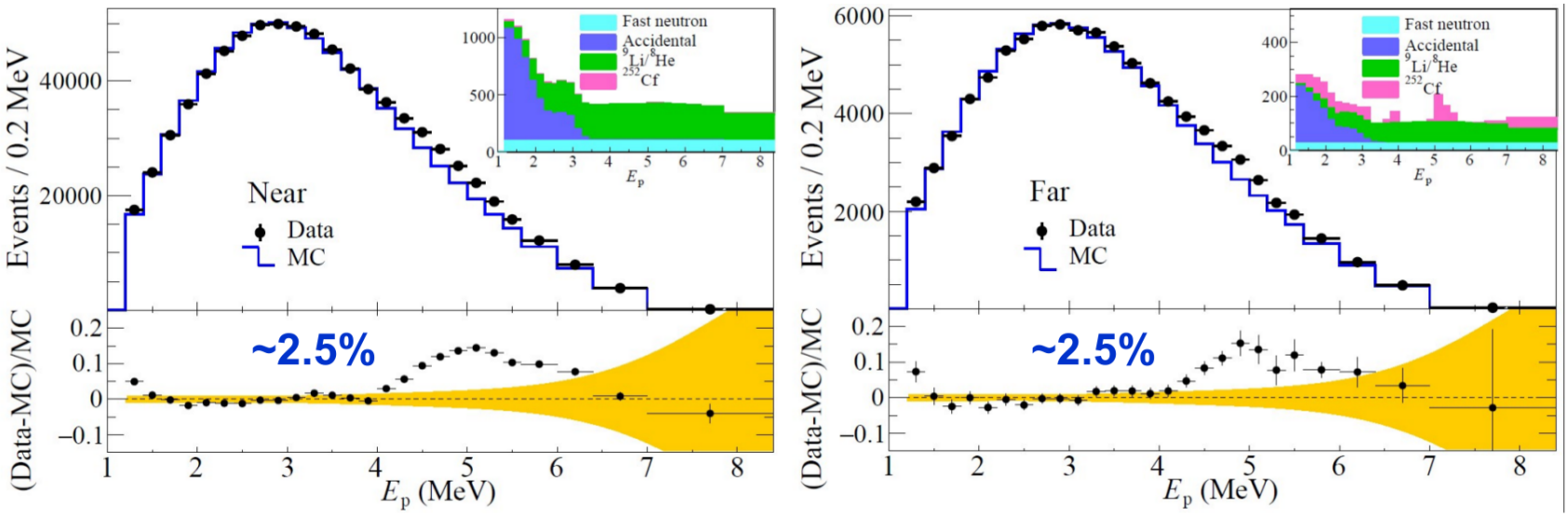


$$P_{ee} \approx 1 - c_{13}^4 \sin^2 2\theta_{21} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$

Feng Peng An et al. (Daya Bay) PRL 116, 061801 (2016)

Flux excess at 5 MeV

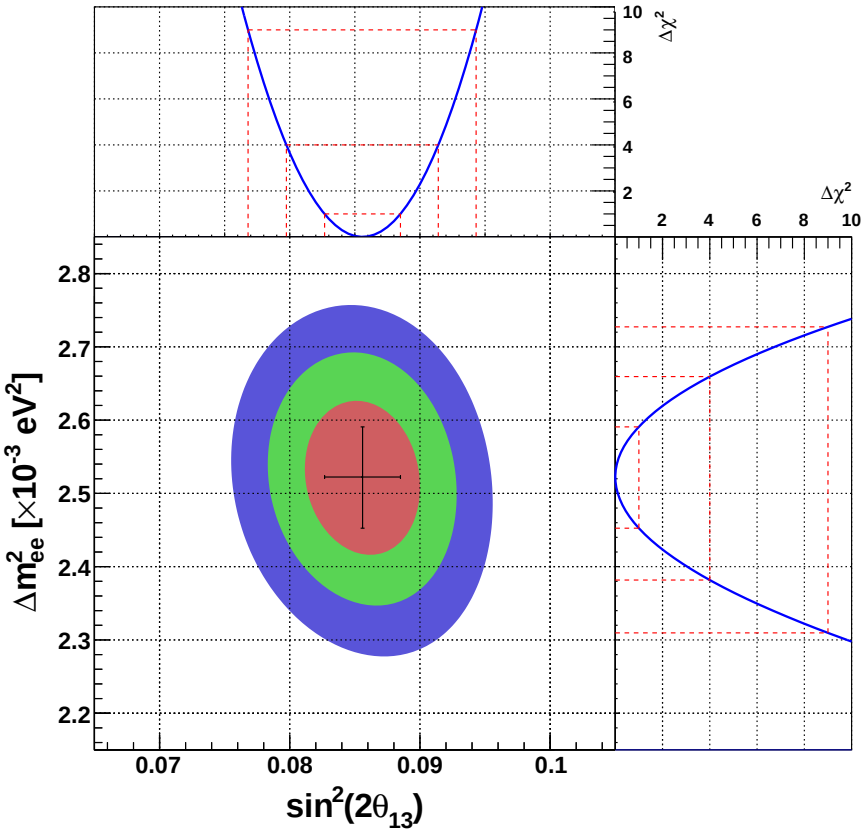
- The ratio of measured over the predicted flux shows an excess at 5 MeV.
- The excess is present in all experiments.
- The "bump" is time-independent and it is correlated with the reactor power.
- The near/far detector arrangement $\rightarrow \theta_{13}$ depends on the relative uncertainties.

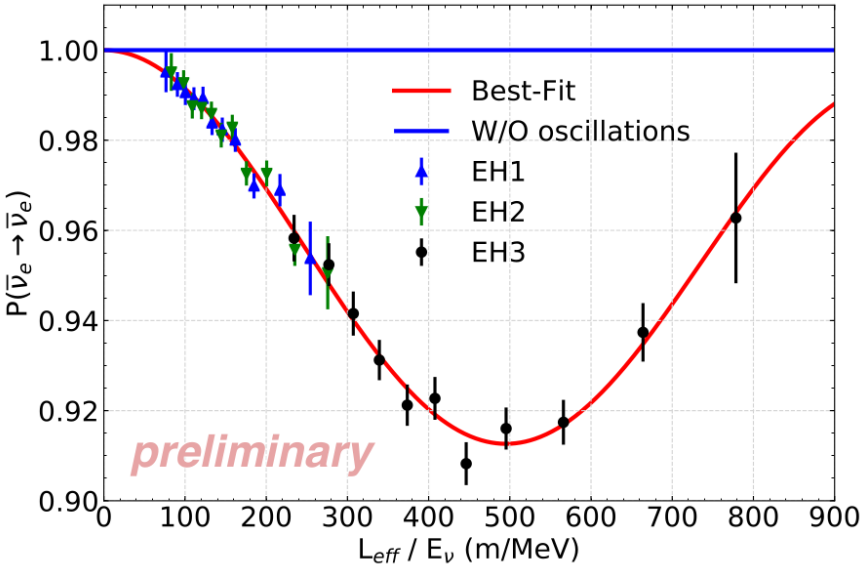


I. Tu (RENO), NEUTRINO 2018

Reactor neutrinos: determination of Δm^2_{31}

- A spectrum analysis determines Δm^2_{31}
- Near detector imposes an upper bound over Δm^2_{31}.
- The oscillations measured at the far detector impose lower bound on θ_{13} and Δm^2_{31}



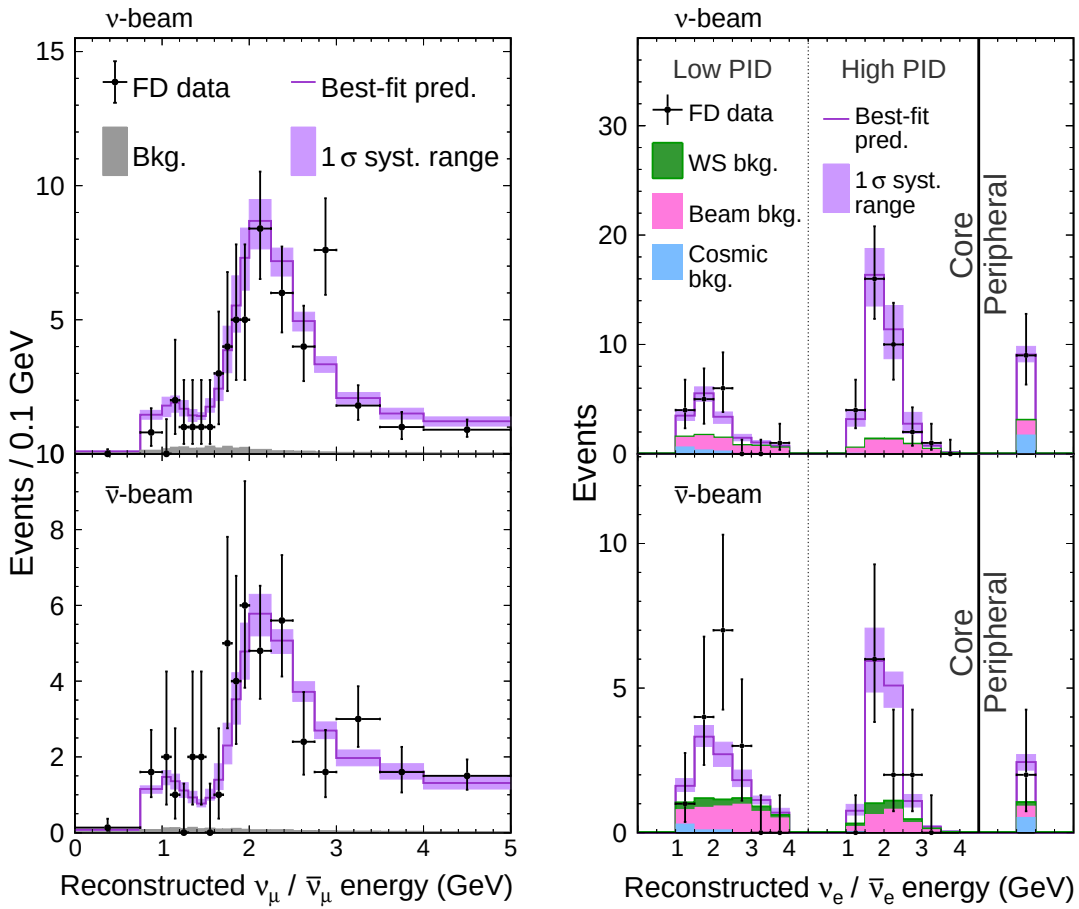


D. Adey et al. (Daya Bay) Phys.Rev.Lett. 121 (2018) J.P. Ochoa-Recoux (Daya Bay), NEUTRINO 2018

Long-baseline experiments

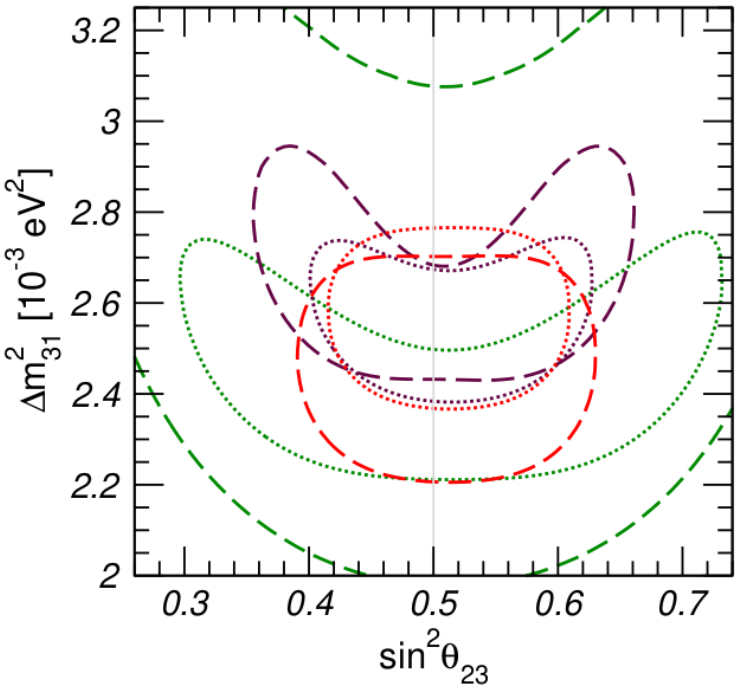
Long-baseline accelerators

- $\nu_e/\overline{\nu}_e$ and $\nu_\mu/\overline{\nu}_\mu$ with $E_\nu \in [0.6 - 7]$ GeV (T2K:$\sim$ 0.6 GeV, NOνA: $\sim$ 2 GeV, MINOS: $\sim$ 3 GeV, MINOS+: $\sim$ 7 GeV)
- The baseline is $\sim$ 100 km (T2K:$\sim$ 295 km, NOνA: $\sim$ 810 km, MINOS/MINOS+: $\sim$ 735 km)



M.A. Acero et al. (NOvA) Phys.Rev.Lett, 123 (2019)

Long-baseline accelerators



NuFIT 4.1 (2018)

$\nu_\mu \to \nu_\mu$ is senstive to $\Delta m^2_{\mu\mu}$ and $\theta_{\mu\mu}$ [1]

$$P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \frac{\Delta m^2_{\mu\mu} L}{4E}$$

where

$$\sin^2\theta_{\mu\mu} = \cos^2\theta_{13}\sin^2\theta_{23}$$

$$\Delta m^2_{\mu\mu} = \sin^2\theta_{12}\Delta m^2_{31} + \cos^2\theta_{12}\Delta m^2_{32} + \cos\delta_{cp}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}\Delta m^2_{21}$$

- $P_{\mu\mu}$ is symmetric around $\theta_{23} \sim 45$ (maximal mixing or not)
  - Discriminate between maximal mixing or not.
- $P_{\mu\mu}$ is sensitive to Δm^2_{31}

[1] Nunokawa, Parke and Funchal, Phys. Rev. D72 (2005) 013009

Long-baseline accelerators

ν_e apperance channel

$$P_{\nu_\mu \to \nu_e} \approx 4 \sin^2\theta_{13} \sin^2\theta_{23}(1+2oA) - C \sin\delta_{cp}(1+oA)$$
$$P_{\bar{\nu_\mu} \to \bar{\nu_e}} \approx 4 \sin^2\theta_{13} \sin^2\theta_{23}(1-2oA) + C \sin\delta_{cp}(1-oA)$$

JHEP 09 (2015) 016

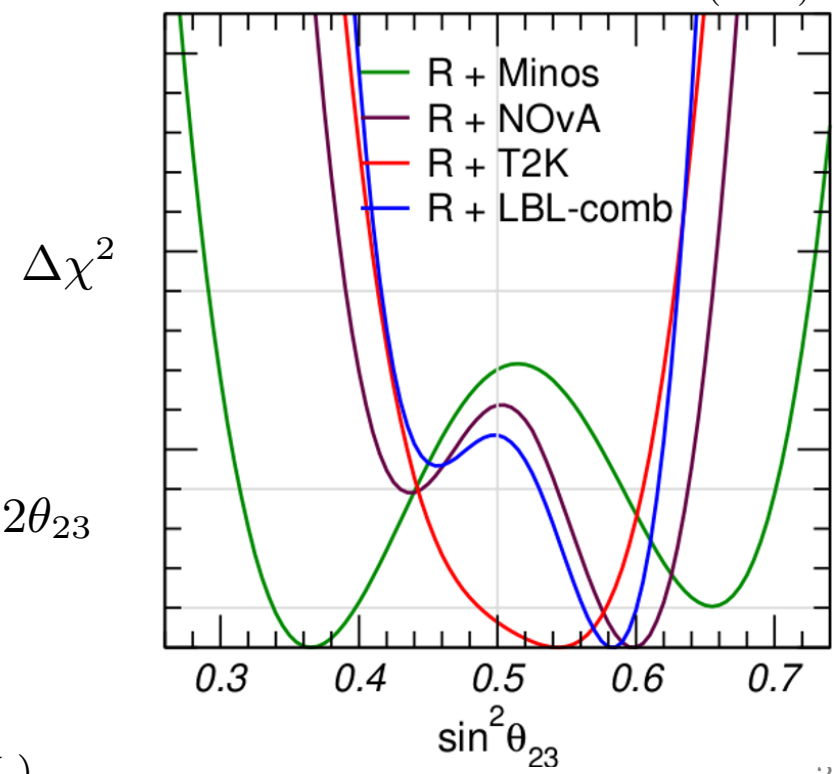
$\nu_\mu \to \nu_e$ is sensitive to

- the θ_{23} octant;
- mass hierarchy;
- δ_{cp};
- θ_{13}.

where

$$C = \Delta m^2_{21} L/4E \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}$$
$$o = sign(\Delta m^2_{31})$$
$$A = |2EV/\Delta m^2_{31}|$$

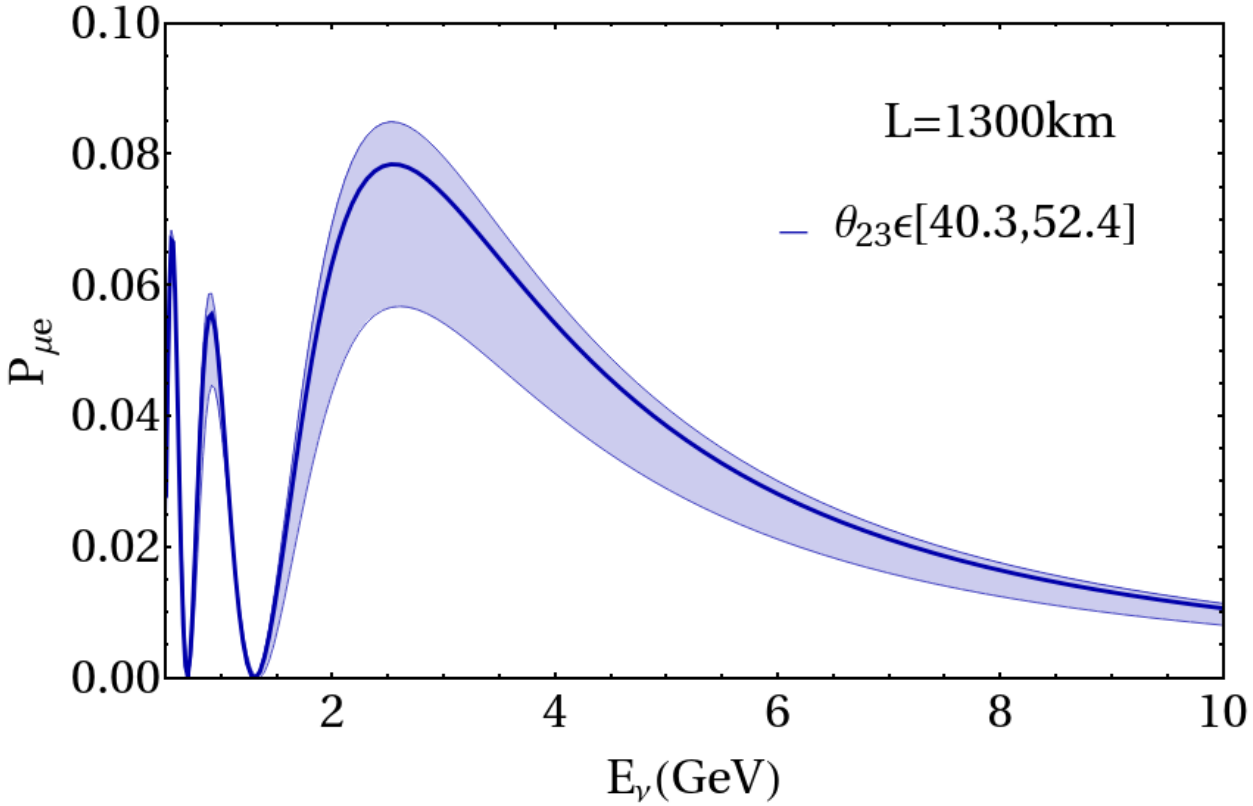
NuFIT 4.0 (2018)



Long-baseline accelerators

Impact of θ_{23}

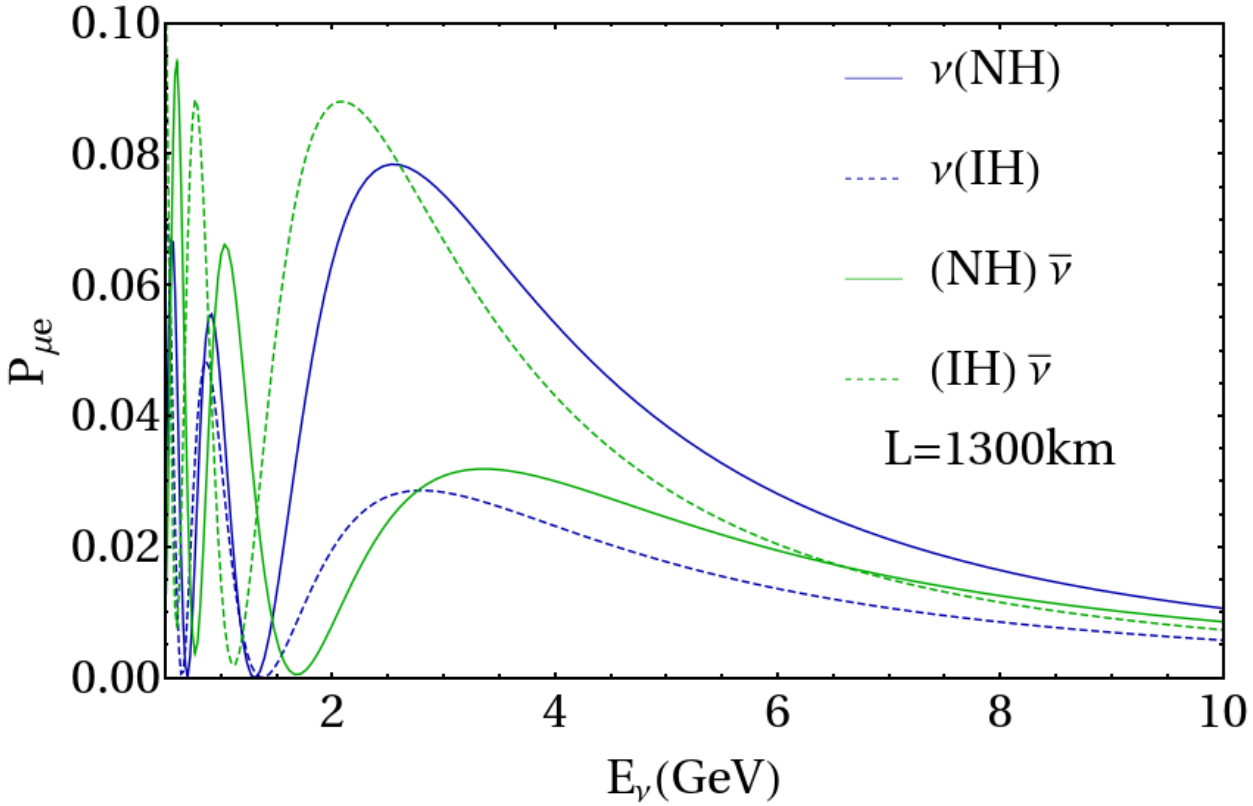
$$P_{\nu_\mu \to \nu_e} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 + 2oA) - C \sin \delta_{cp} (1 + oA)$$
$$P_{\bar{\nu_\mu} \to \bar{\nu_e}} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 - 2oA) + C \sin \delta_{cp} (1 - oA)$$



Long-baseline accelerators

Impact of mass hierarchy

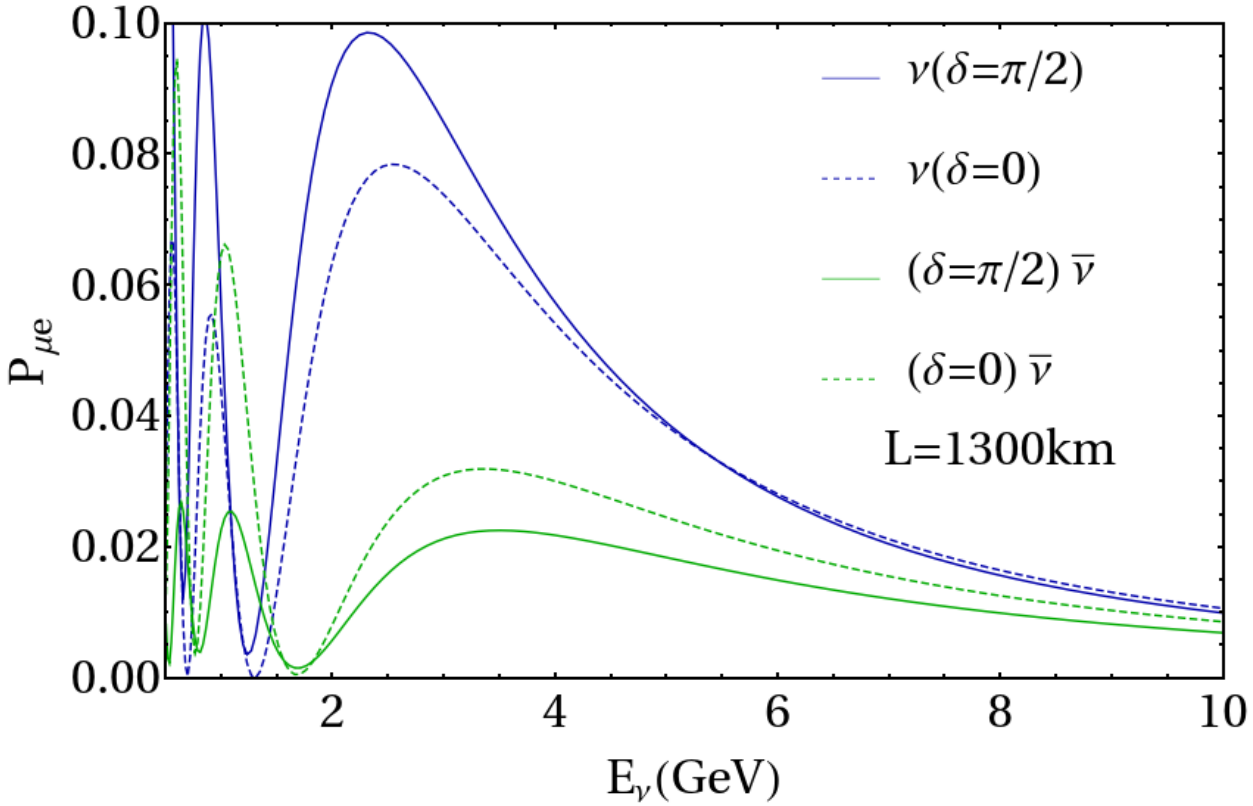
$$P_{\nu_\mu \to \nu_e} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 + 2oA) - C \sin \delta_{cp} (1 + oA)$$
$$P_{\bar{\nu_\mu} \to \bar{\nu_e}} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 - 2oA) + C \sin \delta_{cp} (1 - oA)$$



Long-baseline accelerators

Impact of δ_{cp}

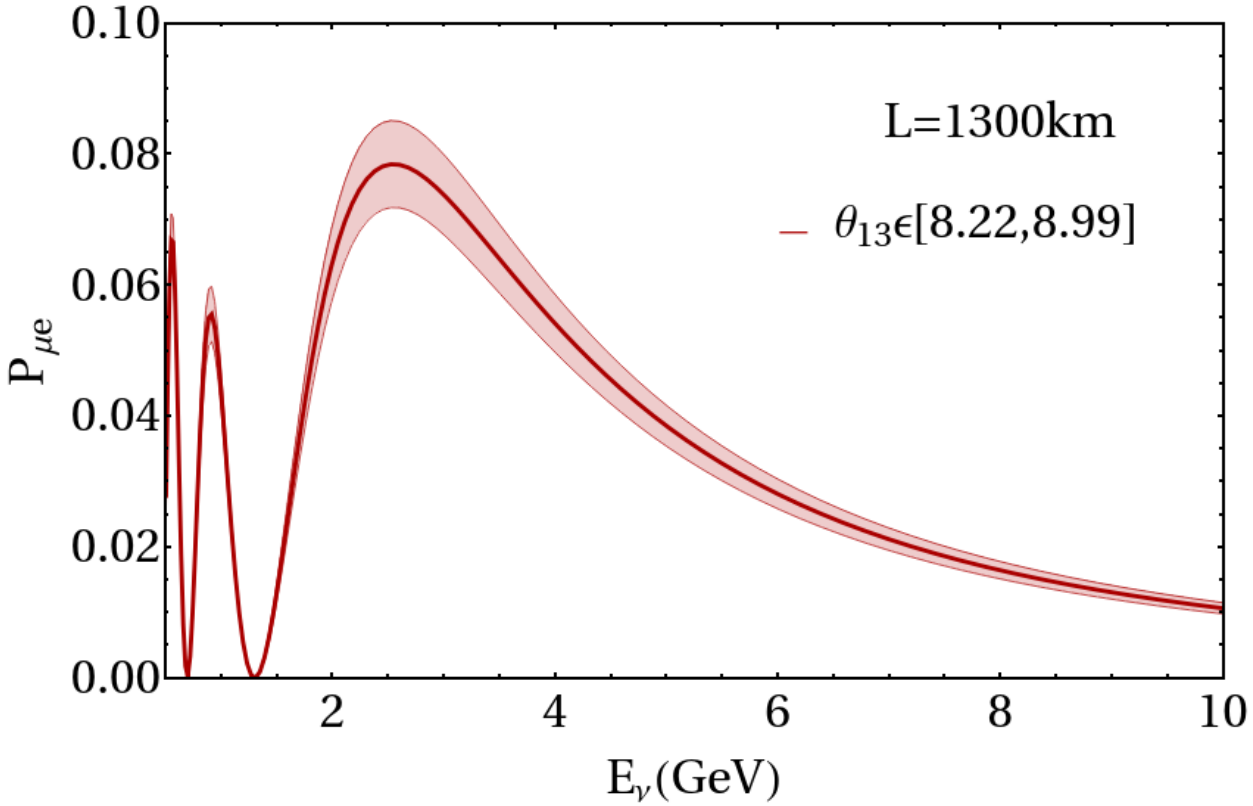
$$P_{\nu_\mu \to \nu_e} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 + 2oA) - C \sin \delta_{cp} (1 + oA)$$
$$P_{\bar{\nu_\mu} \to \bar{\nu_e}} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 - 2oA) + C \sin \delta_{cp} (1 - oA)$$



Long-baseline accelerators

Impact of θ_{13}

$$P_{\nu_\mu \to \nu_e} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 + 2oA) - C \sin \delta_{cp} (1 + oA)$$
$$P_{\bar{\nu_\mu} \to \bar{\nu_e}} \approx 4 \sin^2 \theta_{13} \sin^2 \theta_{23} (1 - 2oA) + C \sin \delta_{cp} (1 - oA)$$



Combining Reactor and LBL experiments

The degeneracies between the oscillation parameters are important once you combine all the available information.

Partial combination of the χ^2's

$$\Delta\chi^2(\delta_{CP}, \theta_{23}, \Delta m^2_{3l})_{\mathrm{LBL}+\theta_{13}} =$$
$$\min_{\theta_{13}} \left(\chi^2_{LBL}(\delta_{CP}, \theta_{23}, \Delta m^2_{3l}, \theta_{13}) + \min_{\Delta m^2_{3l}} \chi^2_{REA}(\Delta m^2_{3l}, \theta_{13}) \right)$$

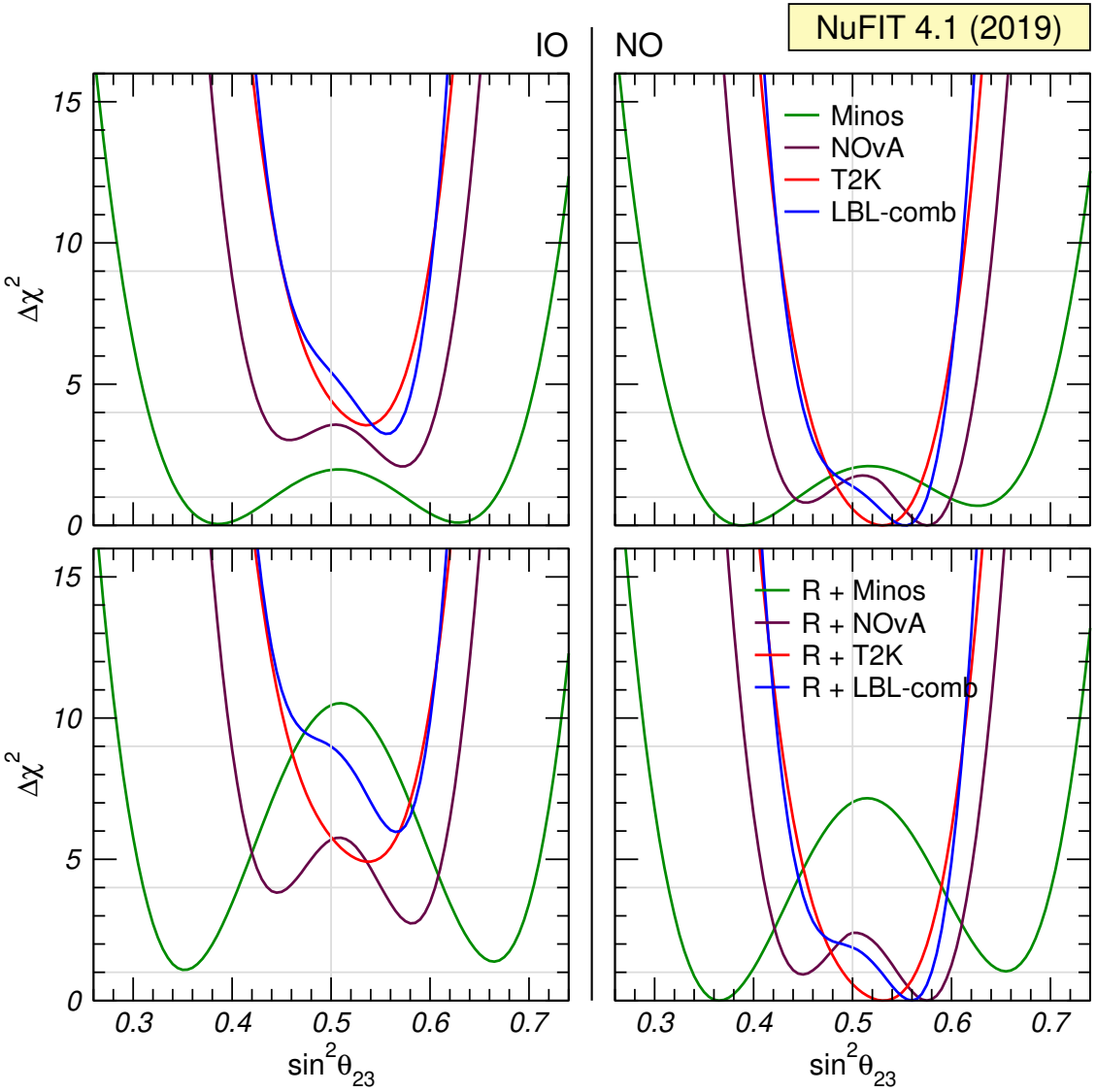
Full combination of the χ^2's

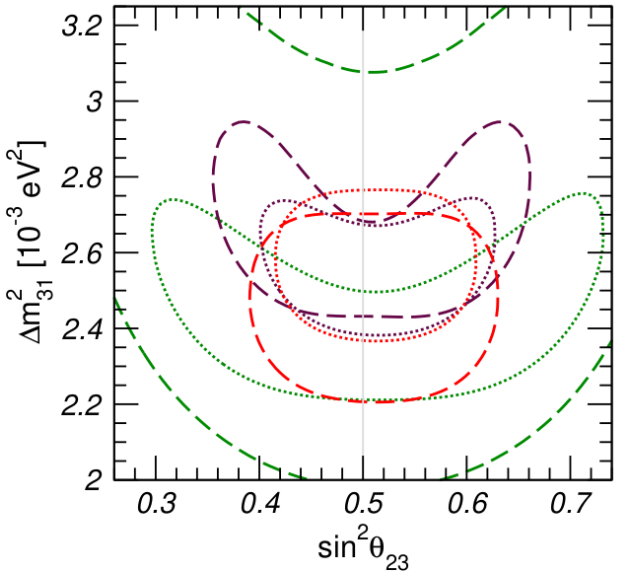
$$\Delta\chi^2(\delta_{CP}, \theta_{23}, \Delta m^2_{3l})_{\mathrm{LBL\ +\ REA}} =$$
$$\min_{\theta_{13}} \left(\chi^2_{LBL}(\delta_{CP}, \theta_{23}, \Delta m^2_{3l}, \theta_{13}) + \chi^2_{REA}(\Delta m^2_{3l}, \theta_{13}) \right)$$

We consider the impact on θ_{23}, Δm^2_{3l} and δ_{CP}

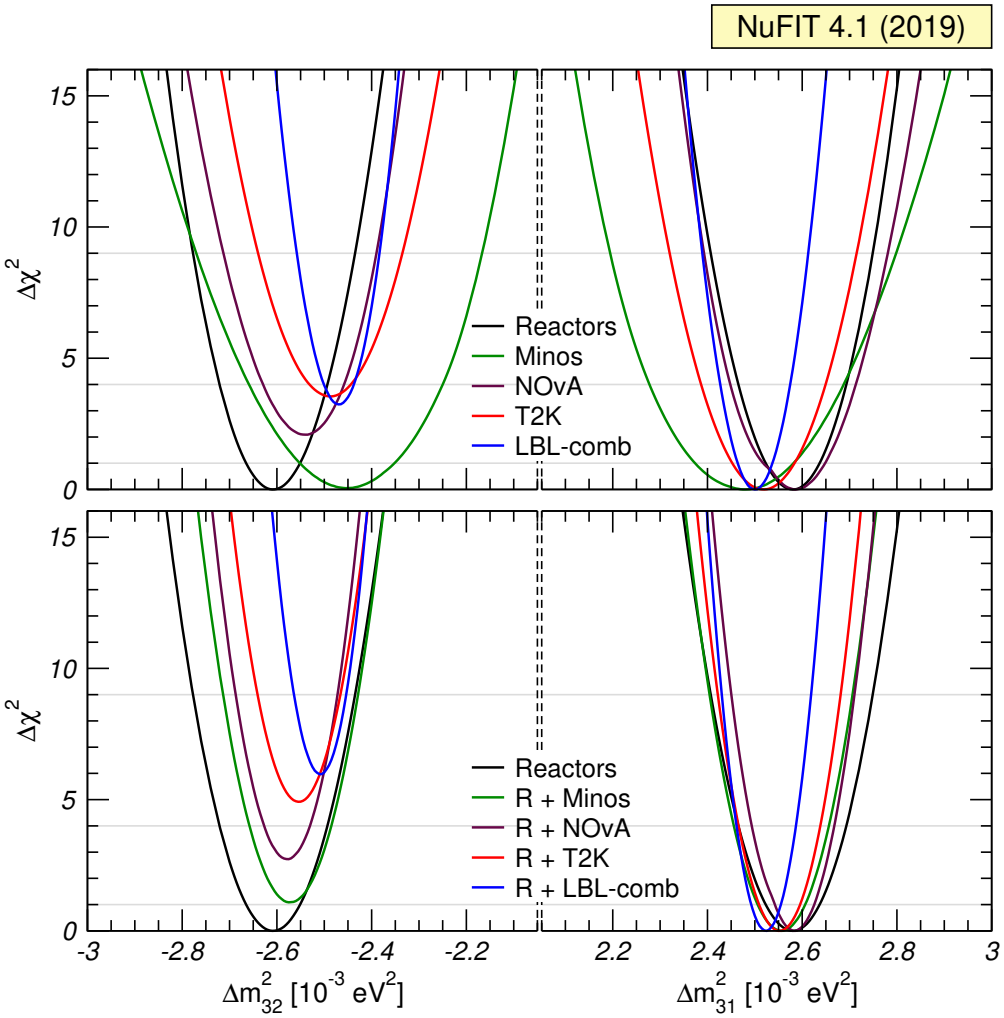
Combining Reactor and LBL experiments: θ_{23}

The preference for large values of Δm^2_{3l} by reactors experiments favor the deviation of maximal mixing.





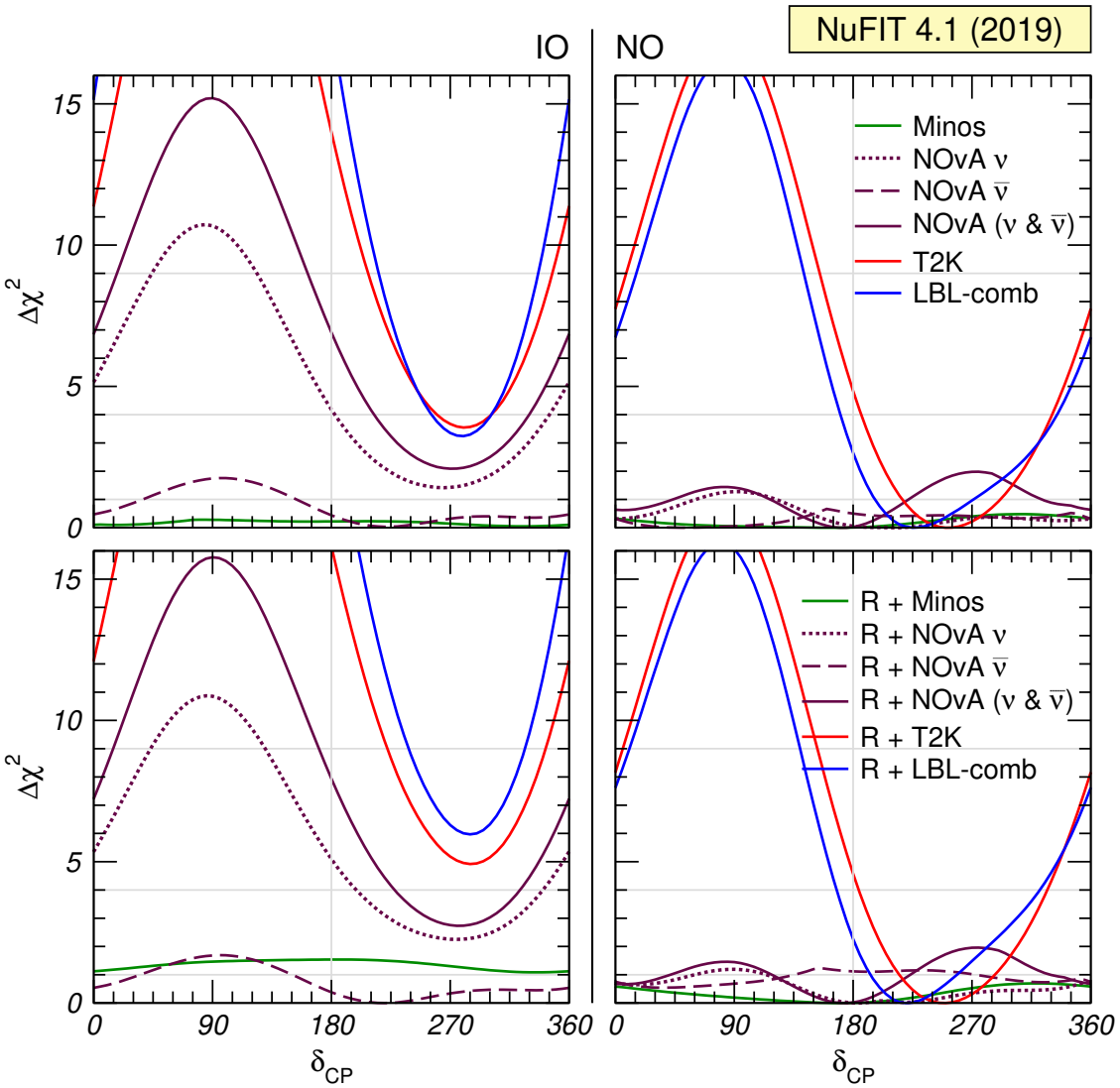
Combining Reactor and LBL experiments: Δm^2_{3l}



$$\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

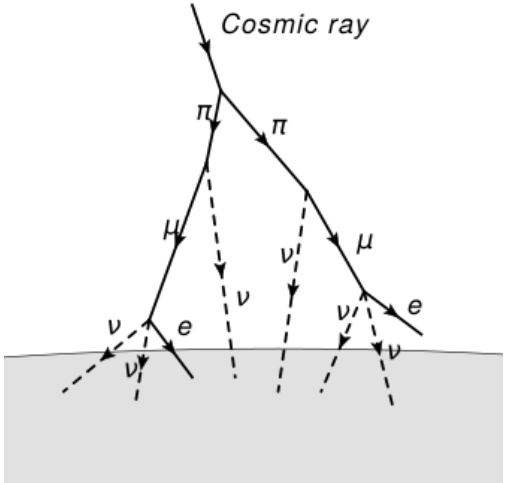
$$\Delta m^2_{\mu\mu} = \sin^2\theta_{12}\Delta m^2_{31} + \cos^2\theta_{12}\Delta m^2_{32}$$
$$+ \cos\delta_{cp}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}\Delta m^2_{21}$$

Combining Reactor and LBL experiments: δ_{cp}



Atmospheric Neutrinos

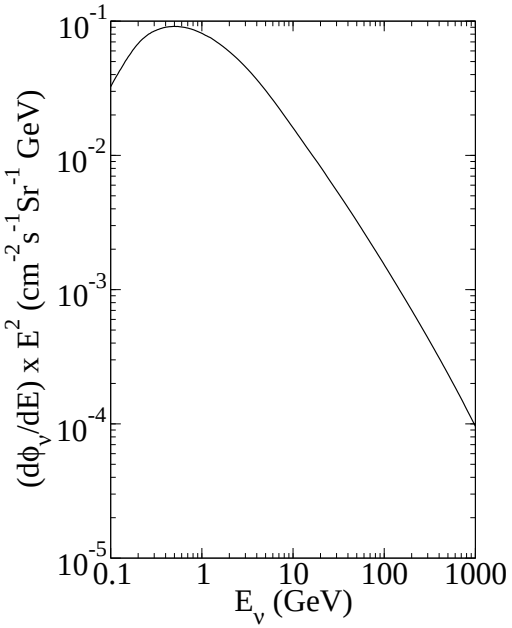
Atmospheric neutrinos



- Created in the collisions of cosmic rays with the atmosphere.

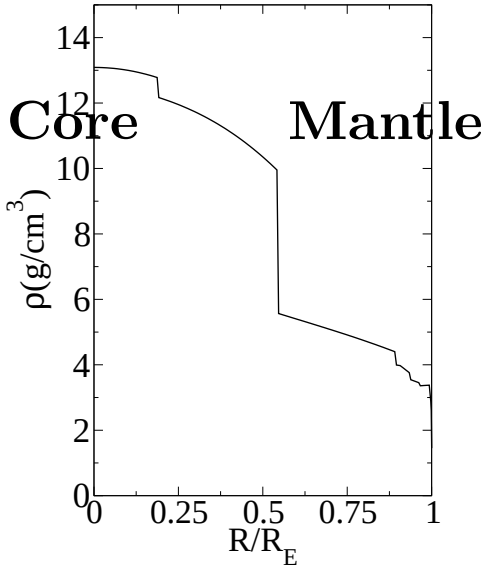
$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\overline{\nu_{\mu}})$$
$$K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\overline{\nu_{\mu}})$$
$$\mu^{\pm} \rightarrow e^{\pm} + \nu_{e}(\overline{\nu_{e}}) + \nu_{\mu}(\overline{\nu_{\mu}})$$

- Atmospheric flux covers a wide range of energies ($\phi_{\nu} \sim E_{\nu}^{-2.7}$).
- The main uncertainties are parametrized by:
  - The normalization.
  - A energy dependence ($\phi_{\nu} \sim \phi_0(E/E_0)^{\gamma}$)
  - The relative contribution of π and K ($R_{\pi/K}$)
  - The ratio between the neutrino and the antineutrino flux ($\phi_{\nu}/\phi_{\bar{\nu}}$)

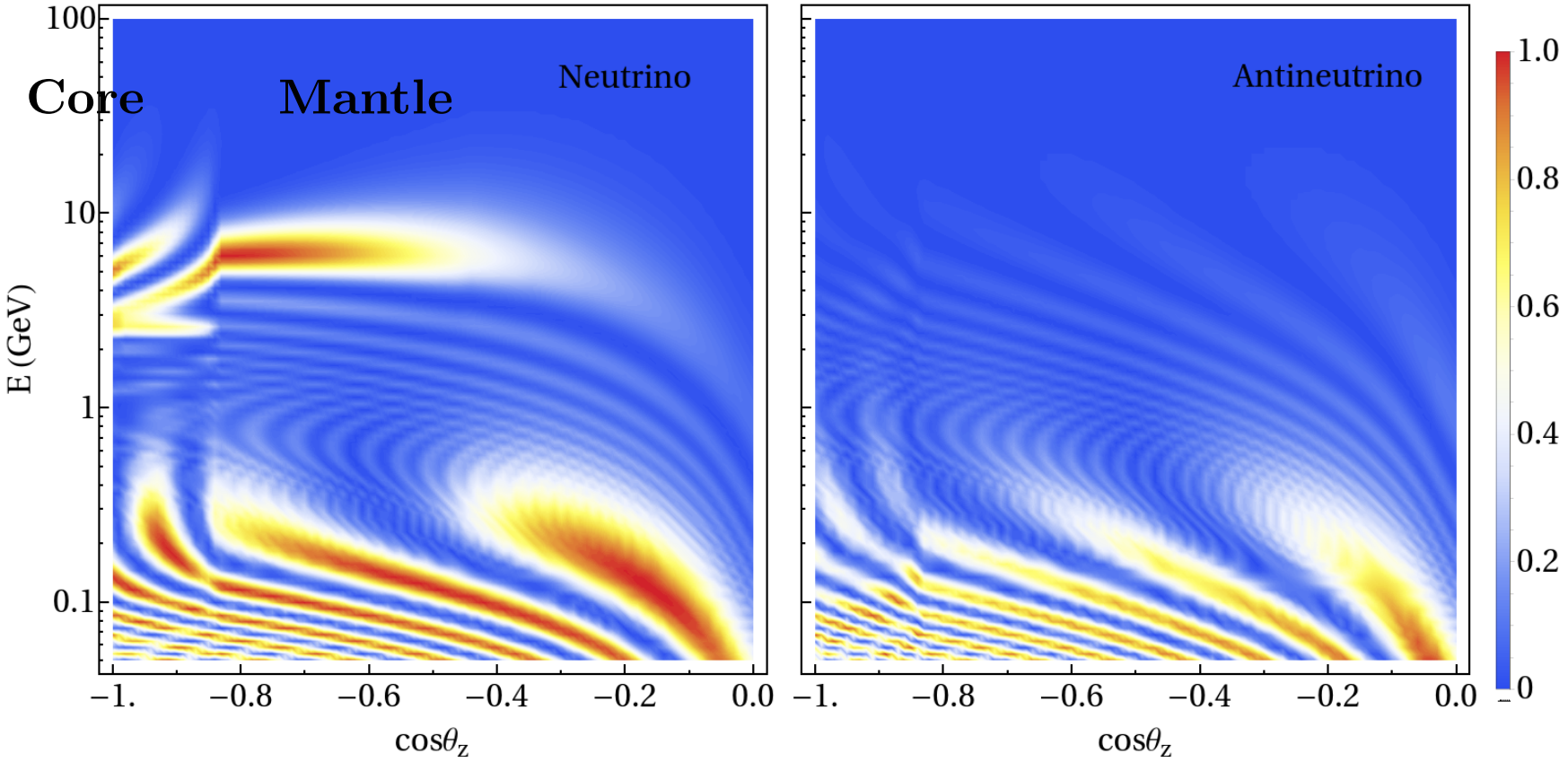


Phys. Rev. D92, 023004 (2015)

Flavor oscillations in the Earth



The Earth matter potential is non-constant.

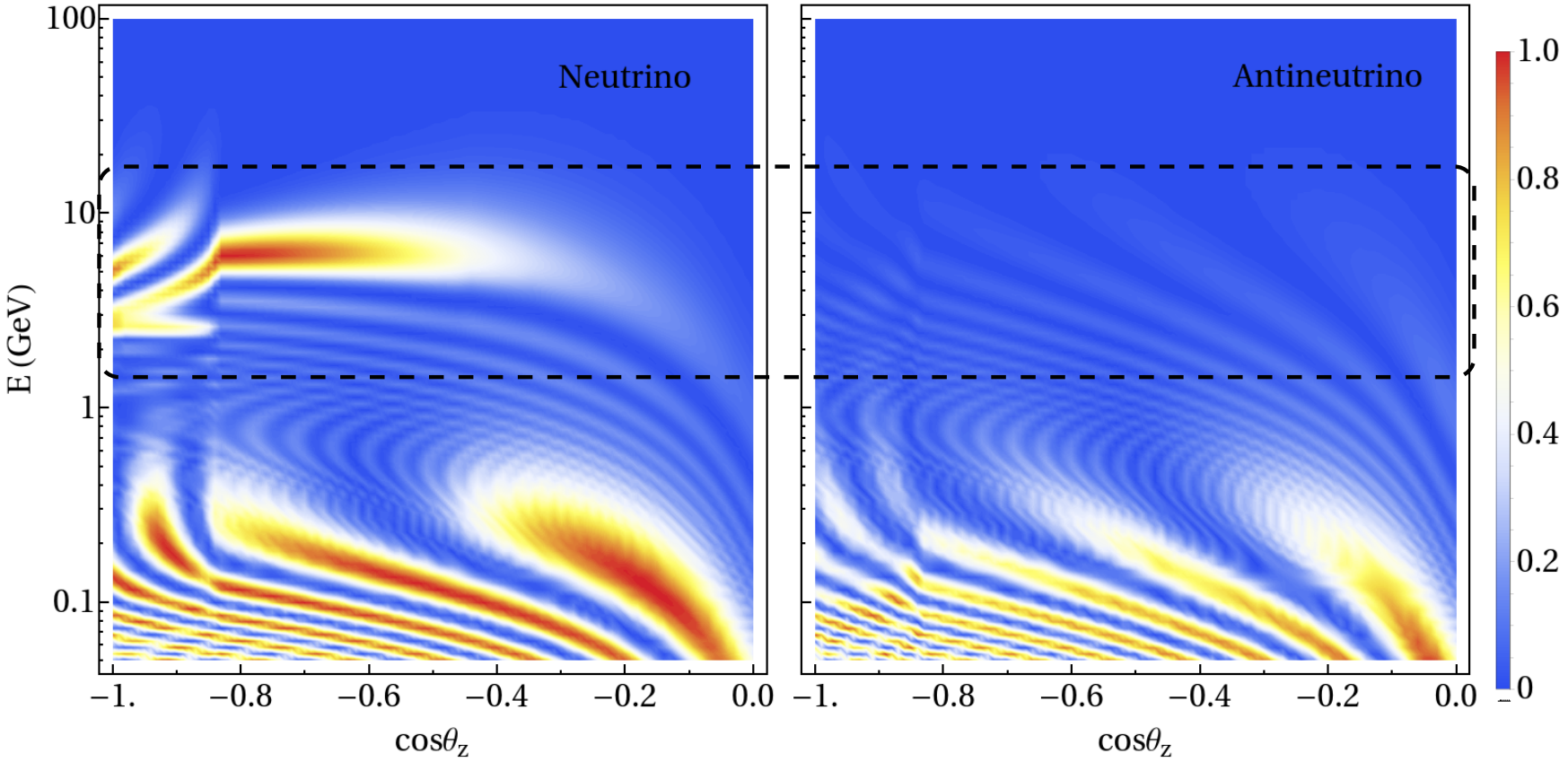


Flavor oscillations in the Earth

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2EV}$$

For 1-3 mixing

- MSW resonance in the mantle due to Δm^2_{atm} at $E_\nu \sim 6$ GeV
- The oscillation length at the resonance is much larger than $R_\oplus$

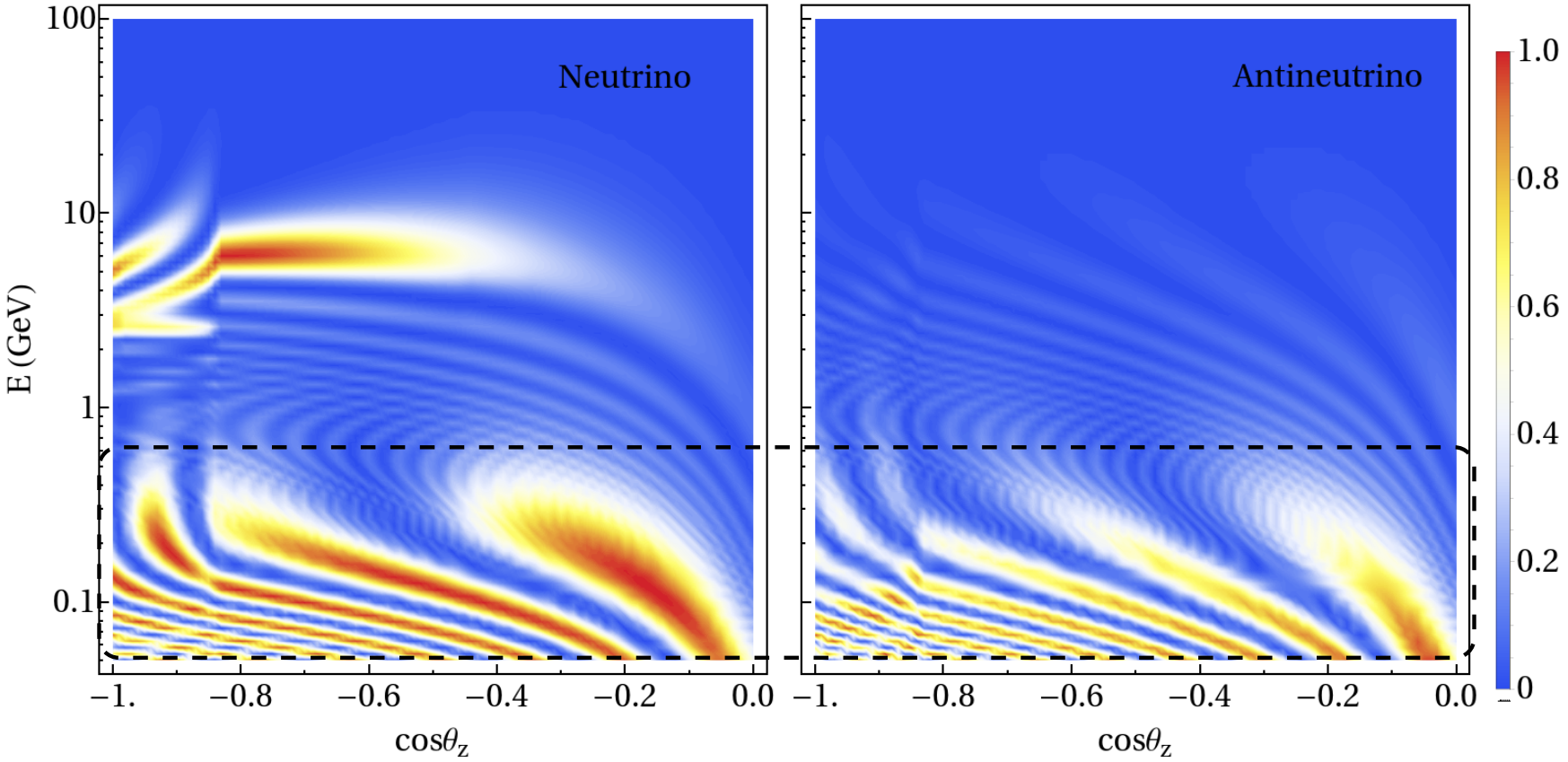


Flavor oscillations in the Earth

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2EV}$$

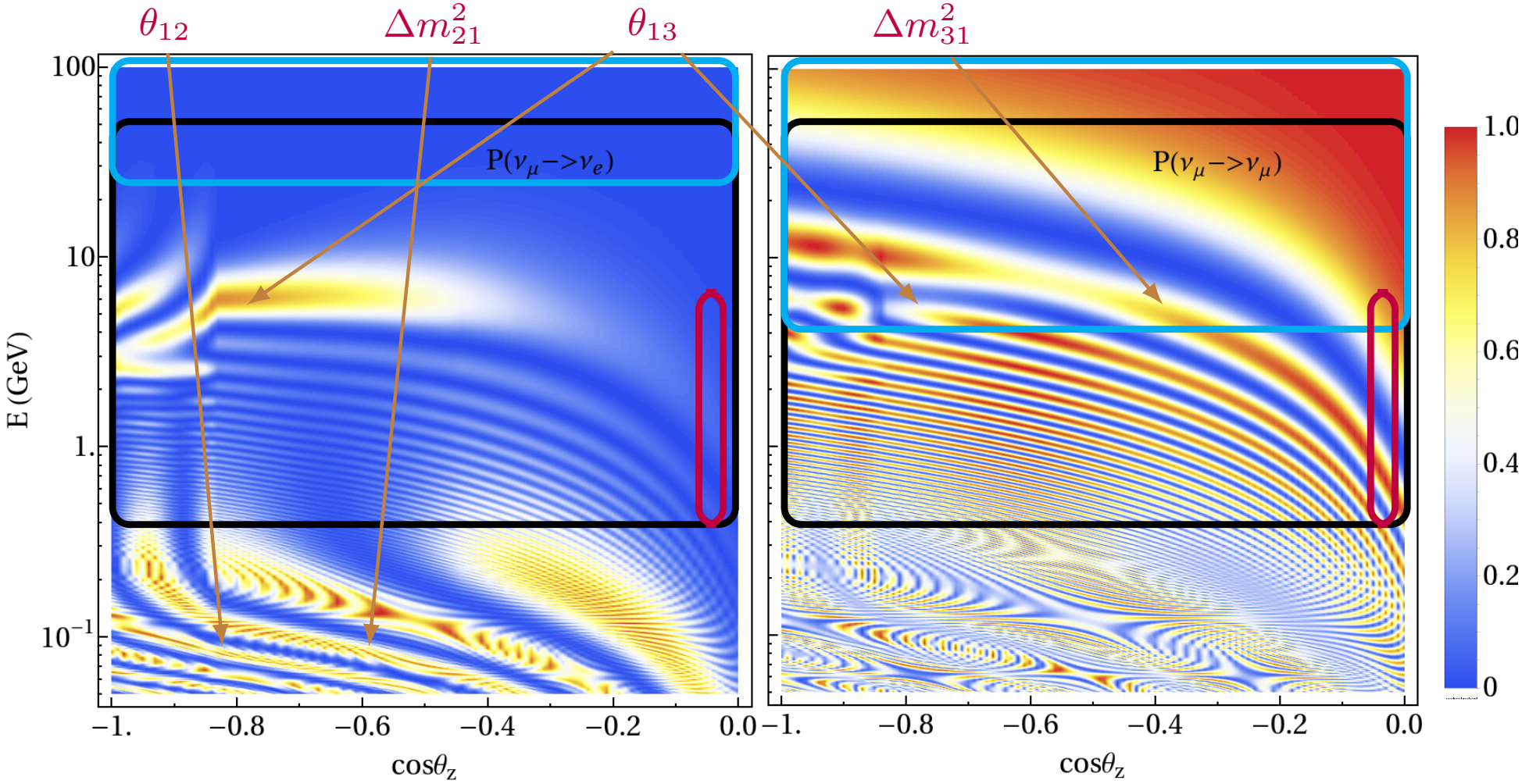
For 1-2 mixing

- 3 MSW resonances in the mantle due to Δm^2_{sol} at $E_\nu \sim 0.1$ GeV
- The oscillation length at the resonance is $L \sim R_\oplus/4$



Atmospheric neutrinos: contribution to the global fit

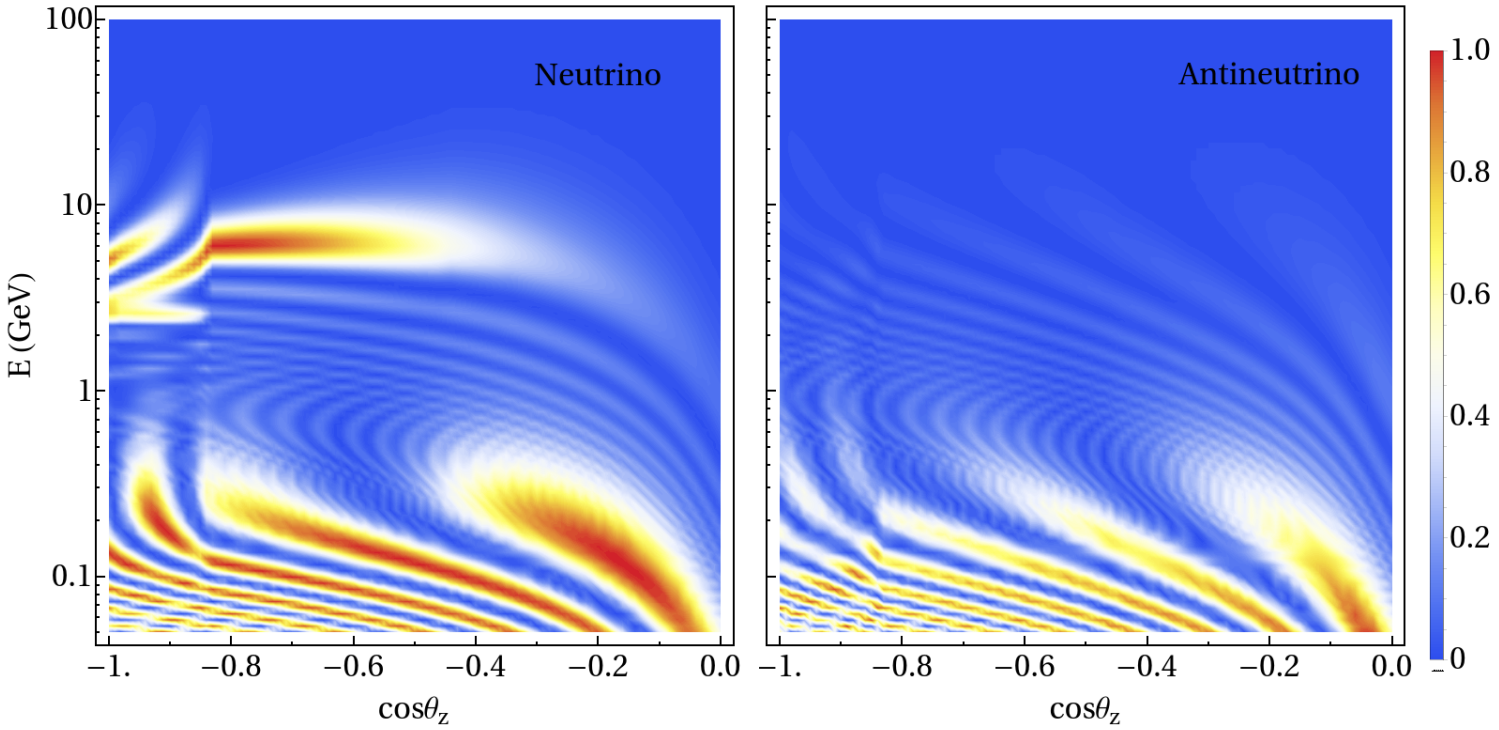
- For $E_\nu \geq 0.5$ GeV and the earth baselines (10^2 - 10^4) km $\Delta m^2_{21} L/E_\nu$ has a subleading effect.
- Atmospheric neutrinos are sensitive to Δm^2_{31}, θ_{23}, θ_{13} and δ_{cp}.



SK IceCube/DeepCore LBL accelerator

Atmospheric neutrinos: mass hierarchy determination

- 1-3 resonance for ν crossing the Earth with energies $E_\nu \in [2-10]$ GeV.
- For NO (IO) there is a resonance in the ν-channels ($\overline{\nu}$-channels).
- Atmospheric experiment cannot distinguish ν from $\overline{\nu}$
  - Cherenkov radiation.
- The number of events contains a contribution of $\nu + \overline{\nu}$.
  - The neutrino contribution is four times bigger
- Statistical determination of the mass hierarchy.

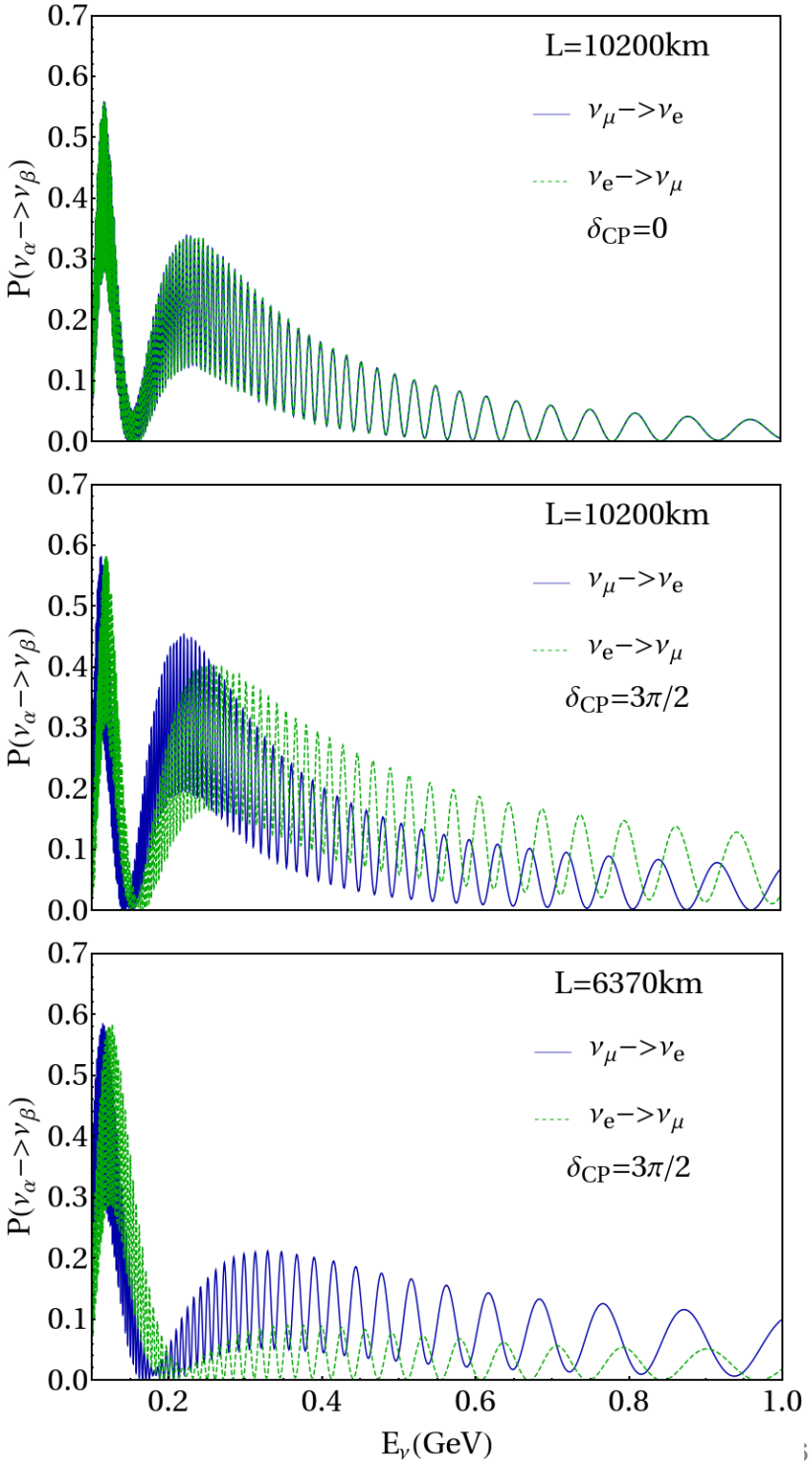


Atmospheric neutrinos: CP-violation phase determination

- For $E < 1$ GeV, the CP-violation term is enhanced due to the development Δm^2_{21}

$$P_{CP} = -8 J_{CP}^{max} \sin\delta_{CP} \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32}$$

- Several baselines contribute to the measurement of δ_{cp}



Kelly, Machado, IMS, Parke and Perez-Gonzalez, Phys.Rev.Lett. 123 (2019) 8, 081801 [1904.02751]

IMS and Minakata PTEP 2019 (2019) 7, 073B07 [1904.07853]

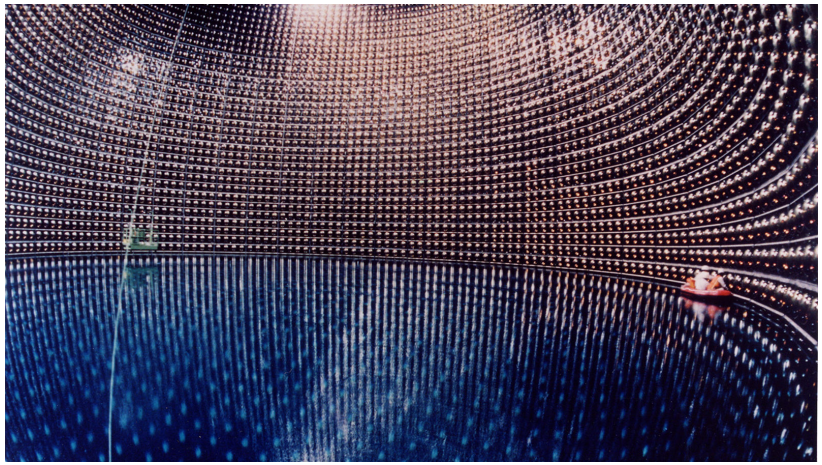
Super-Kamiokande and DeepCore

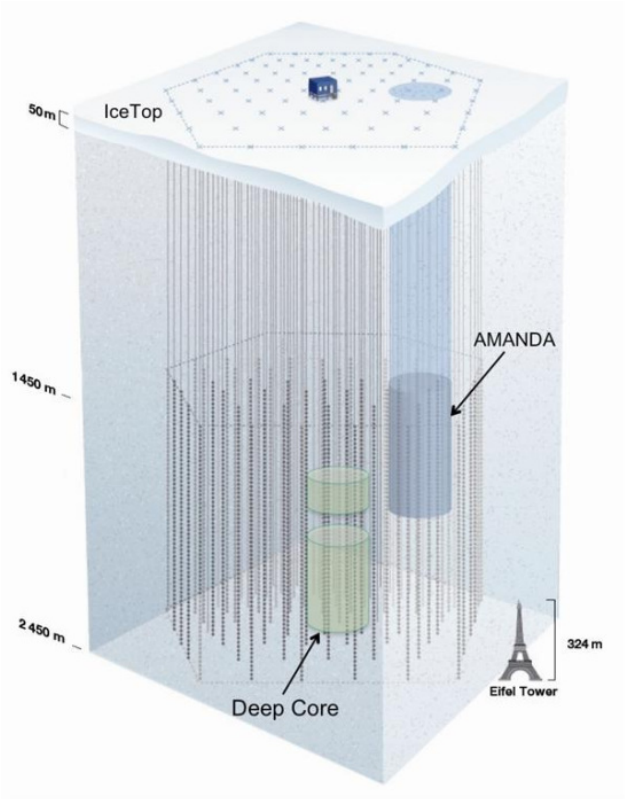
Super-Kamiokande:

- 50 kton of ultra pure water
- Measure neutrinos from sub-GeV to multi-GeV

DeepCore:

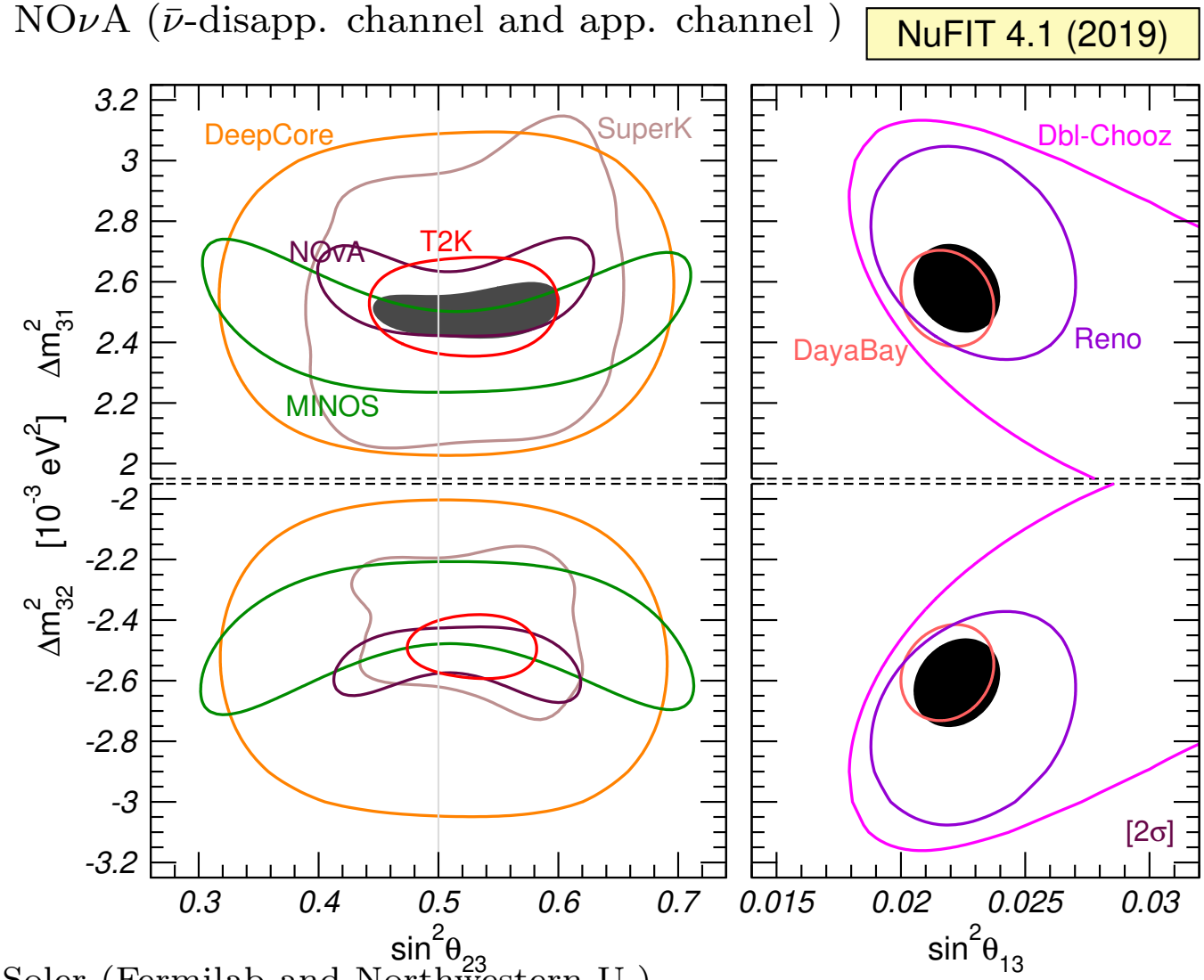
- ~ 10 Mton of ice
- Measure neutrinos from $\sim 5 - 10$ GeV to 100 GeV





Determination of θ_{23} and Δm^2_{31}

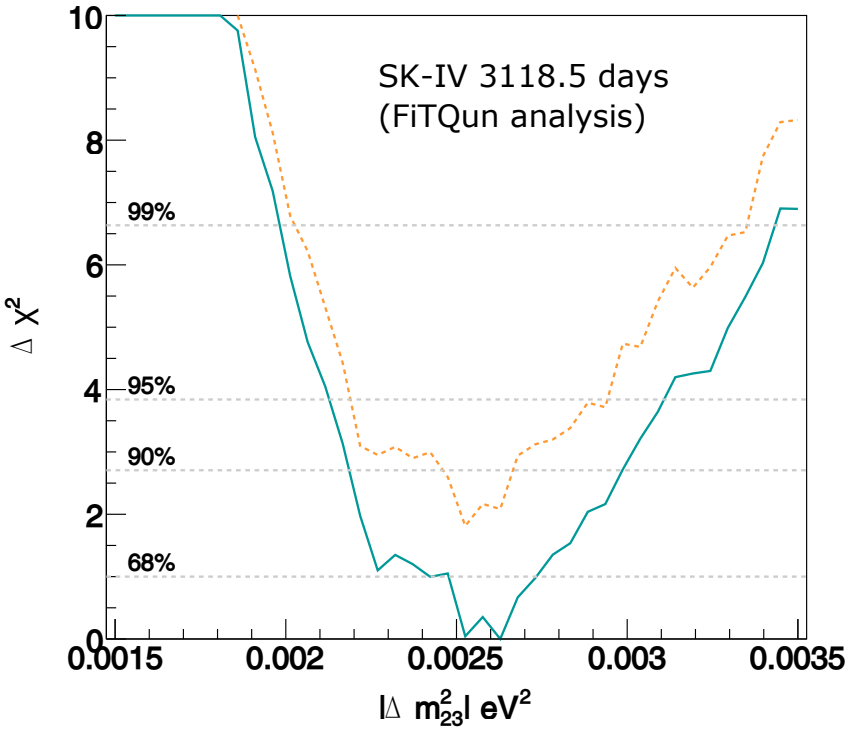
- Preference for the second octant of θ_{23}
- Maximal mixing is disvafor by:
  - T2K (app. channel)
  - NOνA ($\bar{\nu}$-disapp. channel and app. channel)

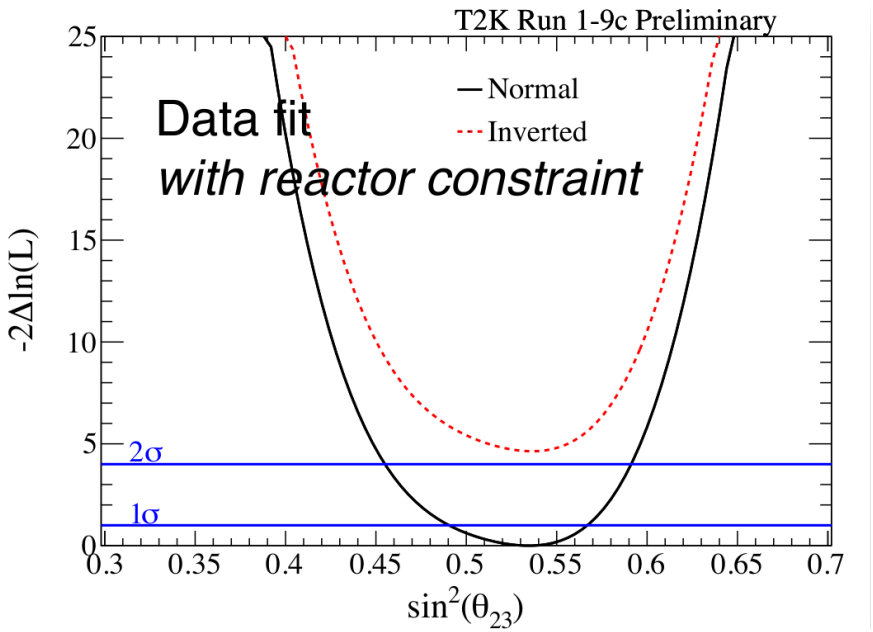


Determination of mass hierarchy

Preference for NH $\sim 3\sigma$.

- SK (SK + T2K) (bounds θ_{13} from reactors): favor NH $\sim 2.1\sigma (\sim 2.3\sigma)$
- T2K (adding θ_{13} from reactors): NH favored $\sim 2\sigma$
- NOνA: NH favored $\sim 1.3\sigma$
- MINOS + MINOS+: very weak preference for NH $\sim 0.6\sigma$.





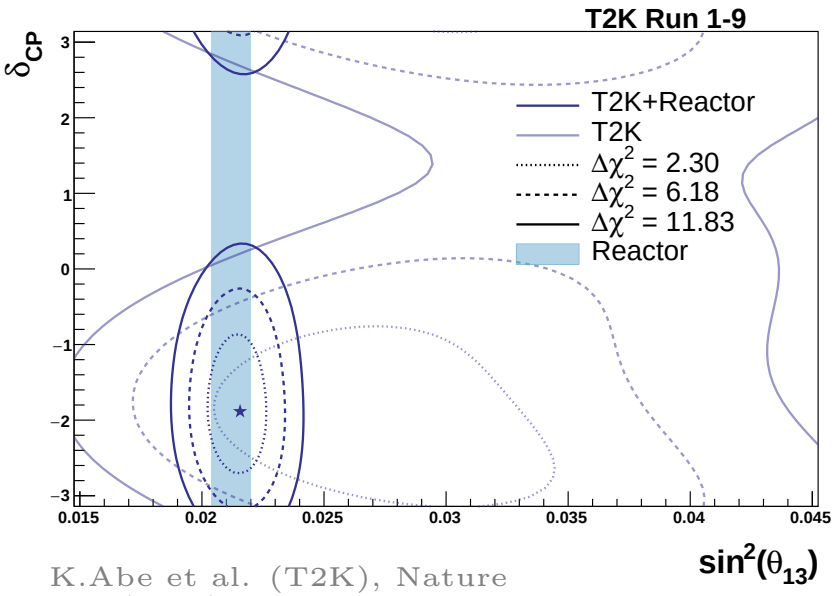
M. Wascko (T2K), NEUTRINO 2018

K. Abe et al. (Super-Kamiokande), PTEP 2019 (2019) 5

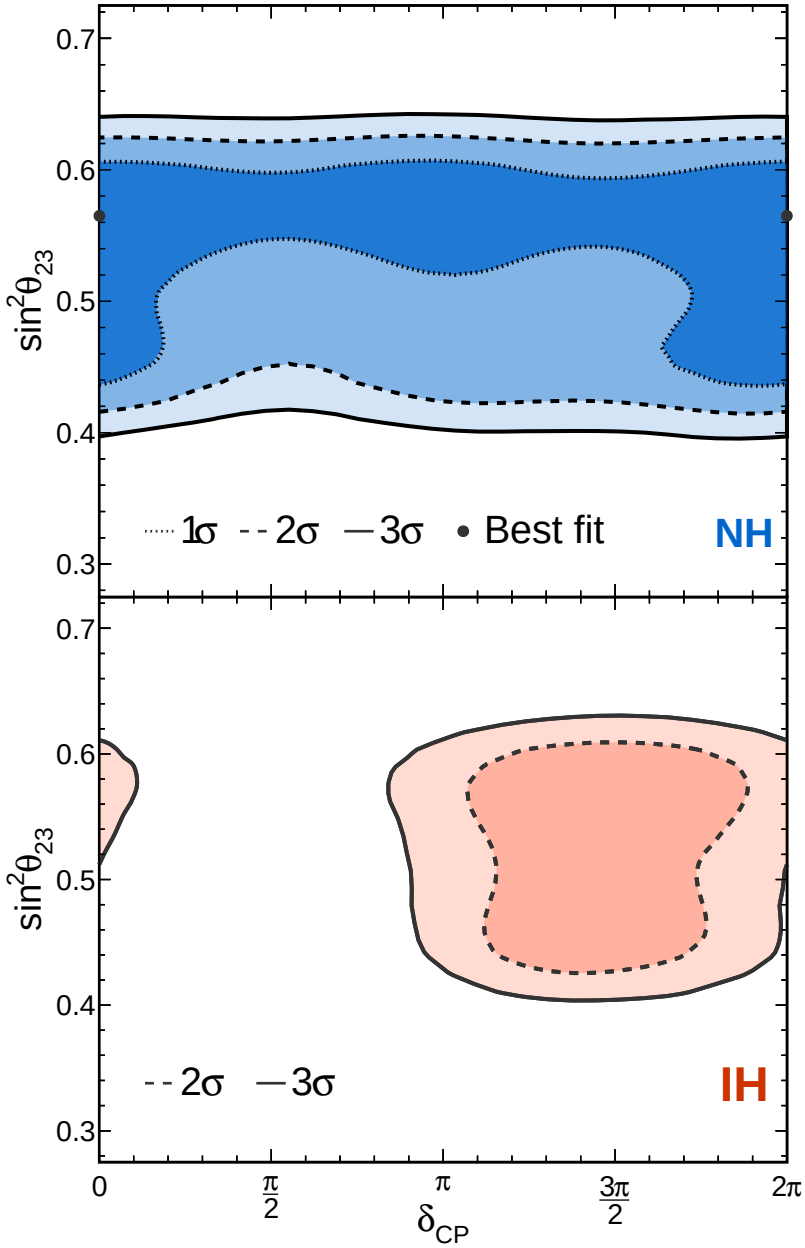
Determination of CP-violation phase

M.A. Acero et al. (NOvA), Phys.Rev.Lett. 123 (2019) 15

- The best-fit of LBL shows $\delta_{cp} = 215°$
  - T2K and NOνA (IO) prefer maximal CP violation ($\delta_{CP} \sim 270°$)
  - NOνA (NO) prefer $\delta_{CP} \sim 30°$
- SK favor $\delta_{cp} = 270°$



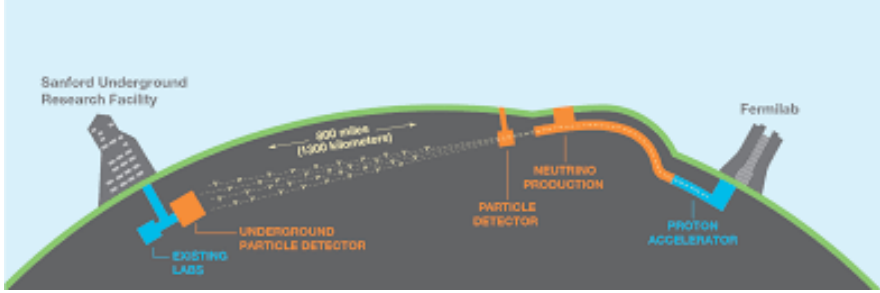
K.Abe et al. (T2K), Nature 580 (2020) 7803



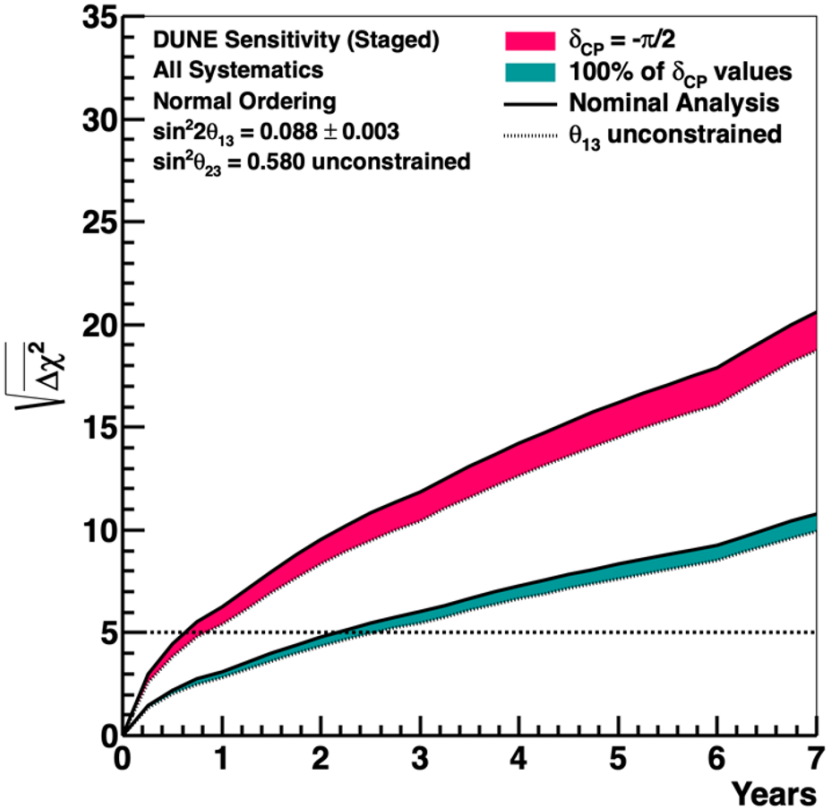
Sub-GeV Atmospheric Neutrinos and CP-Violation in DUNE

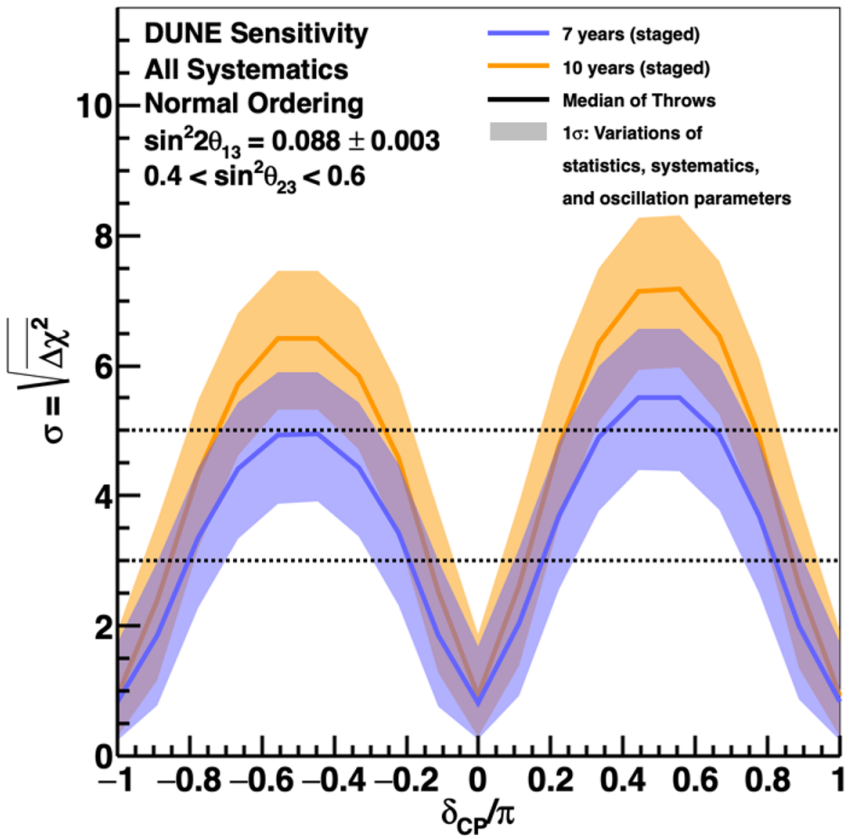
Based on: Kelly, Machado, IMS, Parke and Perez-Gonzalez, Phys.Rev.Lett. 123 (2019) 8, 081801 [1904.02751]

DUNE

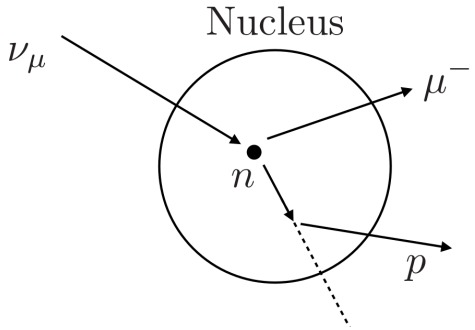


- Precision oscillation parameters (δ_{cp}, Δm^2_{31}, θ_{23})
- Supernova neutrinos
- Solar neutrinos, DSNB
- BSM

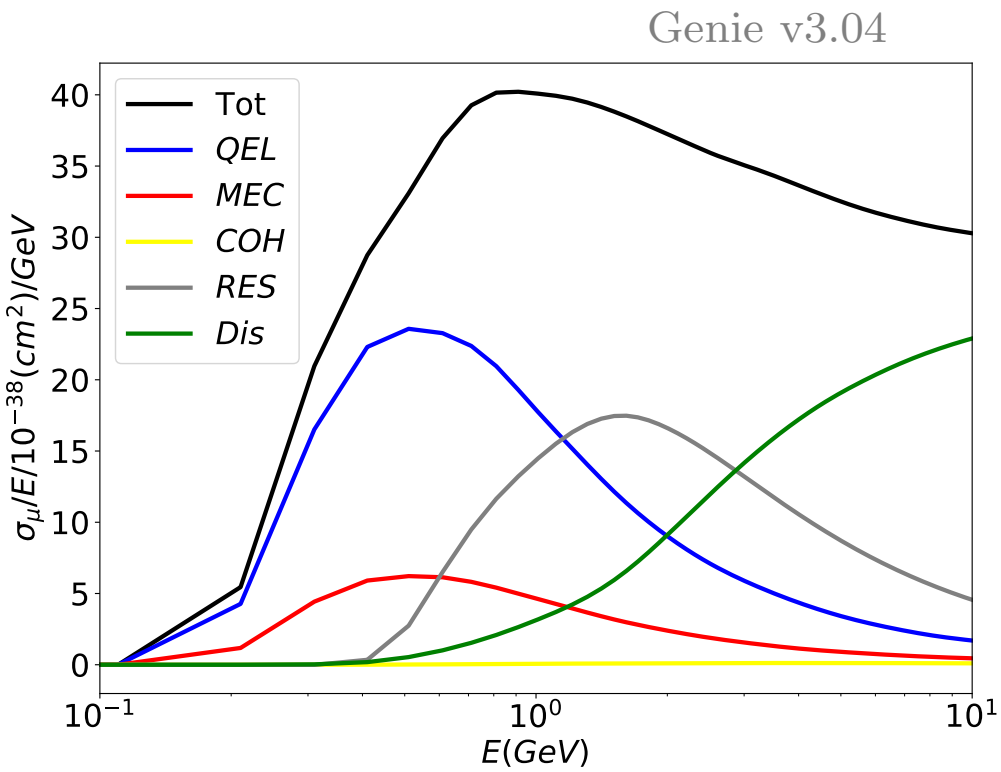




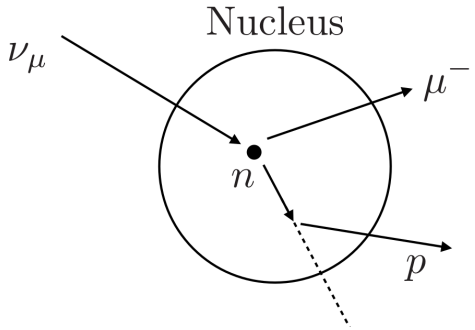
Sub-GeV Atmospheric neutrinos in DUNE



- Atmospheric neutrinos with $E \leq 1$ GeV
- Neutrino interaction is dominated by quasielastic scattering.



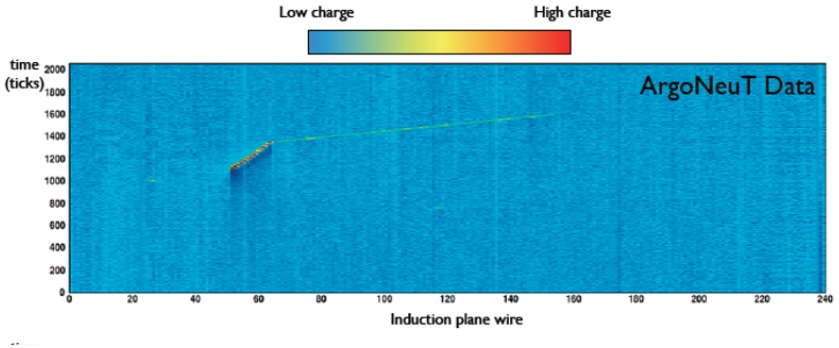
Sub-GeV Atmospheric neutrinos in DUNE



LArTPCs excellent capabilities in the reconstruction of charged particles

- Neutrino direction reconstructed from the event topology

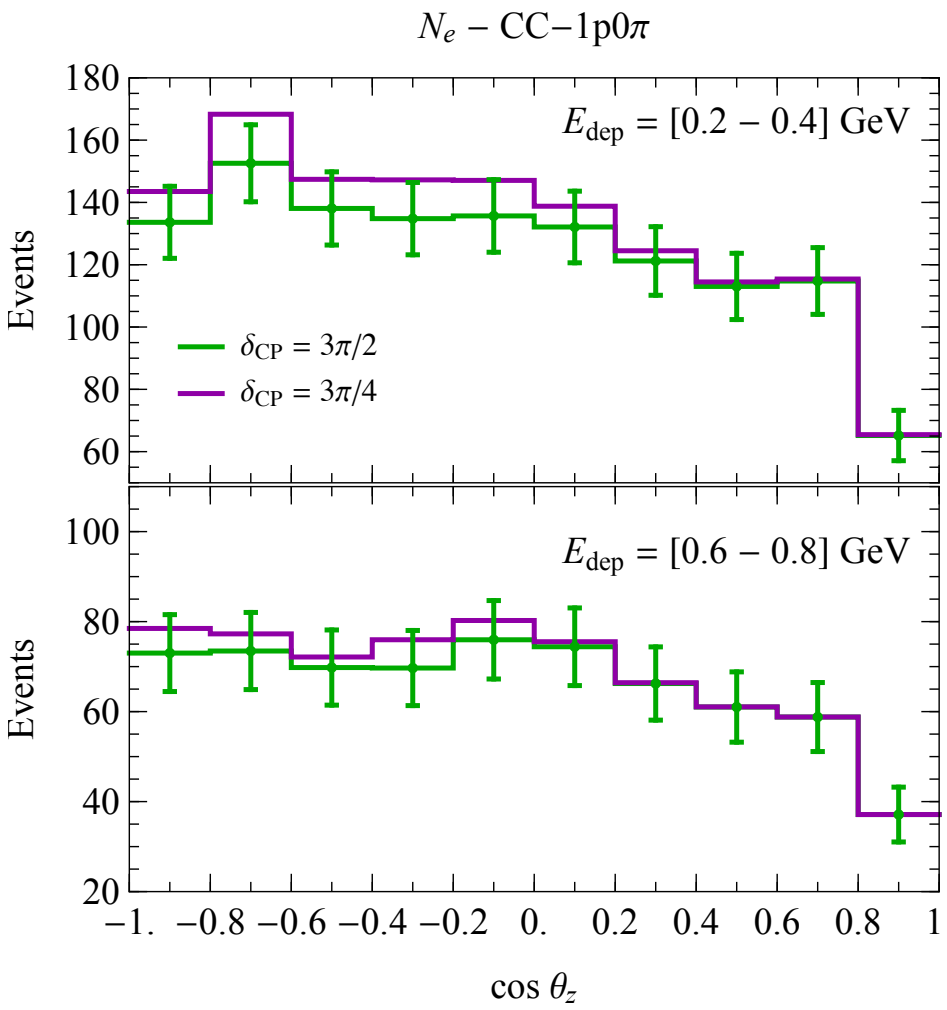
We consider events topologies with 1 charged lepton and up to 2 outgoing protons (CC-Np0π)



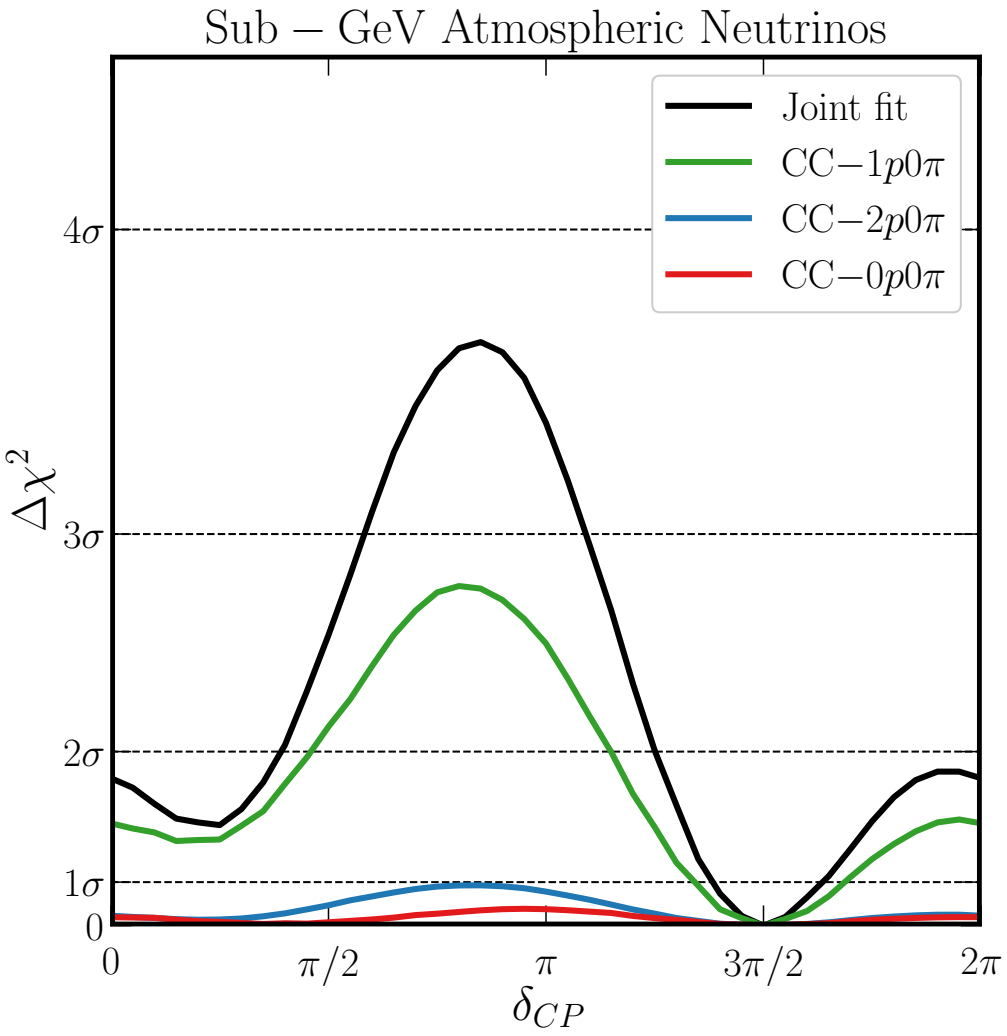
C. Anderson et al. (ArgoNeuT), JINST 7 (2012) P10019

Sub-GeV Atmospheric neutrinos in DUNE

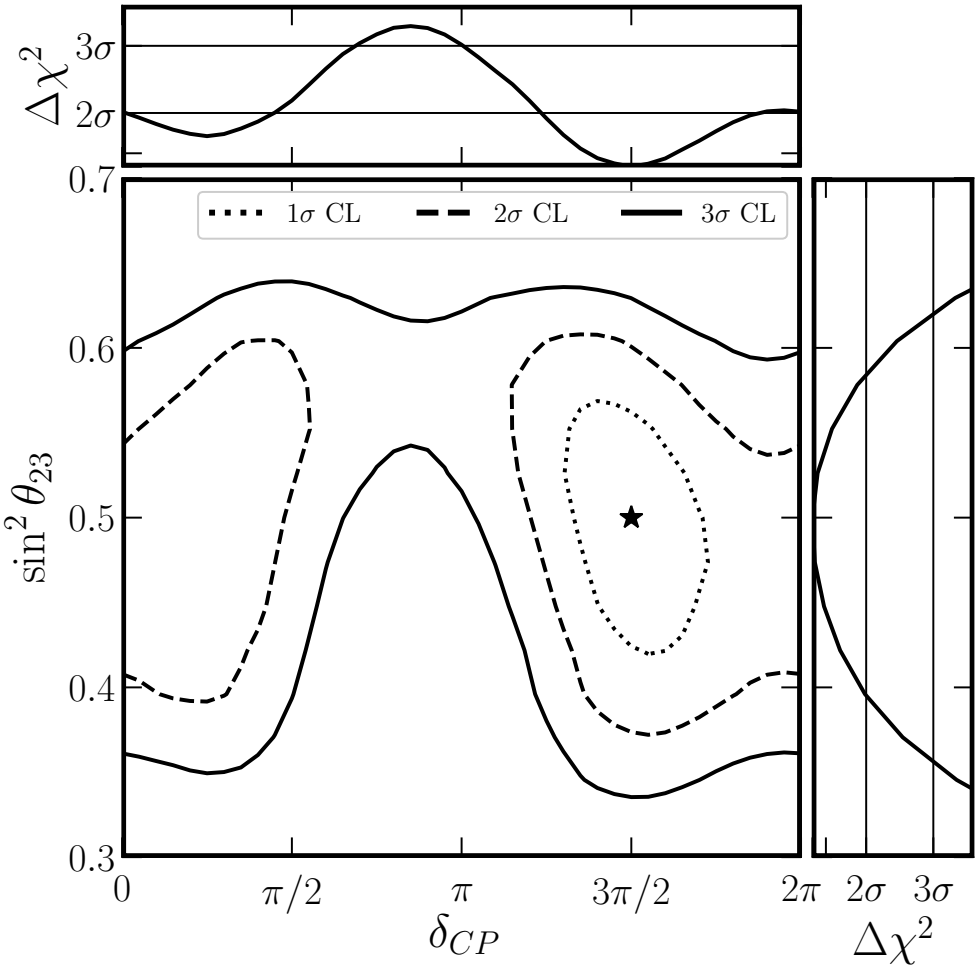
- We distribute the events in bins of 0.2 in $\cos\theta_z$ and 100 MeV in neutrino energy
- Total number of events $\sim 10^4/10$ years.



Sub-GeV Atmospheric neutrinos in DUNE



Sub-GeV Atmospheric neutrinos in DUNE



Conclusions

Most of the neutrino oscillation data can be explained in the framework of 3 neutrino mixing.

The least known oscillation parameters:

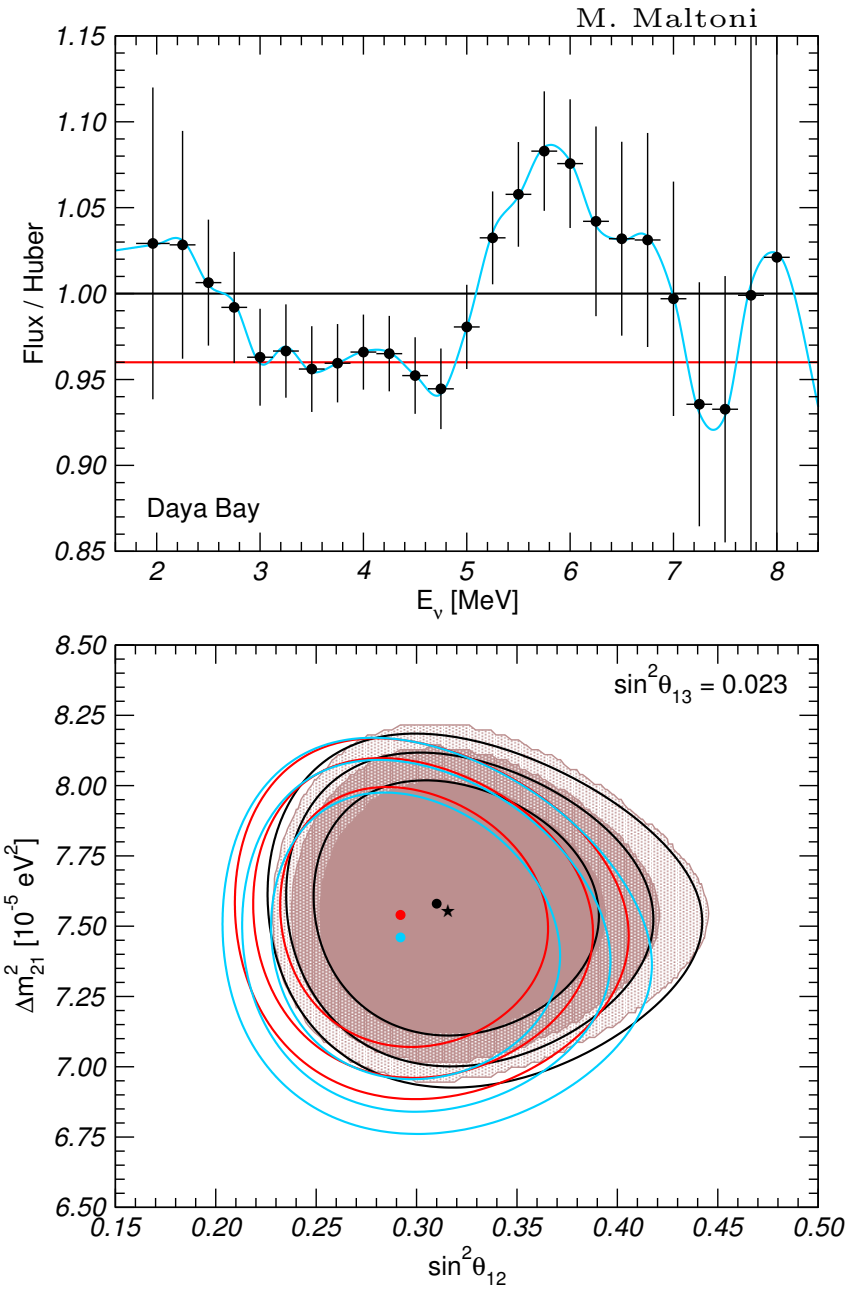
- δ_{CP} (Recent result push to non-maximal violation)
- The octant of θ_{23} (preference for the second octant)
- The mass hierarchy (preference for NO at more than 3σ)

Sub-GeV atmospheric neutrinos in DUNE can be used to probe CP-violation

Thank you!

Backup: KamLAND and the 5 MeV excess

- There is no a near detector in KamLAND.
- The flux can be affected by the excess around $E_\nu \sim 5$ MeV.
- There is a small impact on the determination of Δm^2_{21}



Backup: Evaluation of the Confidence levels

The bounds are derived assuming $\Delta\chi^2$ follows a χ^2 distribution (Wilks theorem).

- The distribution function follows by $\Delta\chi^2$ is affected by:
- the available statistics
- Degeneracies between the parameters
- The ordering is a discrete parameter

Backup: Evaluation of the Confidence levels

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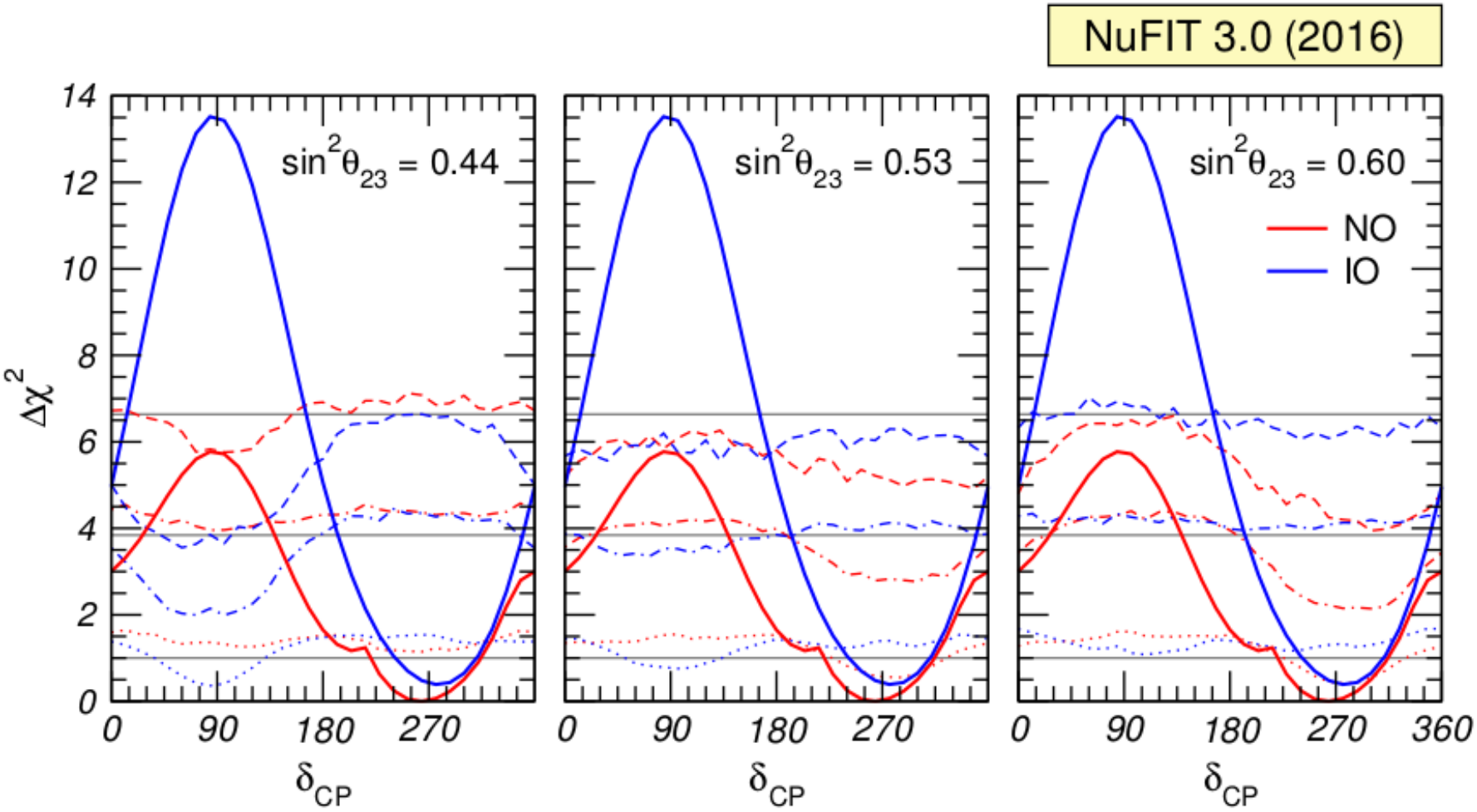
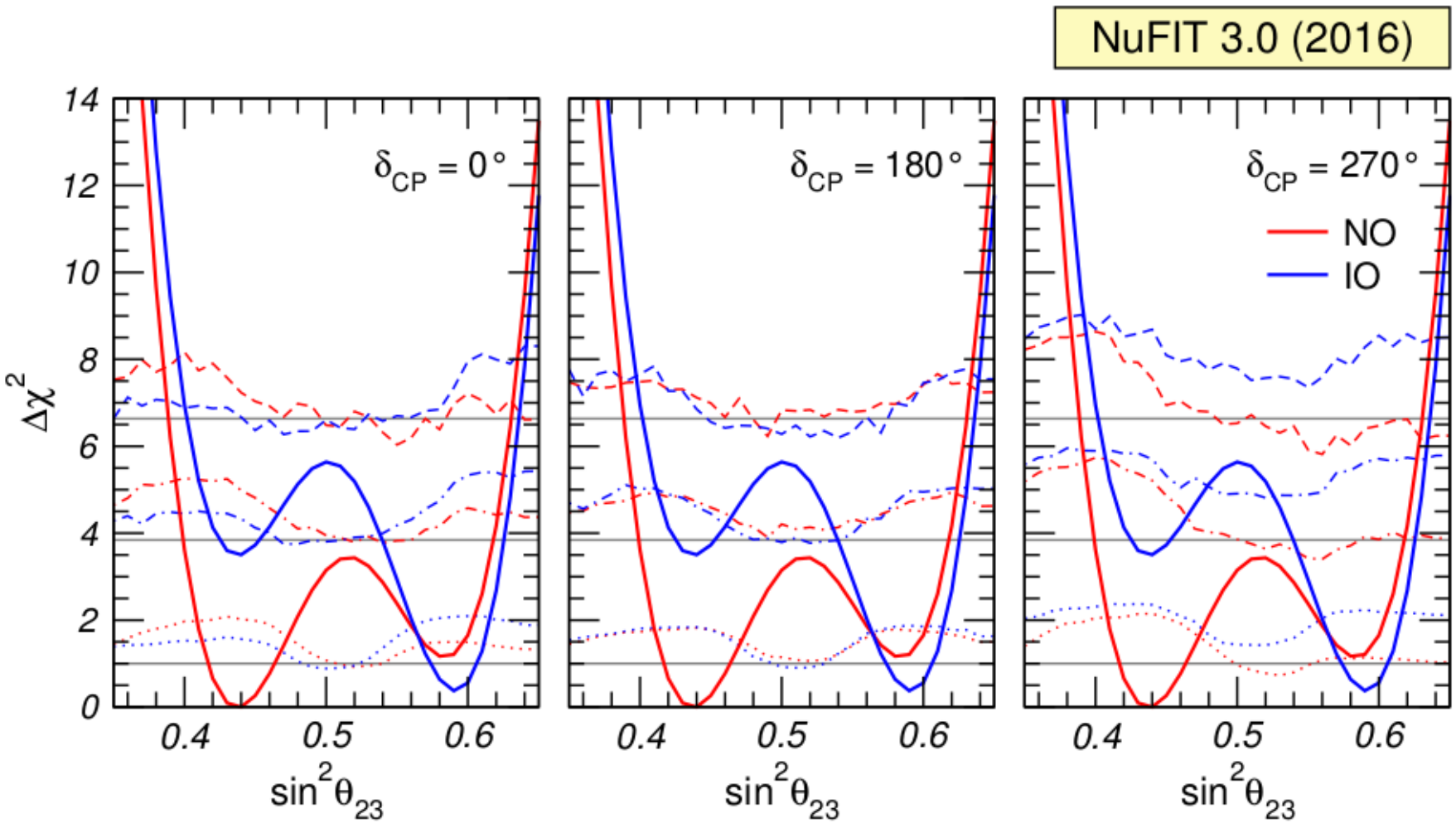
- the available statistics
- Degeneracies between the parameters
- The ordering is a discrete parameter

The confidence levels can be checked via a **Monte Carlo** simulation of the experiments (Daya Bay, T2K, MINOS and NOνA).

Backup: Evaluation of the Confidence levels



Backup: Evaluation of the Confidence levels