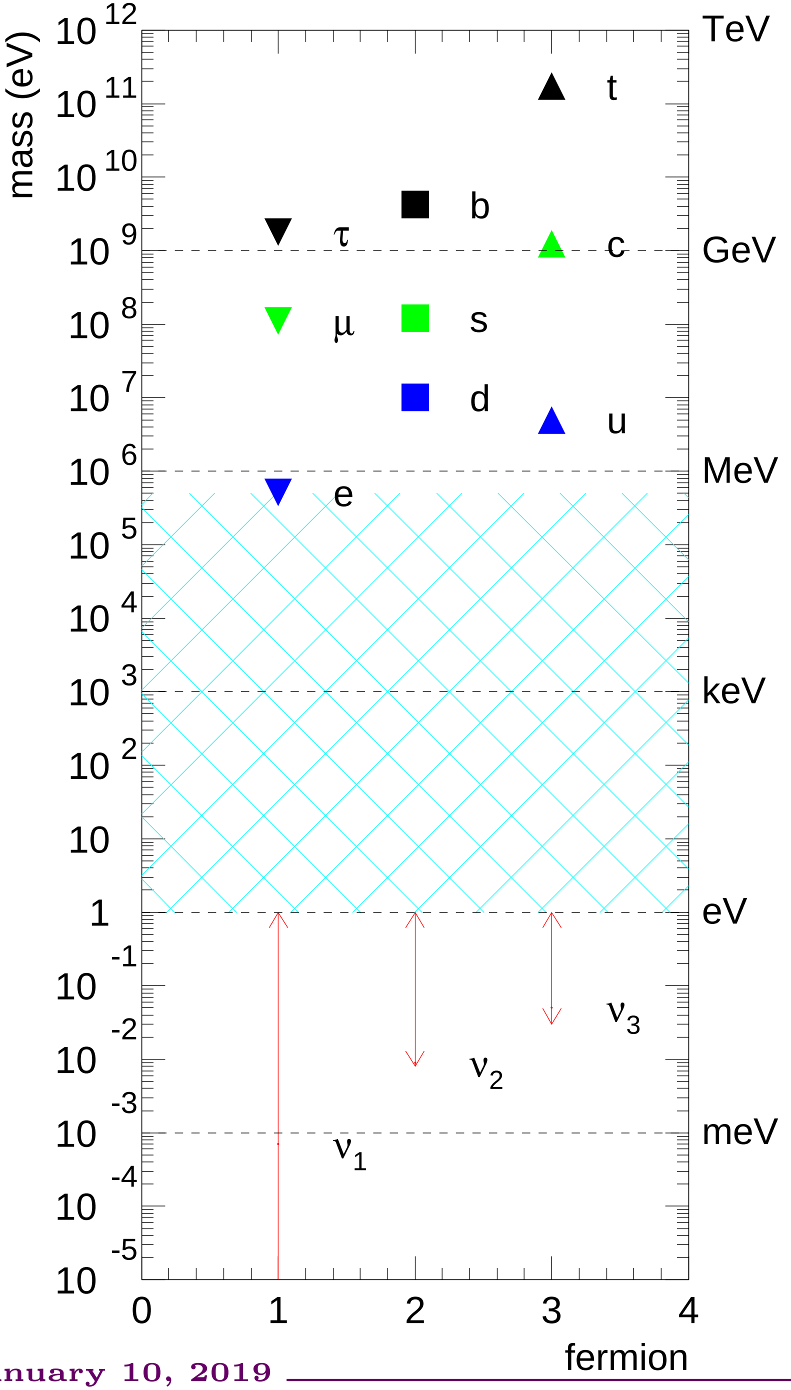
Majorana or Dirac? That is the Question

André de Gouvêa – Northwestern University

Neutrino Seminar – Fermilab

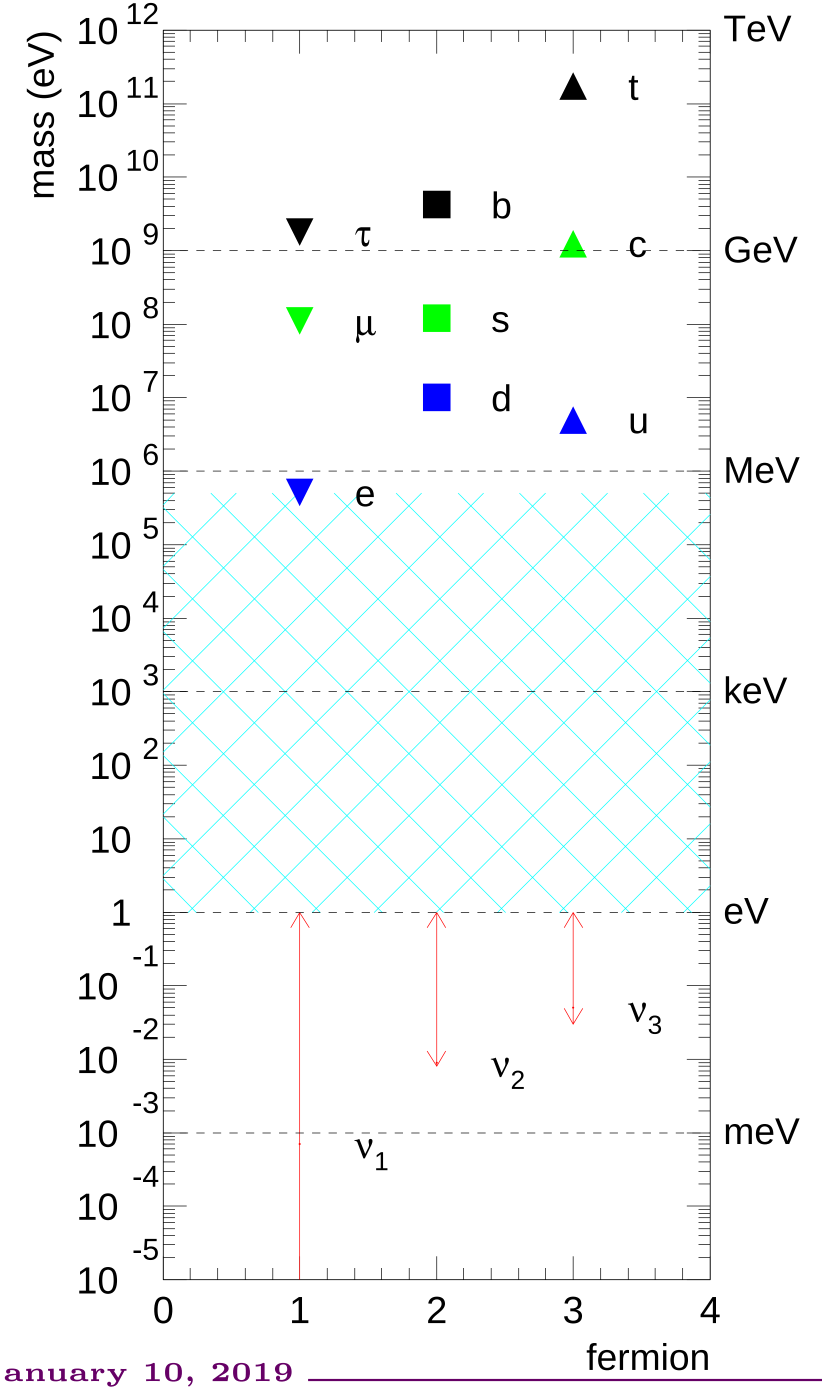
January 10, 2019



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

$\boxed{\textbf{So What?}}$



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

$\boxed{\text{So What?}}$

$\Downarrow$

$\boxed{\text{NEW PHYSICS}}$

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

Piecing the Neutrino Mass Puzzle

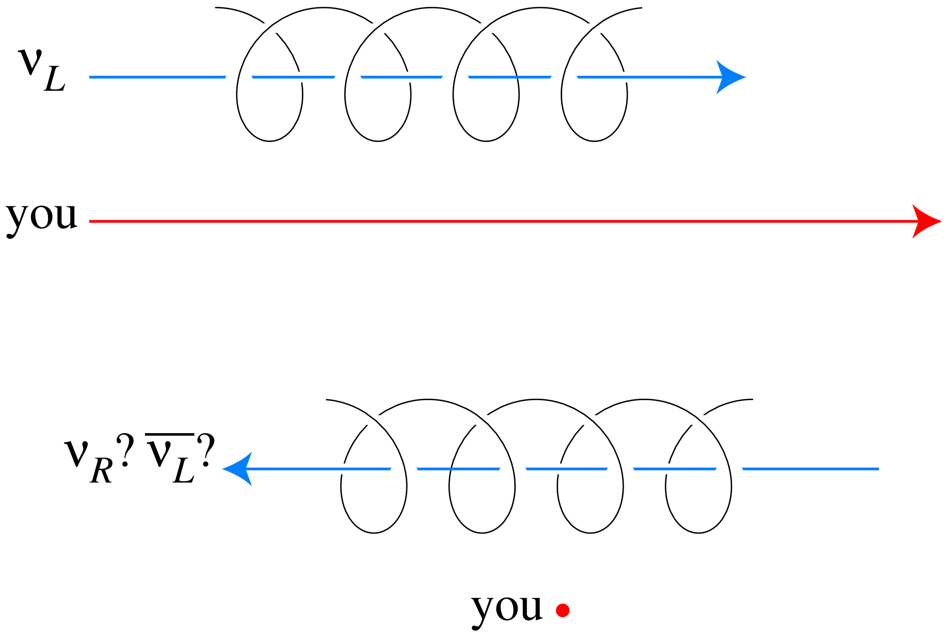
Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including ...

- **understanding the fate of lepton-number**. Neutrinoless double beta decay.
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- precision studies of charged-lepton properties ($g-2$, edm), and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

Fork on the Road: Are Neutrinos Majorana or Dirac Fermions?

[9 of 10 theorists agree: “Burningest” Question in Neutrino Physics Today!]

Posing the Question – Are Neutrinos Majorana Fermions?



A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{Lorentz}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{“DIRAC”}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

“MAJORANA”

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer (Yet)?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu / E$.

The "smoking gun" signature is the observation of LEPTON NUMBER VIOLATION. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.

The deepest probes are searches for Neutrinoless Double-Beta Decay.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \quad \text{followed by} \quad \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

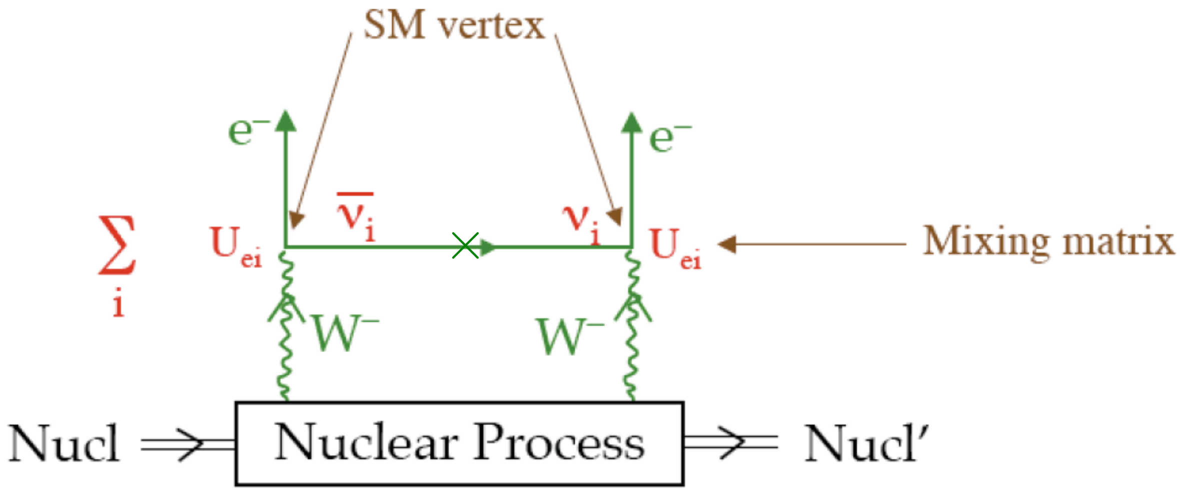
Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1\ \text{eV}/100\ \text{MeV})^2 = 10^{-18}$. VERY Challenging!

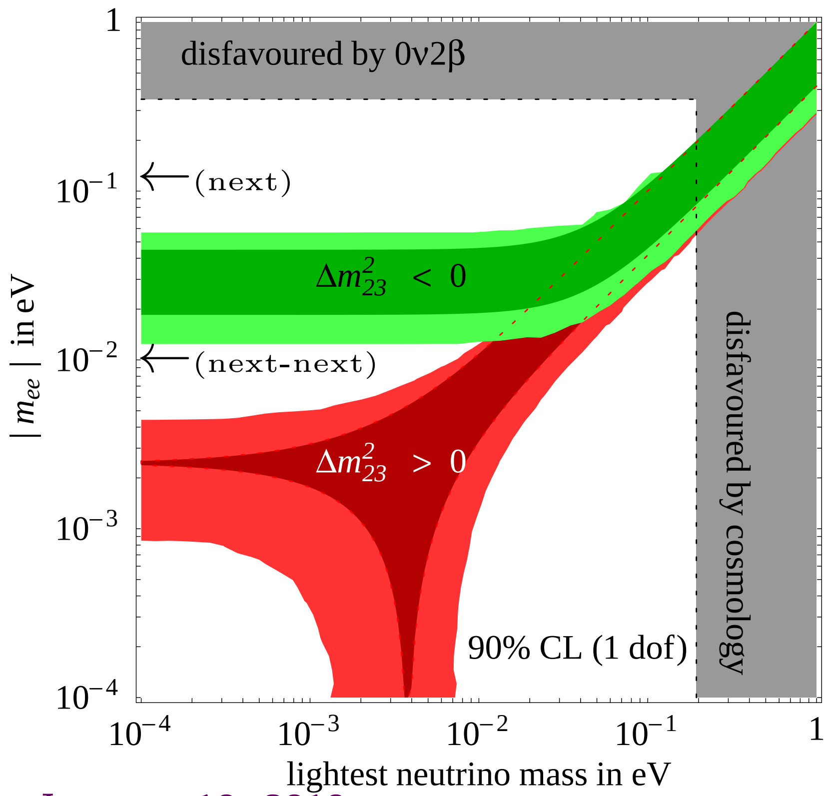
Search for the Violation of Lepton Number (or $B - L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $\boxed{Z \rightarrow (Z+2)e^- e^-}$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

$\Leftarrow$ **no longer lamppost physics!**

Again: Why Don't We Know the Answer?

Neutrino Masses are Very Small*! [e.g. $|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle \simeq\rangle \sim |L\rangle$]

In fact, except for neutrino oscillation experiments, no consequence of a nonzero neutrino mass has ever been observed in any experiment. As far as all non-oscillation neutrino experiments are concerned, neutrinos are massless fermions.

*Very small compared to what? Compared to the typical energies and momentum transfers in your experiment. Another way to think about this: neutrinos are always **ultrarelativistic** in the lab frame.

There are two ways around it:

1. Find something that only Majorana fermions know how to do [e.g. violate lepton number] or
2. **find some non-ultrarelativistic neutrinos to work with!**

The Burden of Working with Non-Ultrarelativistic Neutrinos

In a nutshell: there aren't too many of them, and the weak interactions are weak. Remember, at low energies

$$\sigma \propto E \quad \text{(or worse)}$$

On the other hand, telling Majorana From Dirac neutrinos is "trivial." Indeed, it is an order one effect.

Example: The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

Assuming the Standard Model Cosmology, at least two of the three neutrinos are mostly non-relativistic today:

$$T_\nu \sim 2K \sim 2 \times 10^{-4}\ \text{eV}.$$

Furthermore, it turns out that hitting a Majorana neutrino at rest is two times as easy as hitting a Dirac neutrino at rest, assuming the weak interactions.

In words, the reason is as follows. When you interact with a polarized neutrino at rest, it will either choose to behave like the left-chiral component or the right-chiral component, with the same probability. In the Dirac case, the right-chiral component is sterile, i.e., it does not participate in the weak interactions and you can't interact with it. In the Majorana case, the right-chiral component is the object we usually refer to as the antineutrino. In this case, we get a hit either way!

Example: The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

This means that if we ever observe the cosmic neutrino background, we can determine the nature of the neutrino. If all neutrinos were at rest, for the same neutrino (+ antineutrino, in the Dirac case) flux, we expect twice as many events in the experiment if the neutrinos are Majorana fermions. Oe can easily include finite temperature effects, effects related to the neutrino mass ordering, a potential primordial lepton asymmetry, etc.

Some challenges:

- We have never detected the cosmic neutrino background! (see, however, PTOLEMY [arXiv:1808.01892] for a great idea that may work one day);
- We measure flux times cross-section. While we know the average neutrino number density of the universe very well from the Standard Model of Cosmology, we don't know the number density of neutrinos *here* very well [Uncertainty around 100%?].

Example: Neutrinos Near Threshold?

We looked at

$$e\gamma \to e\nu\bar{\nu}$$

at sub-eV energies, because it can be done, in principle (electron at rest, infrared photon). Best to do it in the mass basis! Using the Fermi theory…

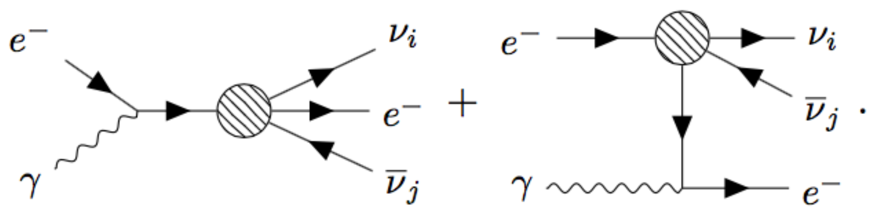
$$\mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} = -\sqrt{2}G_F\left(\bar{\nu}_j\gamma^\mu P_L\nu_i\right)\left[\bar{\ell}_\alpha\gamma_\mu\left(g_V^{\alpha\beta ij}\mathbb{1} - g_A^{\alpha\beta ij}\gamma_5\right)\ell_\beta\right], \tag{II.4}$$

where we introduce the vector and axial couplings

$$g_V^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\left(1 - 4\sin^2\theta_W\right)\delta_{ij}\delta_{\alpha\beta}, \qquad g_A^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\delta_{ij}\delta_{\alpha\beta}. \tag{II.5}$$

Since the only charged leptons considered in this work are electrons, we will make the simplification $g_{V,A}^{ij} \equiv g_{V,A}^{eeij}$.

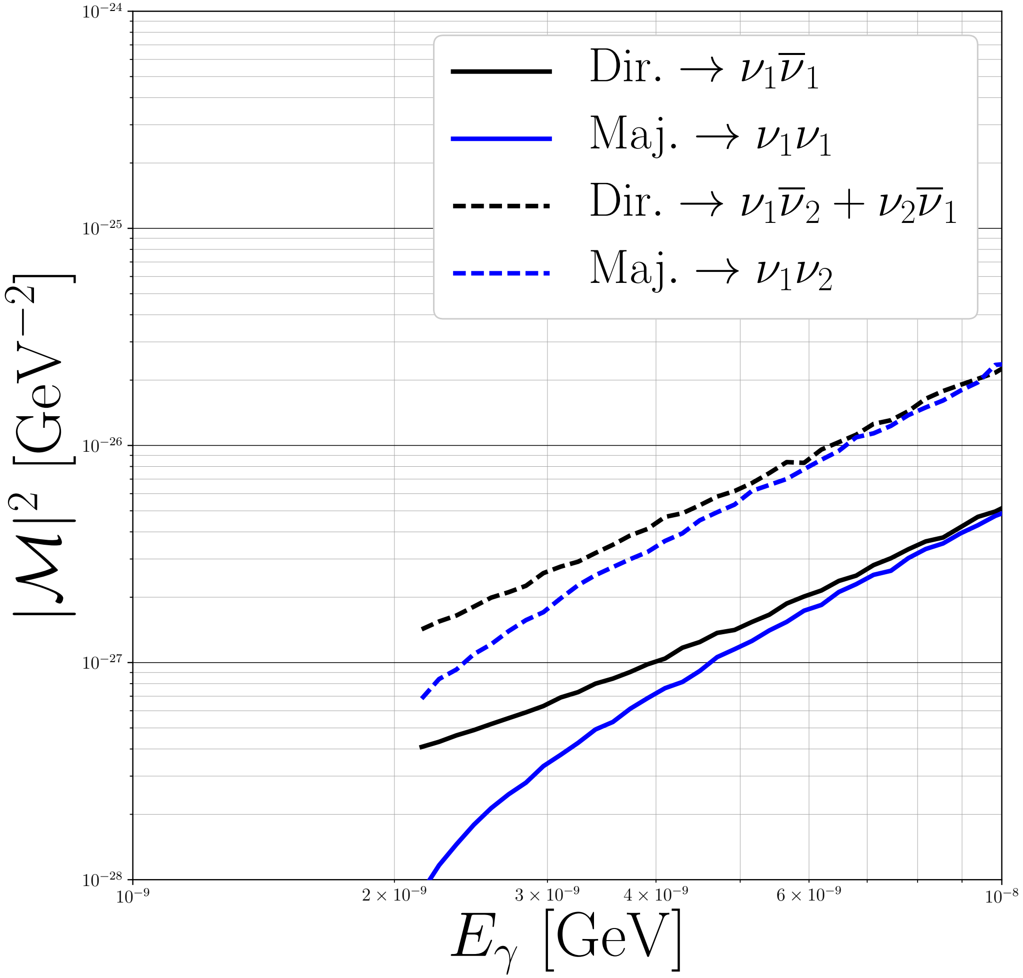
The following diagrams are relevant to the evaluation of the amplitude:

$i\mathcal{M} \approx$  (II.6)

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]

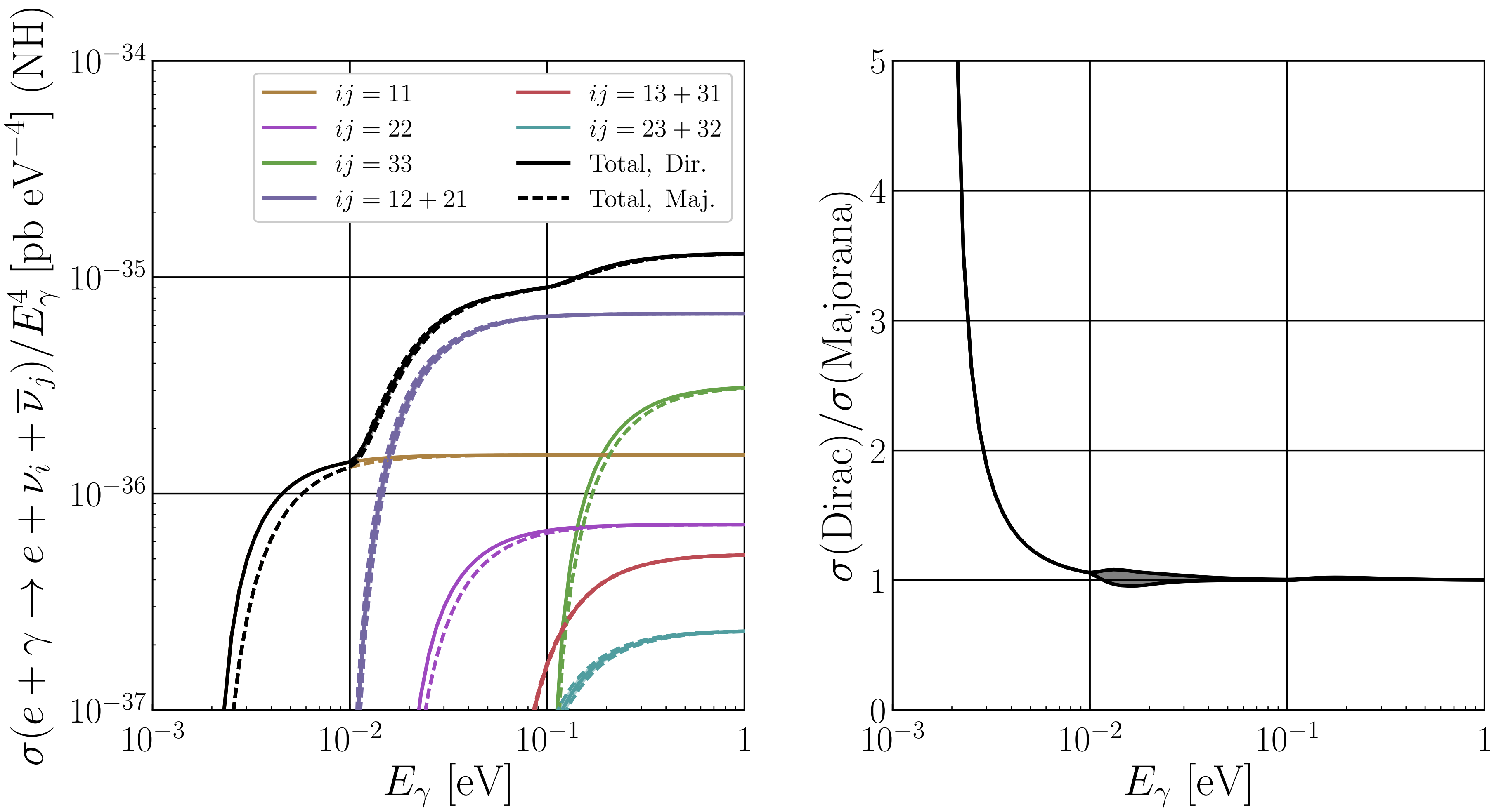
Example: Neutrinos Near Threshold?

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Example: Neutrinos Near Threshold?

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Another Example of Neutrinos Near Threshold (Brief)

Atomic process: $A^* \to A\gamma$, where A (A^*) is a neutral atom (in some excited state). Now replace the γ with an off-shell Z, which manifests itself as two neutrinos:

$$A^* \to A\nu\bar{\nu}.$$

It is easy to imagine sub-eV energies and hence the neutrinos are not ultra-relativistic.

For all the details including rates – tiny – and difference between Majorana and Dirac neutrinos – large – see, for example, Yoshimura, hep-ph/0611362, Dinh *et al.*, arXiv:1209.4808, and Song *et al.* arXiv:1510.00421, and references therein.

Neutrino Decay (Hint – Only Massive Particles Decay)

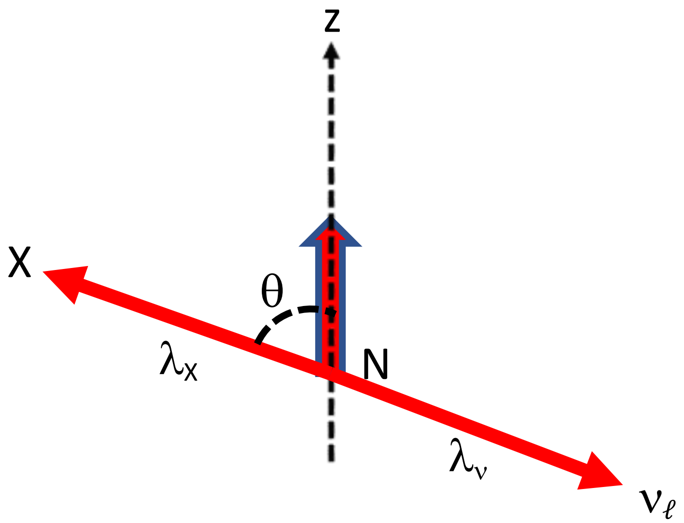
[Balantekin, AdG, Kayser, arXiv:1808.10518]

The two heavy neutrinos are expected to decay. E.g., if the neutrino mass ordering is normal, the decay modes $\nu_3 \to \nu_1\gamma$ and $\nu_3 \to \nu_1\nu_2\bar{\nu}_1$ are not only kinematically allowed, they are mediated by the weak interactions once mixing is taken into account.

Dirac and Majorana neutrinos "decay differently." In particular, the number of accessible final states, and the way in which they can potentially interfere, is such that the partial widths, and the lifetimes are different – assuming the same mixing and mass parameters – if the neutrinos are Majorana or Dirac.

Obvious challenges. $\Gamma \propto (m_\nu)^n$ [n is some positive power] so the neutrino lifetimes are expected to be astronomical. Insult to injury, the $\nu \to \nu$'s decay mode is significant, which renders studying the final products of the decay a rather daunting task. Nonetheless, we proceed ...

CPT invariance [at leading order]



We showed

$$\frac{d\Gamma(N \to \nu_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 + \alpha\cos\theta)$$

$$\frac{d\Gamma(\bar{N} \to \bar{\nu}_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 - \alpha\cos\theta)$$

Since $\alpha = -\bar{\alpha}$, for Majorana neutrinos we get $\alpha = 0$. This result holds for any self-conjugate boson X.

The two-body decay of a Majorana fermion into a self-conjugate final state is isotropic

A.B. Balantekin, B. Kayser, Ann. Rev. Nucl. Part. Sci. **68** (2018) 313-338 (arXiv:1805.00922)
A.B. Balantekin, A. de Gouvêa, B. Kayser, arXiv:1808.10518

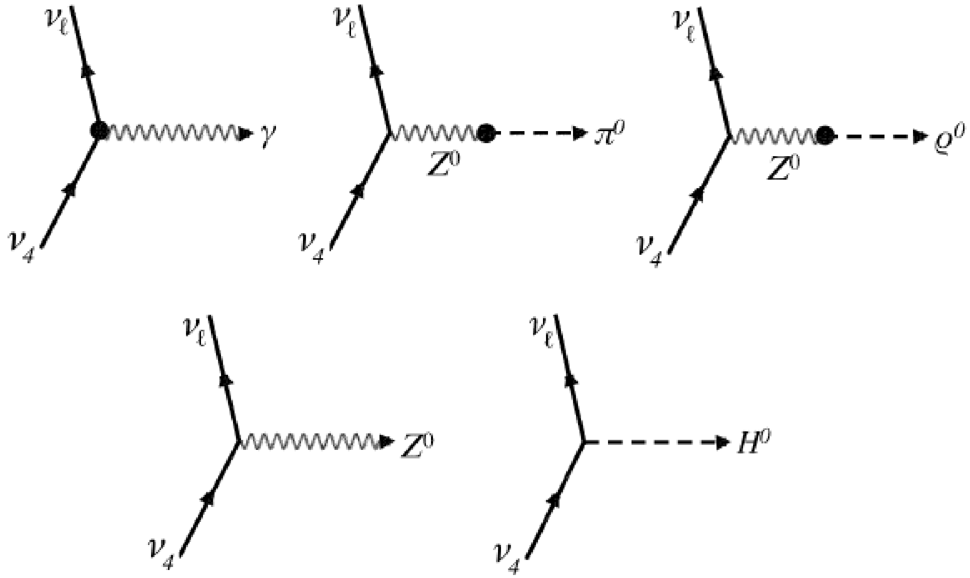
A More Realistic (?) Application – Neutral Heavy Leptons

If a neutral heavy lepton ν_4 is discovered somewhere – LHC, MicroBooNE, ICARUS, DUNE, SuperB Factory, SHiP, etc – in the future, after much rejoicing, we will want to establish whether this fermion is *Majorana* or *Dirac*.

How do we do it?

- Check for lepton-number violation. What does it take?
  - A lepton-number asymmetric initial state (easy). Or an even-by-event lepton number "tag" of the neutral heavy lepton (e.g. LHC environment).
  - Charge identification capability in the detector (sometimes absent or partially absent).
- **Kinematics**. Not only are the decay widths different – not useful, since it requires we know unknown parameters – the kinematics are qualitatively different, as I showed in the last slide.

Heavy Neutral Leptons – More Realistic (?) Application



All of these decays are isotropic for a Majorana parent. Otherwise (weak interactions)...

Boson	γ	π^0	ρ^0	Z^0	H^0
α	$\frac{2\Im(\mu d^*)}{\lvert\mu\rvert^2+\lvert d\rvert^2}$	1	$\frac{m_4^2-2m_\rho^2}{m_4^2+2m_\rho^2}$	$\frac{m_4^2-2m_Z^2}{m_4^2+2m_Z^2}$	1

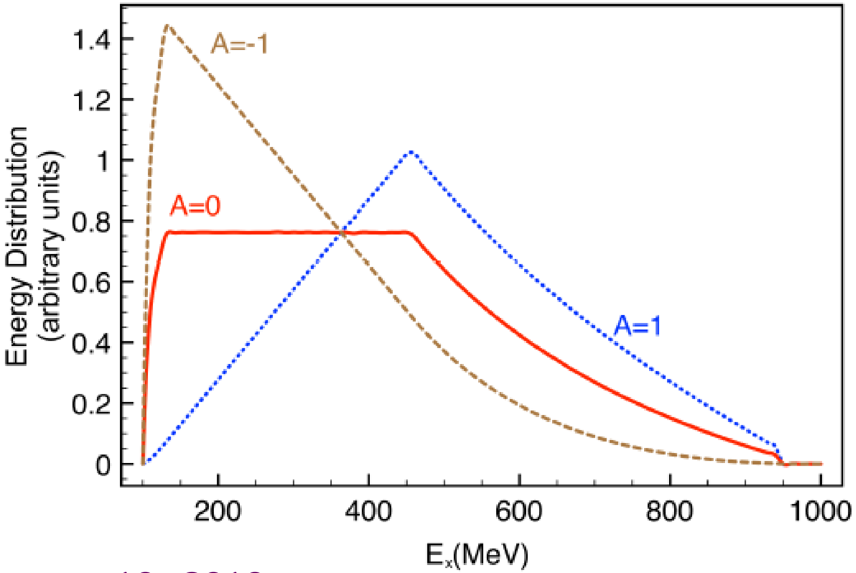
Energy distribution in the Laboratory ("same" as angular distribution)

Parent's rest frame for $N \to \nu_\ell + X$

$$\frac{dn_X}{d\cos\theta_X} \propto (1 + A\cos\theta_X), \quad A = \alpha \times \text{polarization}$$

Lab frame with $r = m_X^2/m_N^2 < 1$

$$\frac{dn_X(E_N, E_X)}{dE_X} \propto \frac{2}{p_N(1-r)}\left[1 + A\left(\frac{2}{(1-r)}\frac{E_X}{p_N} - \left(\frac{1+r}{1-r}\right)\frac{E_N}{p_N}\right)\right]$$



$m_X = 100\,\text{MeV}$

$m_N = 300\,\text{MeV}$

$500\,\text{MeV} < E_N < 100\,\text{MeV}$

Final comment: We Can Use Charged Final States Too!

The two-body final states here all involve a neutrino and a neutral boson. Impossible to reconstruct the parent rest-frame and it requires measuring the properties of a neutral boson, which is sometimes challenging. Can we use the charged final states? E.g.

$$\nu_4 \to \mu^+\pi^-$$

Most of the time, 'yes'! The reason is as follows. CPT invariance (at leading order) implies, for 100% polarized Majorana fermions,

$$\frac{d\Gamma(\nu_4 \to \mu^+\pi^-)}{d\cos\theta} \propto (1+\alpha\cos\theta) \quad \text{while} \quad \frac{d\Gamma(\nu_4 \to \mu^-\pi^+)}{d\cos\theta} \propto (1-\alpha\cos\theta)$$

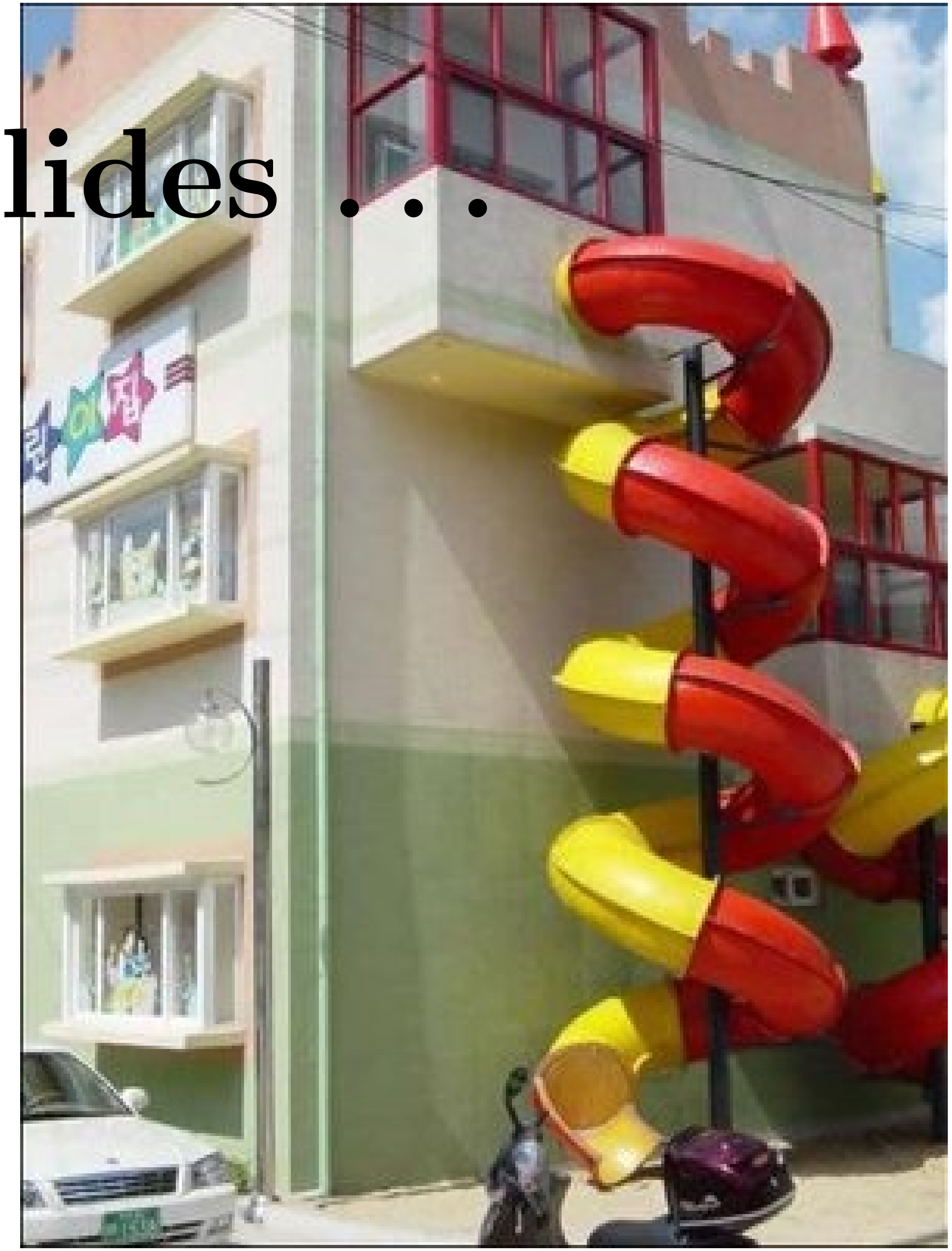
so the **charge-blind sum of the two is also isotropic**. This is not the case for Dirac neutrinos as long as the production of neutrinos and antineutrinos is asymmetric, which is usually the case.

Can this be done in practice? We don't know – homework assignment

Quick Summary

- Majorana and Dirac Fermions are Qualitatively Different;
- However, massless Majorana and Dirac fermions are "the same" – this is a non-question!;
- Challenge for neutrinos. Since they are always ultra-relativistic, it is very difficult to address whether they are Majorana or Dirac since they are massless as far as the experiment is concerned;
- One way around it is to look for phenomena that can only occur if the neutrino is a Majorana fermion (e.g., lepton-number violation). In this case, even if the phenomenon is very rare (like $0\nu\beta\beta$), any deviation from zero would allow one to establish that the neutrinos are Majorana fermions.
- The other way is to find circumstances where the neutrinos are not ultra-relativistic. In this case, the Majorana versus Dirac differences are large. The rates, on the other hand…
- Potential application: neutral heavy leptons (new neutrino states). If they exist, we will want to known: Majorana or Dirac?

Backup Slides . . .



How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 (3×3 unitary matrix)

$\underline{-5}$ (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

$$
\begin{array}{rl}
9 & (3\times 3 \text{ unitary matrix}) \\
\underline{-3\;\;} & (\text{three right-handed charged lepton fields}) \\
6 & (3 \text{ mixing angles and } 3 \text{ CP-odd phases}).
\end{array}
$$

There is CP-invariance violating parameters even in the 2 family case: $4-2=2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \overline{\nu_L^c} (M_\nu) \nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \overline{\nu_L^c} (M_\nu) E^{-i\alpha} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \bar{\nu}_R (M_\nu) \nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \bar{\nu}_R (M_\nu) E^{-i\alpha/2} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$