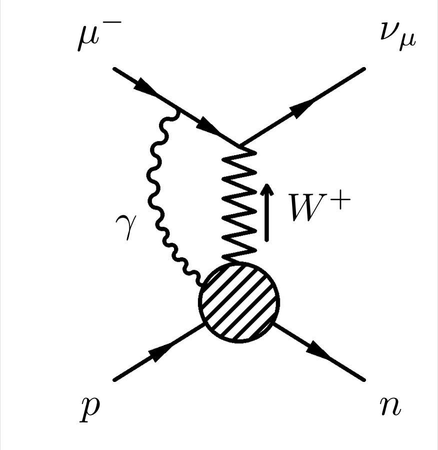

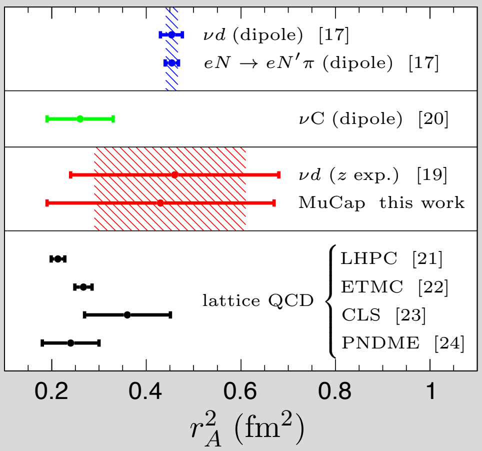

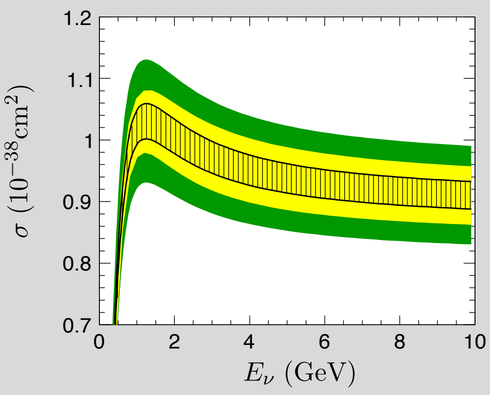

Muon capture and neutrino interactions

RICHARD HILL, U. Kentucky and Fermilab

announcement: U.Kentucky is newest URA member

Overview

- why neutrino interactions

- why muon capture

- nucleon axial radius and neutrino interactions

- muon capture from muonic hydrogen

- implications for neutrino interactions

why neutrino interactions

why bother with neutrino interactions? Isn't this too hard/too different/ somebody else's problem?

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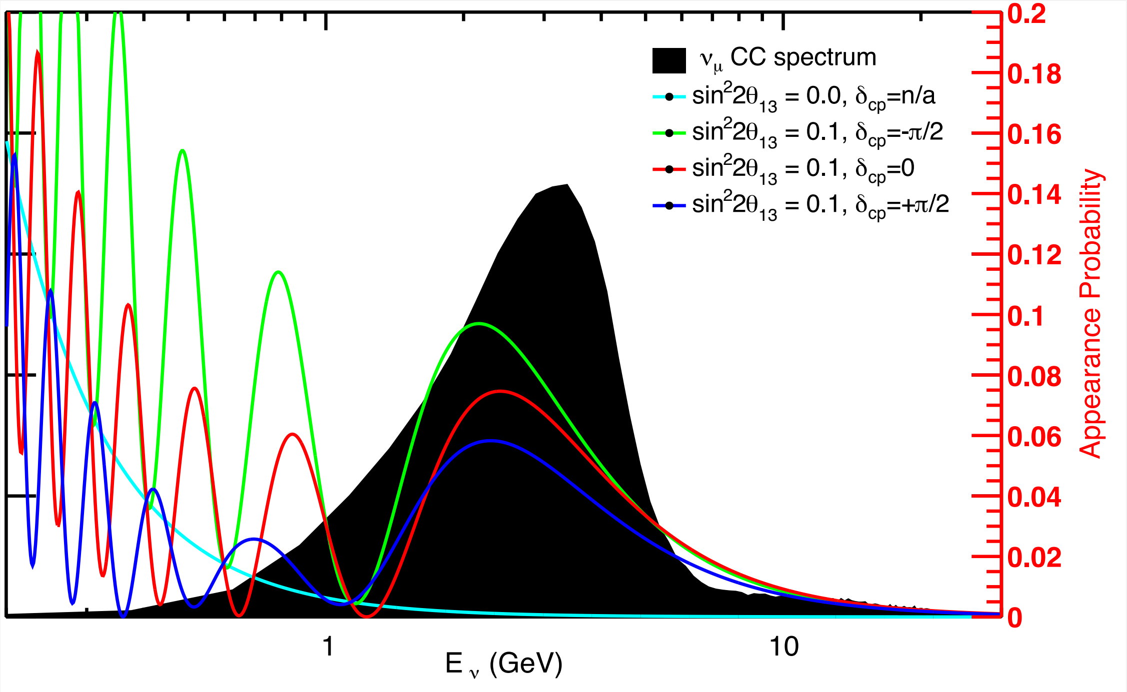
“The good news is that it’s not my problem”

why bother with neutrino interactions? Isn’t this too hard/too different/ somebody else’s problem?

“The goo[illegible]ews is that it’s no[illegible]roblem”

ν_e appearance from a ν_μ beam

long baseline neutrino oscillation experiment is **simple in conception**:



but **difficult in practice:** rely on theory to determine cross sections: e.g. $\sigma(\nu_e)/\sigma(\nu_\mu)$ to a precision of 1%

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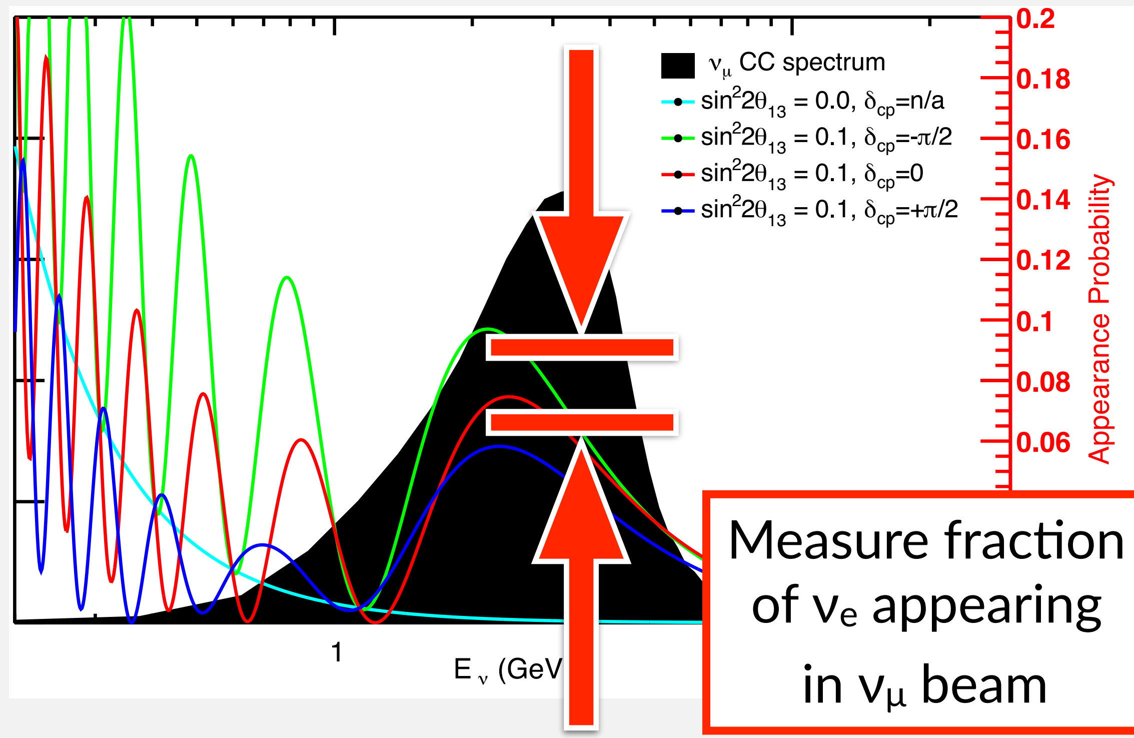

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long baseline neutrino oscillation experiment
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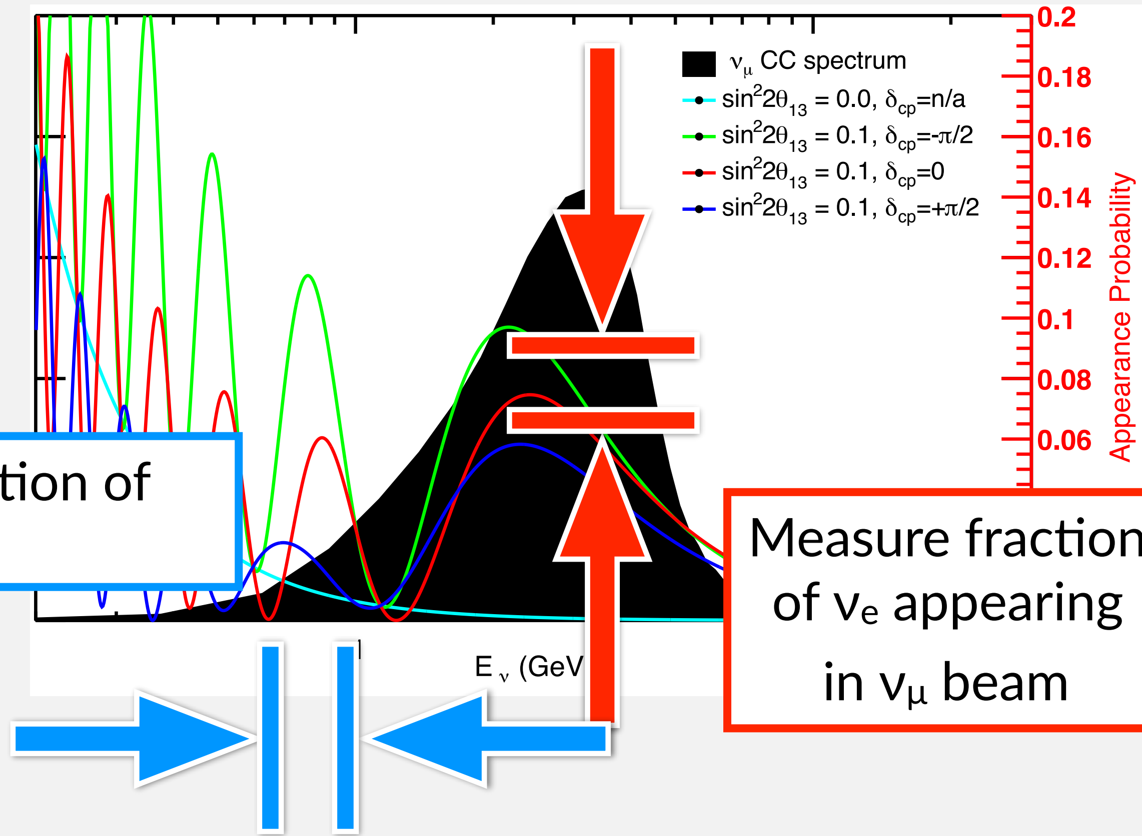
ν_e appearance from a ν_μ beam



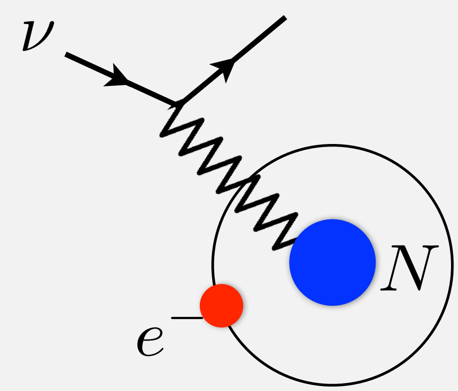
Do it as a function of energy

Measure fraction of ν_e appearing in ν_μ beam

but **difficult in practice:** rely on theory to determine cross sections: e.g. $\sigma(\nu_e)/\sigma(\nu_\mu)$ to a precision of 1%

Complications from the strong interaction

practical experiments use atomic nuclei as targets, introducing the complication of hadronic physics



For example,

DUNE: ^{40}Ar

T2K: ^{16}O

NOvA: ^{12}C

we can and must tame hadronic uncertainty, in order to access fundamental neutrino properties long baseline neutrino oscillation experiment is **difficult** in **practice**:

simple picture is complicated by

- ν_e versus ν_μ cross section differences

need theory for $\sigma_{\nu e}/\sigma_{\nu\mu}$, at ~% precision of measurement

and also

- intrinsic ν_e component of beam
- degeneracy of uncertainty in detector response and neutrino interaction cross sections
- imperfect energy reconstruction

aided by near detector but

- beam divergence and oscillation (near flux$\neq$far flux)

need theory for $\sigma_{\nu\mu}$, at a precision depending on the experimental capabilities current paradigm:

constrain neutrino interactions by

- determining nucleon level amplitudes
- parameterizing/measuring/calculating nuclear modifications

folk paradigms:

constrain neutrino interactions by

- starting at the quark level
- computing nuclear response

“perfect theory”

constrain neutrino interactions by

- starting directly at the nuclear level
- parameterizing and measuring every cross section

“perfect expt.”

in any paradigm:

near detector has access to primarily ν_μ neutrinos

νe appearance signal is directly impacted by ν_μ/ν_e cross section differences

- kinematics
- 2nd class currents (G parity violation)
- radiative corrections (QED and EW)
- signal definition

having talked the talk, do some walking:

- ν_μ/ν_e in the time reversal process ($\mu\ p \rightarrow \nu\ n$)
- nucleon input uncertainty (e-p, $\nu\ d \rightarrow \nu\ n$)

tautology: no nuclear cross section can be more precise than inputs to the nuclear model

Notes:

beyond neutrino oscillations related applications relying on quantitative nucleon structure:

- fundamental constants (probable 7 sigma shift in Rydberg)
- sigma terms and WIMP-DM direct detection
- g_A and BBN
- …

QED is "easy". But QED + nucleon structure is "hard"

entering a precision realm where percent level corrections to nucleon structure need to be calculated, not just estimated why muon capture

Nucleon properties
capture rate on proton = measurement of nucleon structure

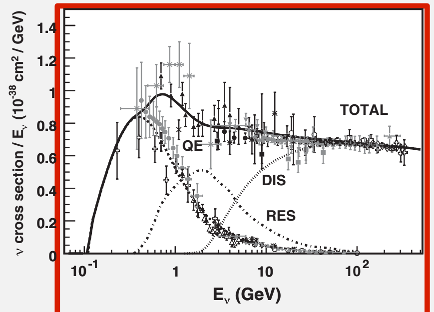
Nuclear properties
capture rate = constraint on nuclear model

Standard candle: experiment
e.g. muon capture as a test source

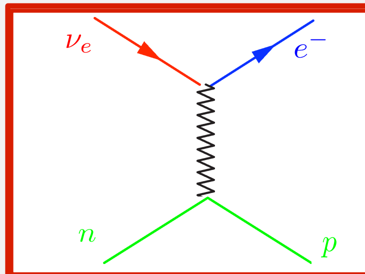
Standard candle: theory
e.g. muon capture as a test source

r_A and ν interactions

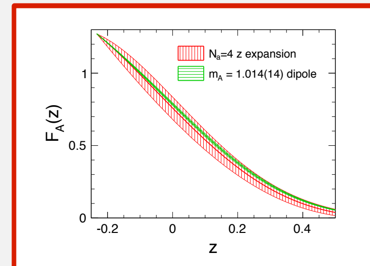
A critical number: the nucleon axial radius



quasi elastic (QE) dominance

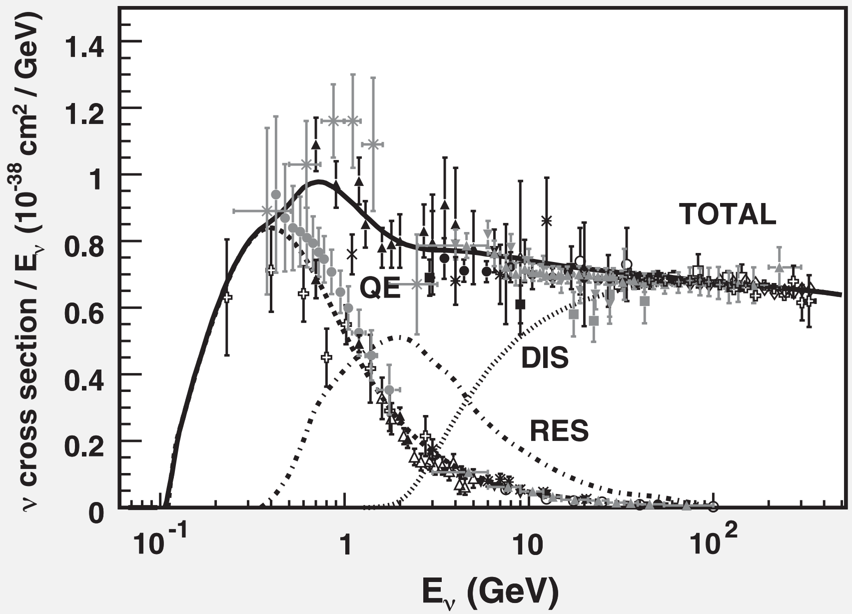


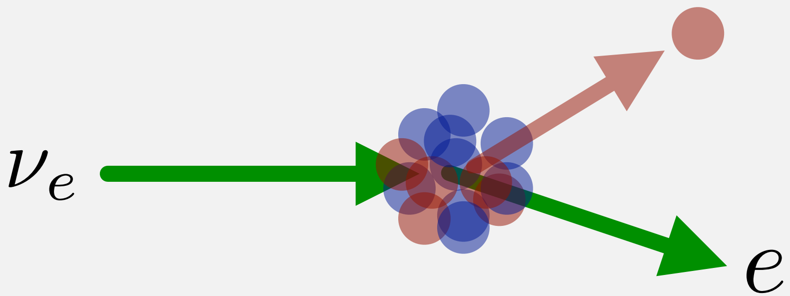
nucleon form factors for QE process



linear dependence of form factors on kinematics

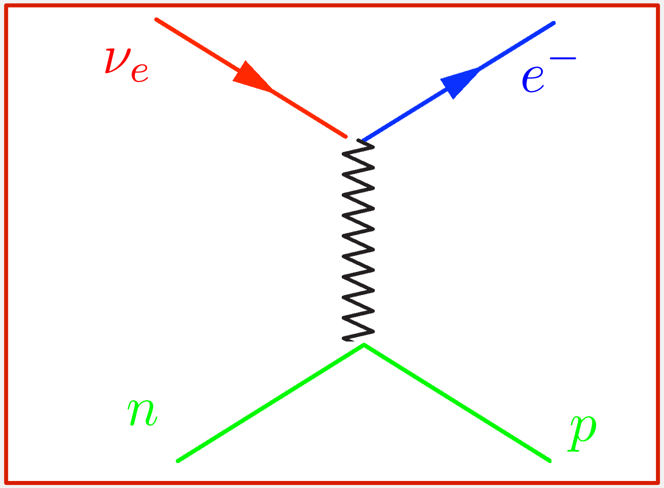
quasi elastic scattering





the dominant reaction mechanism for our beams (~GeV) is quasi-elastic scattering, where the neutrino interacts with an individual nucleon inside the nucleus

Nucleon form factors

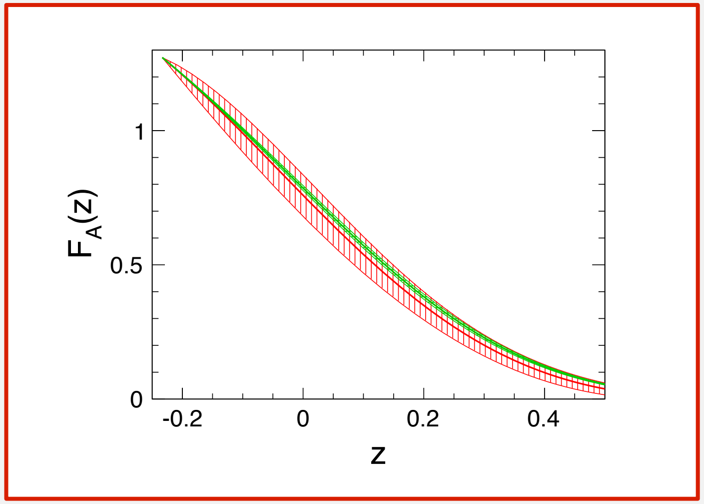


$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p')\Big\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\Big\}u^{(n)}(p)$$

The quasi-elastic process is described by nucleon form factors, most of which can be extracted from electromagnetic processes.

Exception is axial form factor.

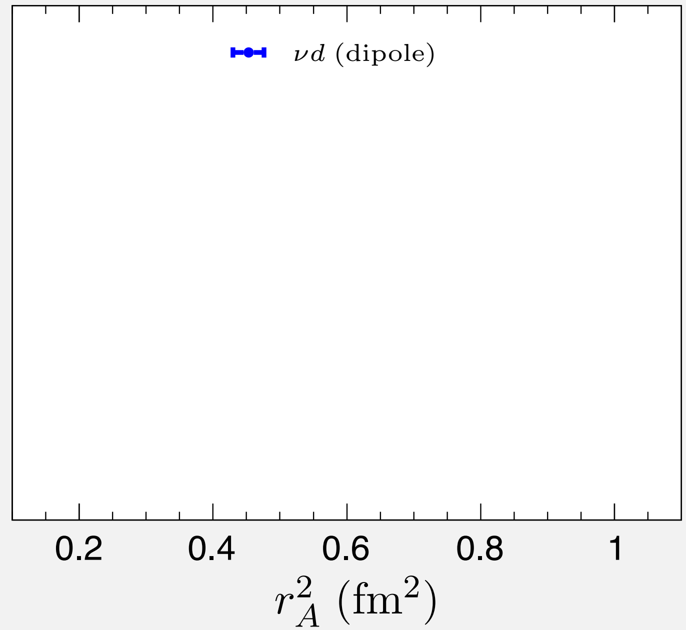
Linearity and the nucleon axial radius



Underlying QCD theory guarantees that a smart choice of variable will linearize the form factor

Normalization of this linear function is measured precisely in neutron beta decay

The slope of this linear function is the critical number for cross section energy dependence



BNL 1981

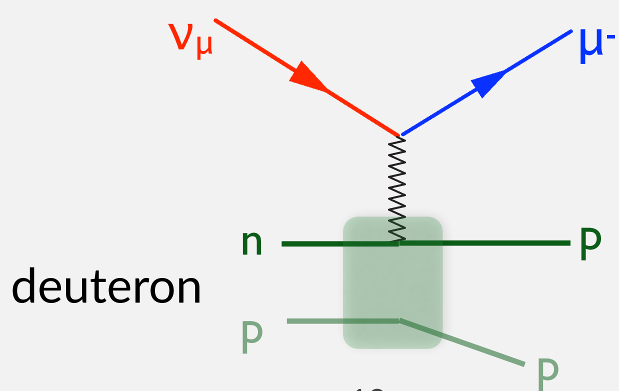
ANL 1982

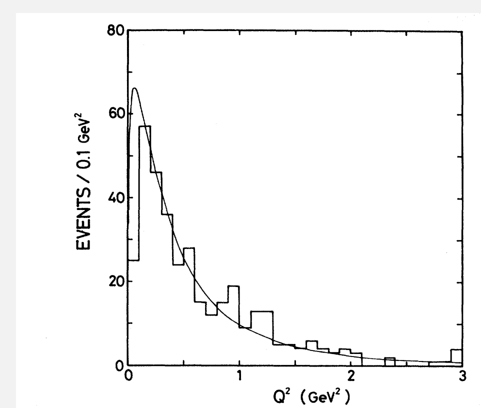
Fermilab 1983

What do we know about this critical number?

The number seemed uncontroversial for decades:

extracted from deuterium bubble chamber data





Kitigaki et al. PRD 28, 436 (1983)

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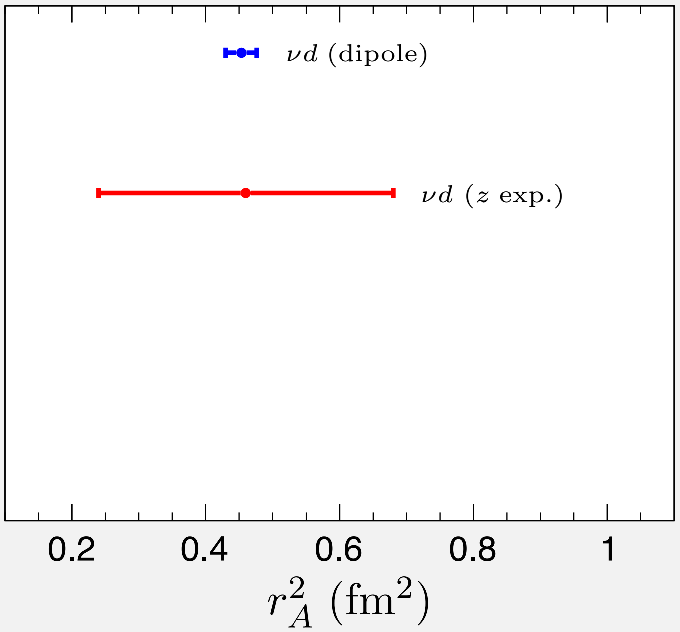
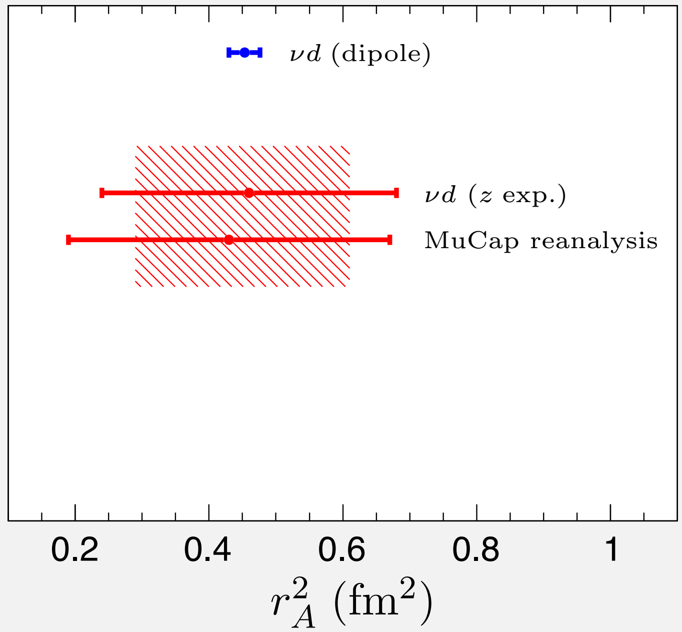


In fact the extraction relied on a hidden model assumption, and the true uncertainty is an order of magnitude larger

Bhattacharya, RJH, Paz 2011

Meyer, Betancourt, Gran, RJH 2016

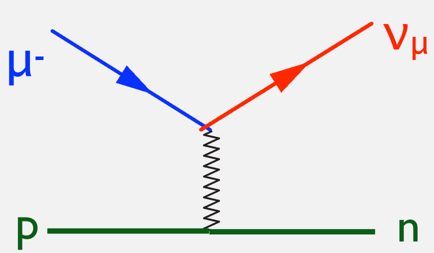
Introduces a ≳10% uncertainty in every neutrino-nucleus cross section. A wrench in the works for oscillation experiments.



What do we know about this critical number?

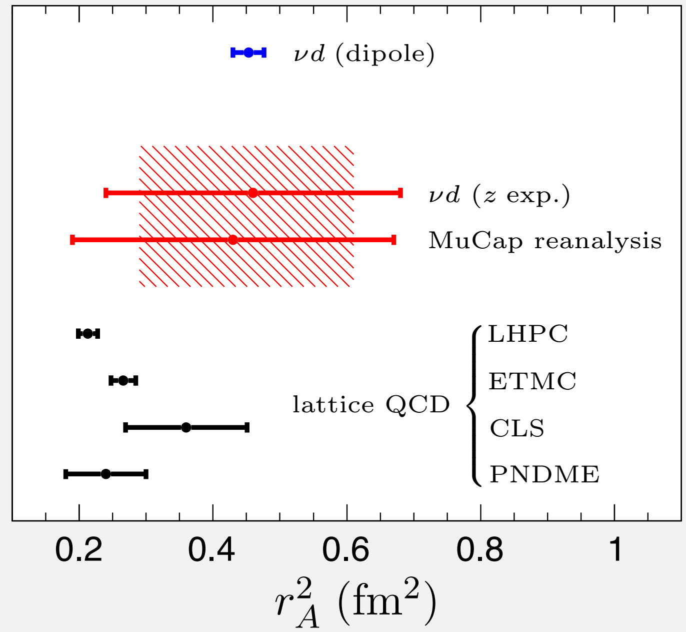
Look at the process in reverse: muon capture from ground state of muonic hydrogen (subject of this talk)

Improved theory analysis and existing data: already competitive with world ν-d data. Significant improvements possible



RJH, Kammel, Marciano, Sirlin 2017

What do we know about this critical number?



Not enough data from elementary target neutrino scattering
(let's figure out what can be done: http://www.int.washington.edu/PROGRAMS/18-2a/18-2a_workshop.html)

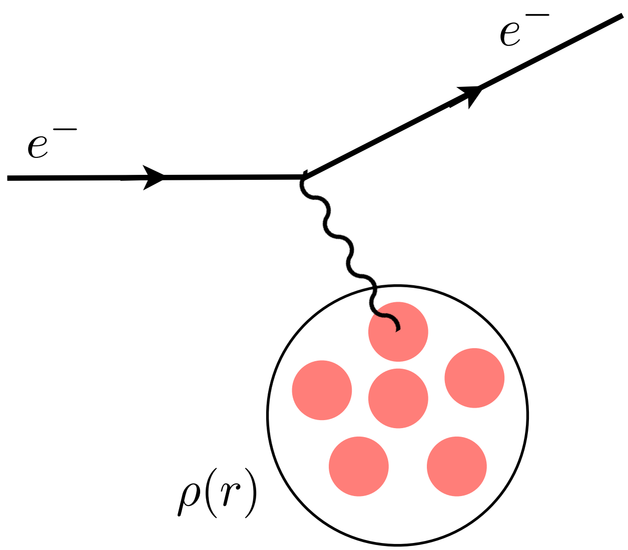
Lattice QCD is embarking on an ambitious, long-range program to answer this challenge

Where does the large uncertainty in F_A from scattering come from?

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$

$$F(q^2) = \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$$
$$= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \ldots\right]\rho(r)$$
$$= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \ldots$$



for the relativistic, QM, case, *define* radius as slope of form factor

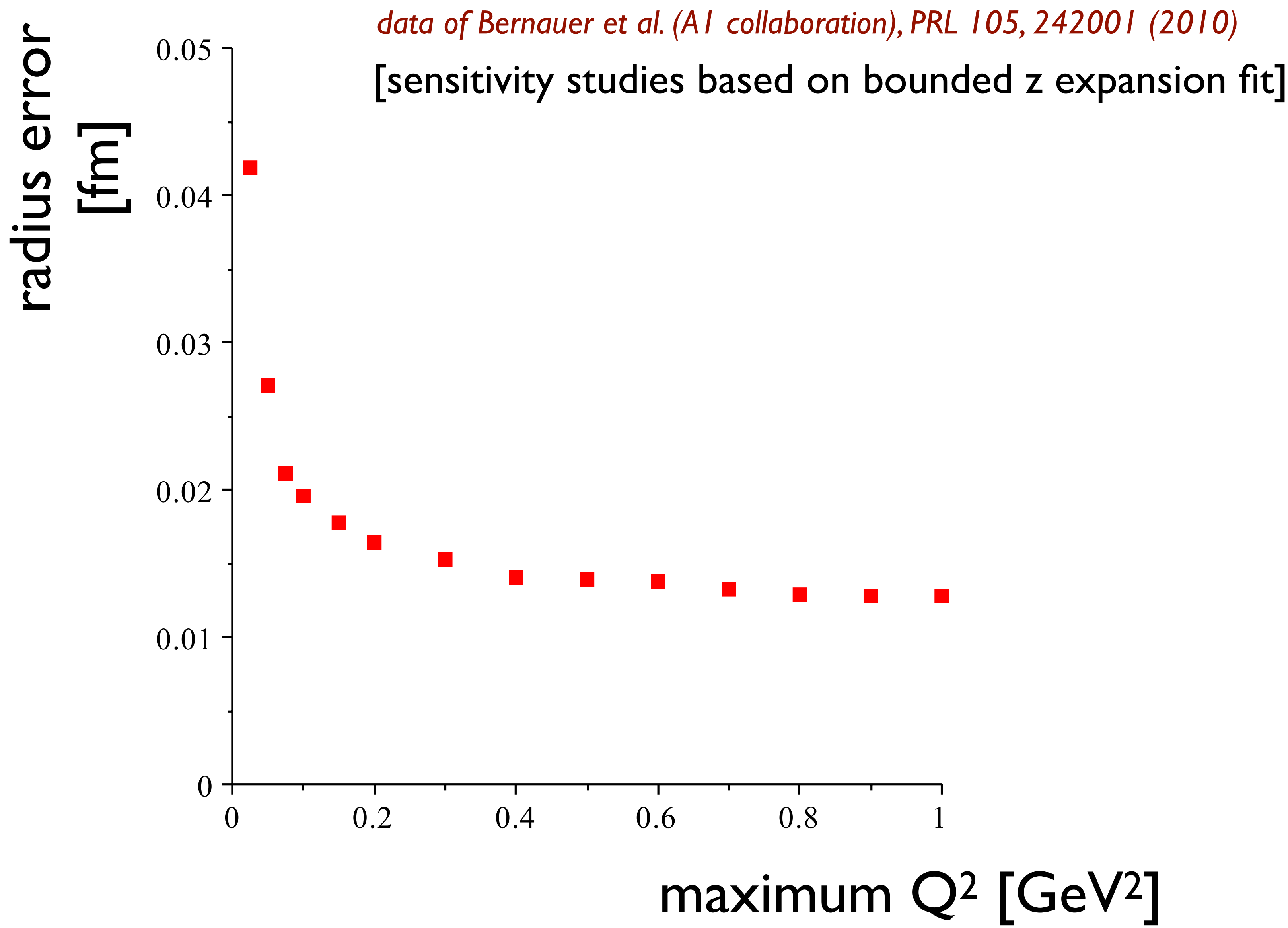
$$\boxed{r_E^2 \equiv 6\frac{d}{dq^2}G_E(q^2)\Big|_{q^2=0}}$$

(up to radiative corrections)

$$\langle J^\mu\rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu}q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2}F_2 \qquad G_M = F_1 + F_2$$

Radius extraction requires data over a Q^2 range where a simple Taylor expansion of the form factor is invalid



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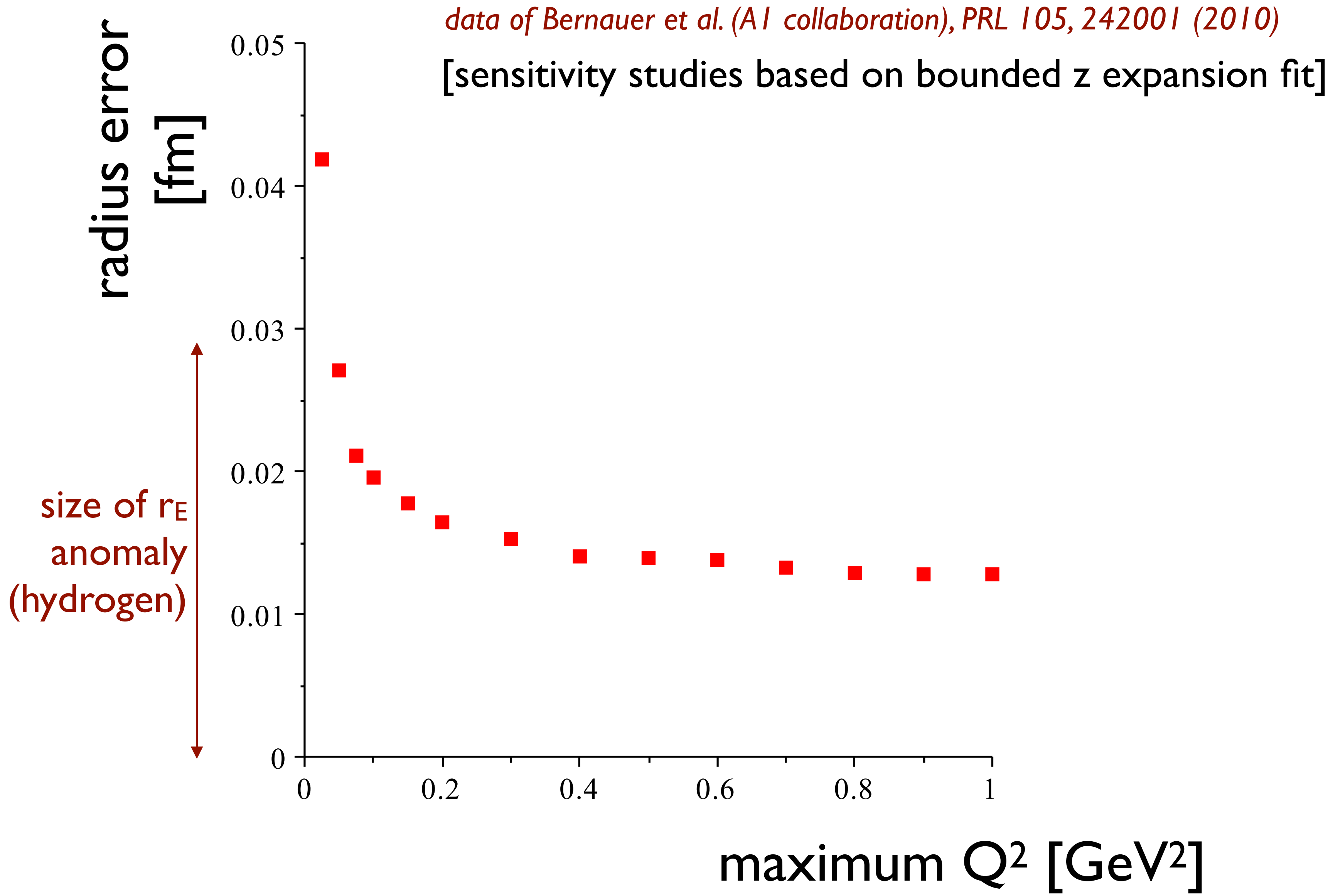

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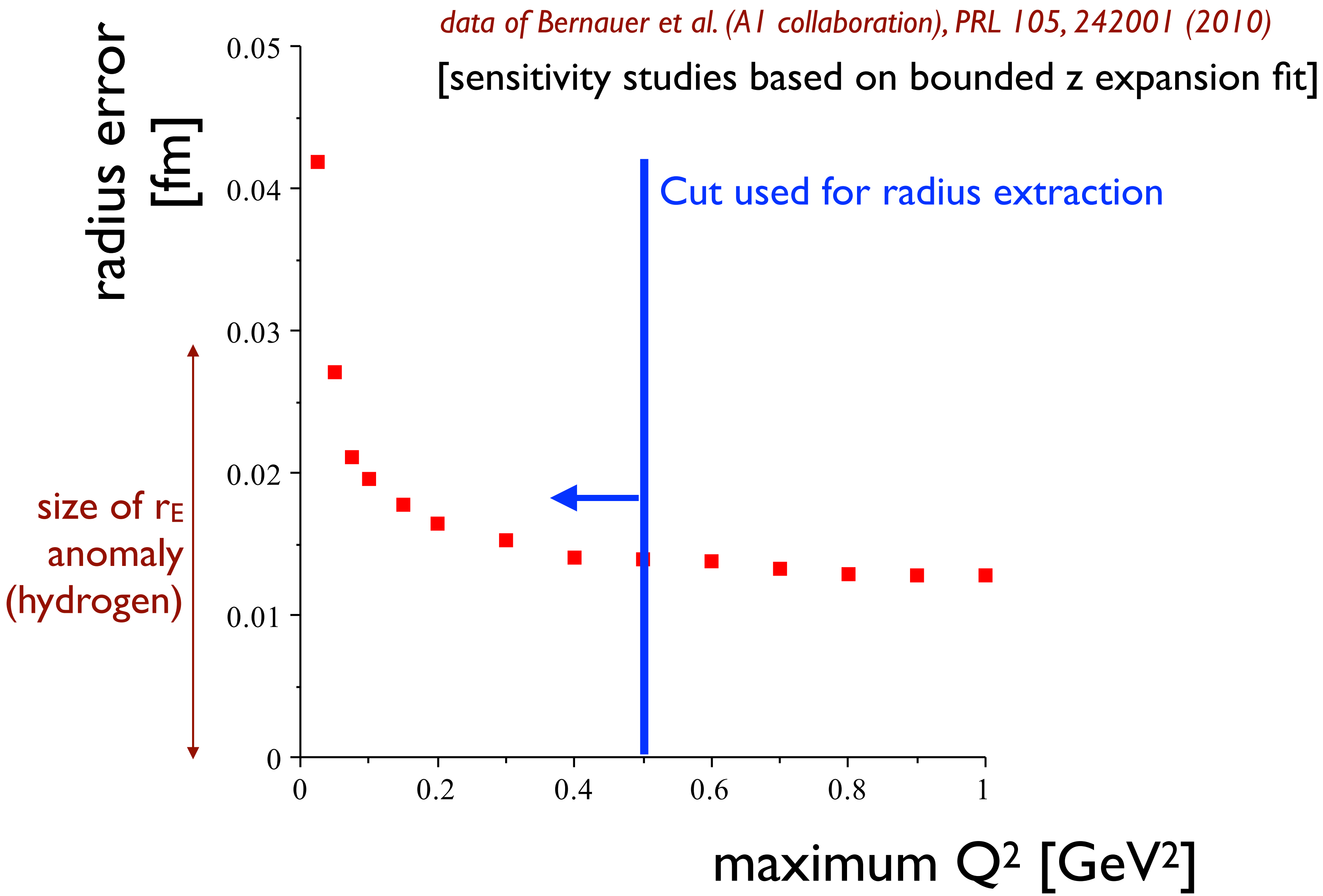

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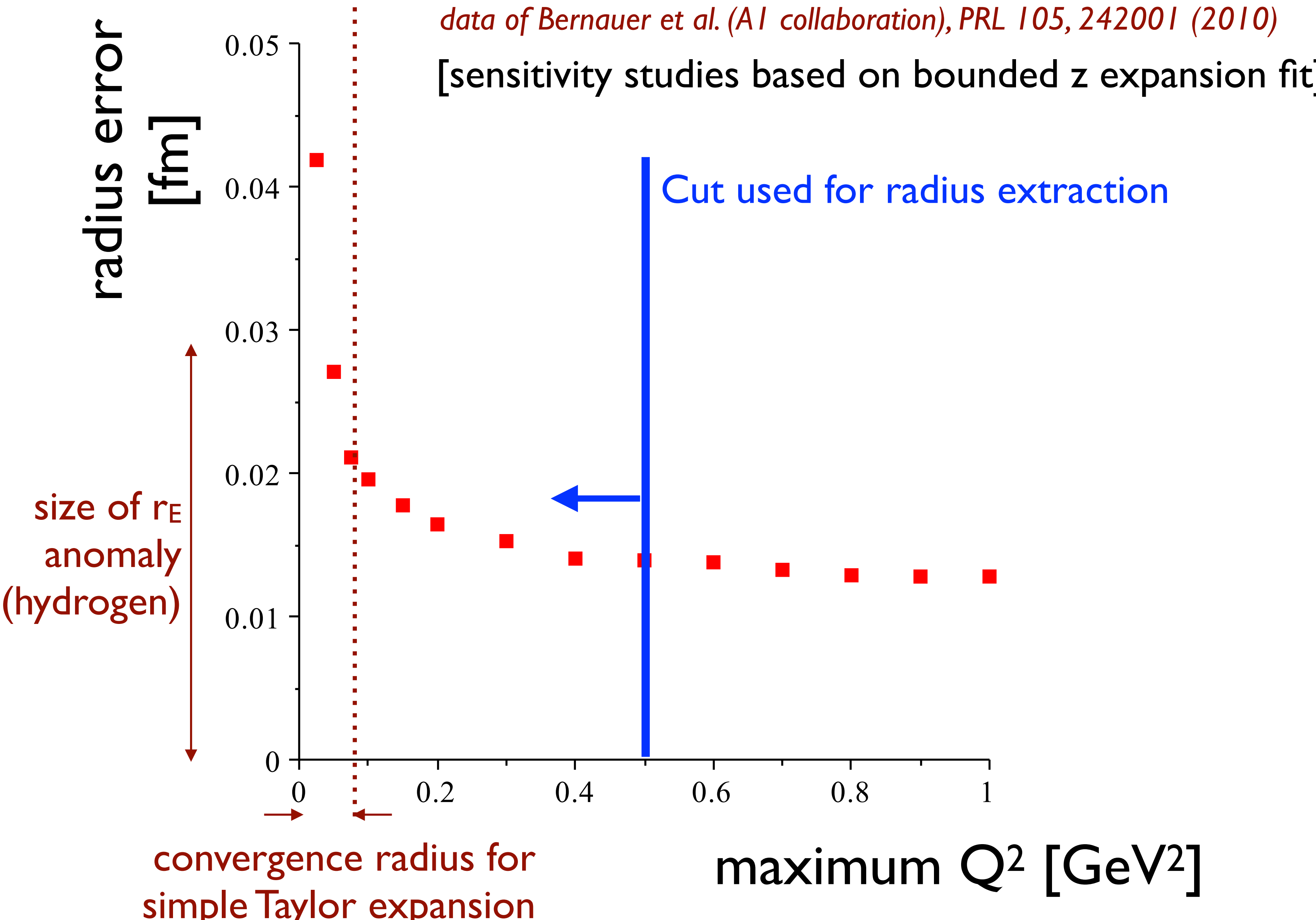
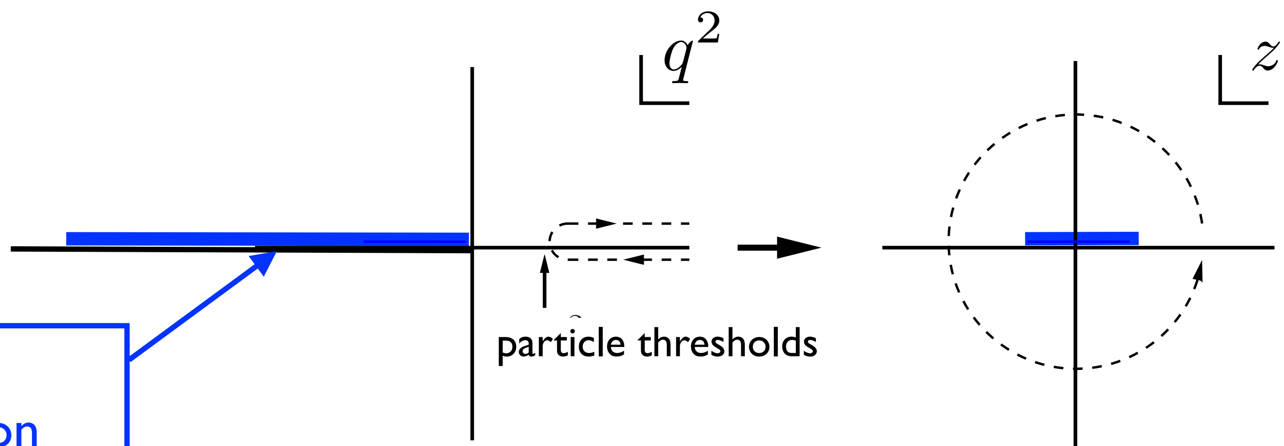


That’s ok: underlying QCD tells us that Taylor expansion of form factor in appropriate variable is convergent



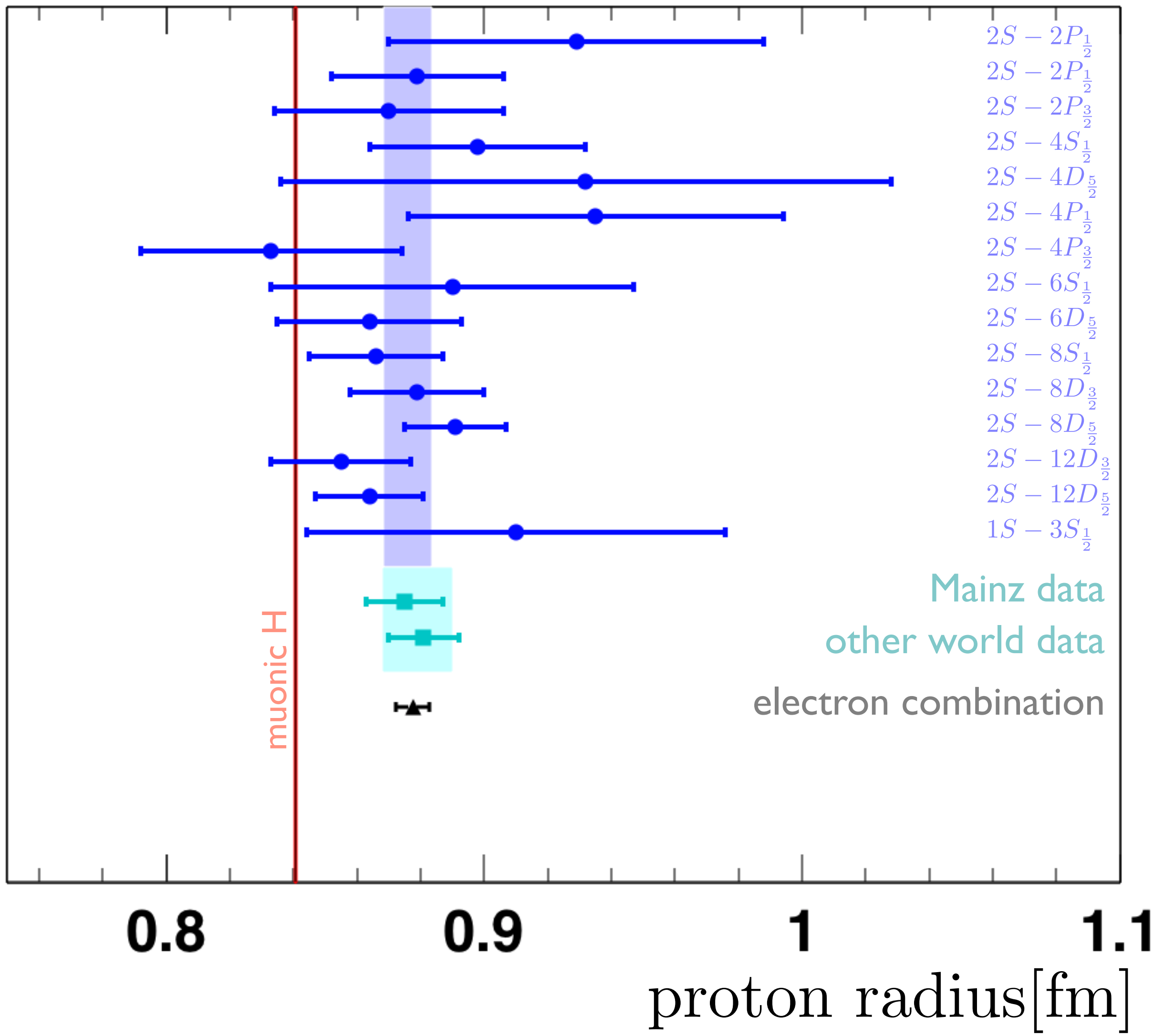
experimental
kinematic region

$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$F(q^2) = \sum_k a_k [z(q^2)]^k$$

coefficients in rapidly
convergent expansion encode
nonperturbative QCD

Reanalysis of scattering data reveals strong influence of shape assumptions



Errors larger, but discrepancy remains

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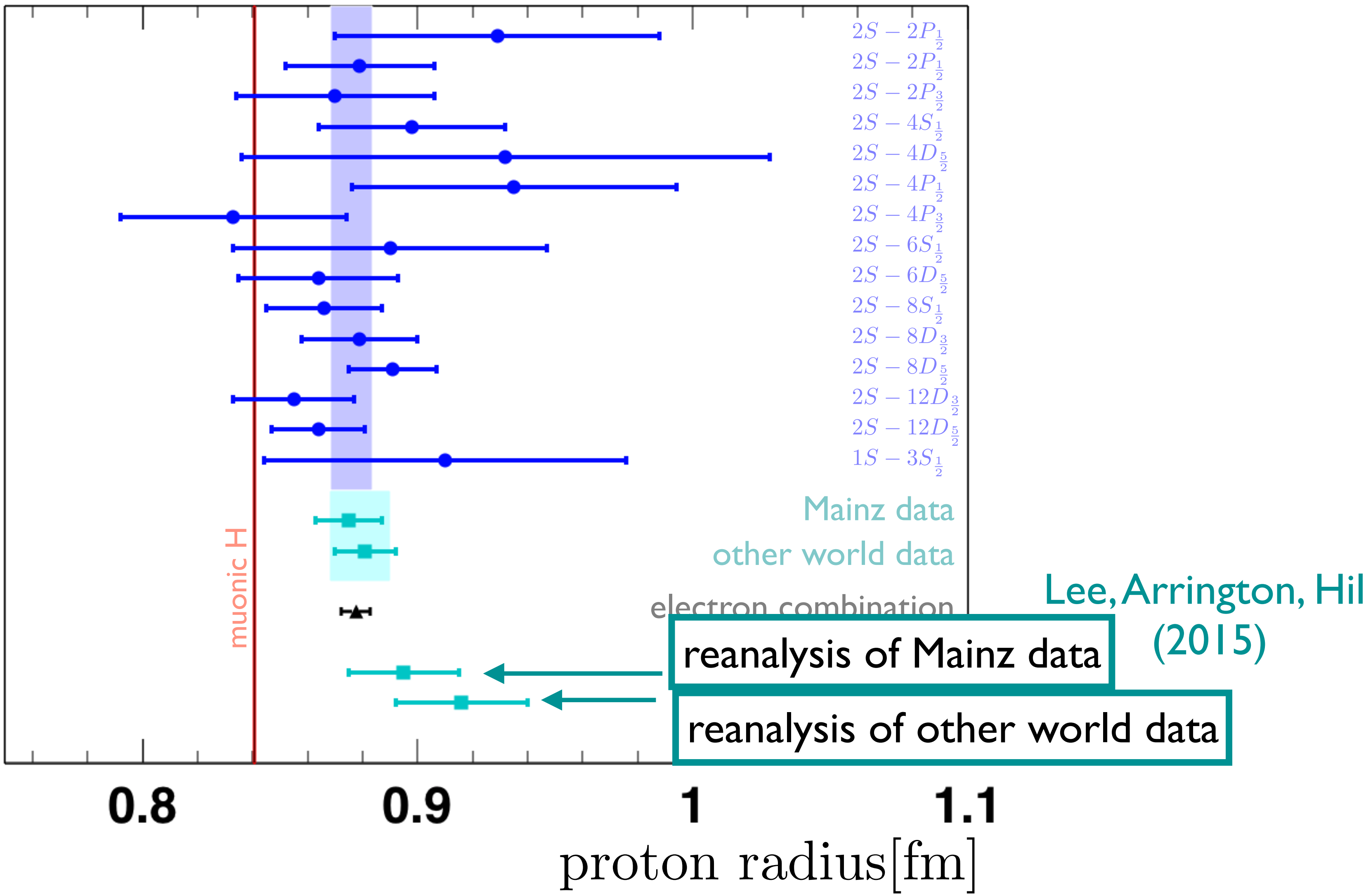

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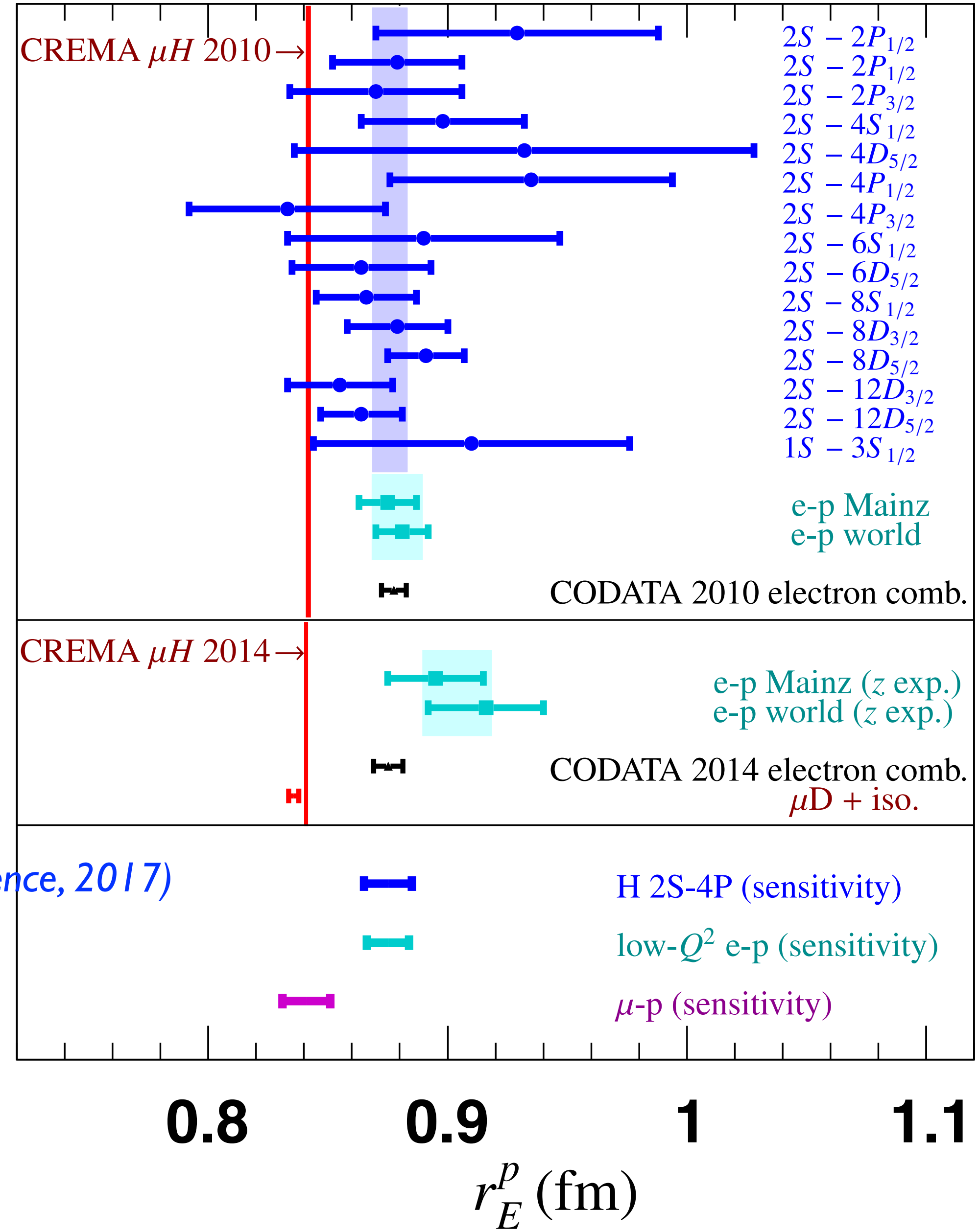
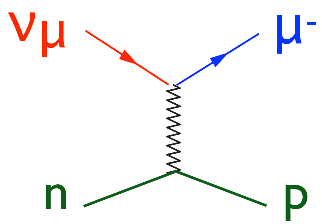


update: Beyer et al. (Science, 2017)

Similar analysis with neutrino scattering:

Start with the basic process



$$\sigma(\nu n \to \mu p) = |\cdots F_A(q^2)\cdots|^2$$

poorly known axial-vector form factor

A common ansatz for F_A has been employed for the last ~40 years:

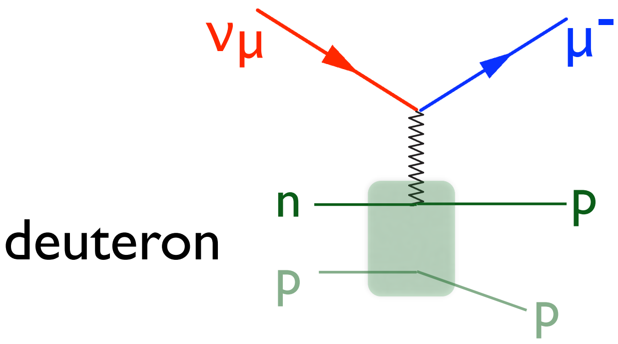
$$F_A^{\mathrm{dipole}}(q^2) = F_A(0)\left(1 - \frac{q^2}{m_A^2}\right)^{-2}$$

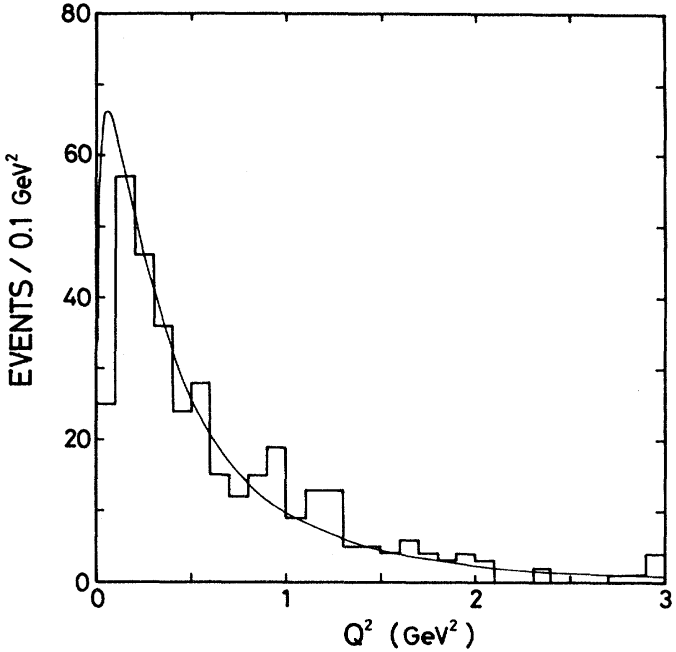
Inconsistent with QCD.

Typically quoted uncertainties are (too) small (e.g. compared to proton charge form factor!)

$$\frac{1}{F_A(0)}\frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6}r_A^2 \qquad r_A = 0.674(9)\ \mathrm{fm}$$

Best source of almost-free neutrons: deuterium





Deuterium bubble chamber data

- small(-ish) nuclear effects
- small(-ish) experimental uncertainties
- small statistics, ~3000 events in world data

Fermilab 15-foot deuterium bubble chamb
PRD 28, 436 (1983)

also:

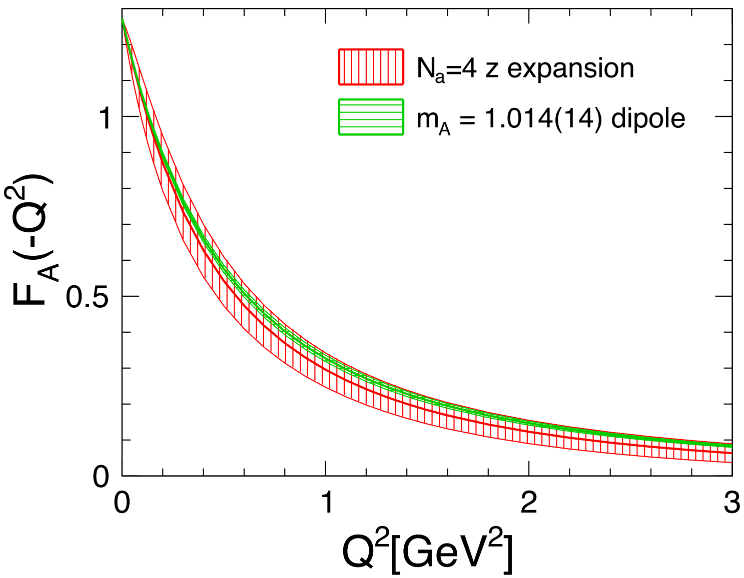
ANL 12-foot deuterium bubble chamber,
PRD 26, 537 (1982)

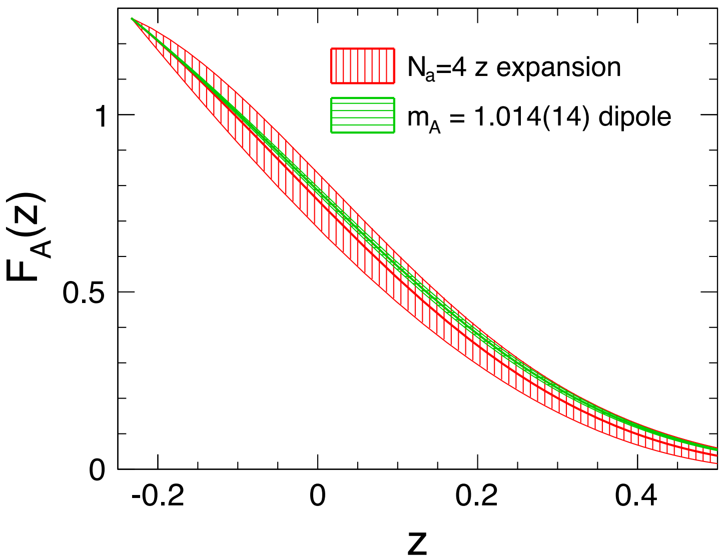
BNL 7-foot deuterium bubble chamber,
PRD23, 2499 (1981)

• F_A with complete error budget:

$$[a_1, a_2, a_3, a_4] = [2.30(13), -0.6(1.0), -3.8(2.5), 2.3(2.7)]$$

$$C_{ij} = \begin{pmatrix} 1 & 0.350 & -0.678 & 0.611 \\ 0.350 & 1 & -0.898 & 0.367 \\ -0.678 & -0.898 & 1 & -0.685 \\ 0.611 & 0.367 & -0.685 & 1 \end{pmatrix}$$





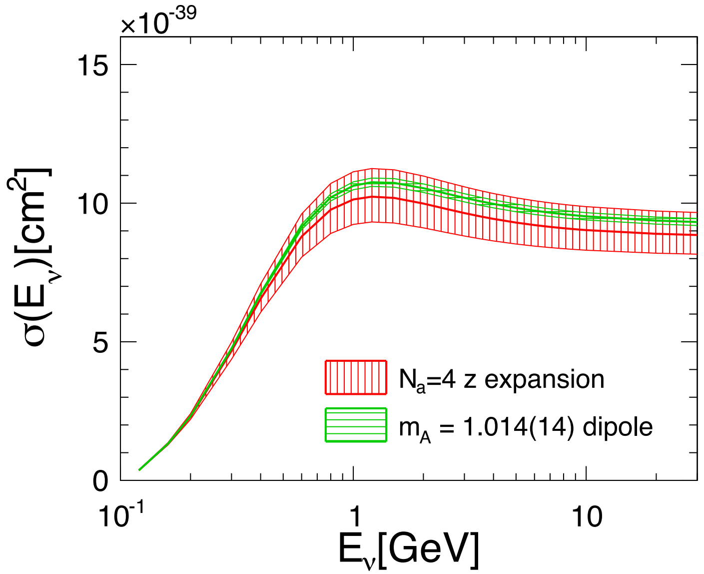
Derived observables: 1) axial radius

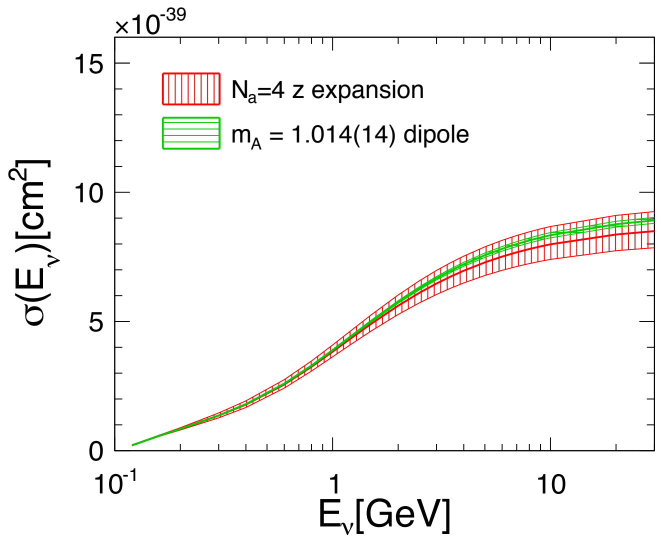
$$\frac{1}{F_A(0)}\frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6}r_A^2$$

$$r_A^2 = 0.46(22)\,\mathrm{fm}^2$$

- order of magnitude larger uncertainty compared to historical dipole fits
- impacts comparison to other data, e.g. pion electroproduction, muon capture

Derived observables: 2) neutrino-nucleon quasi elastic cross sections

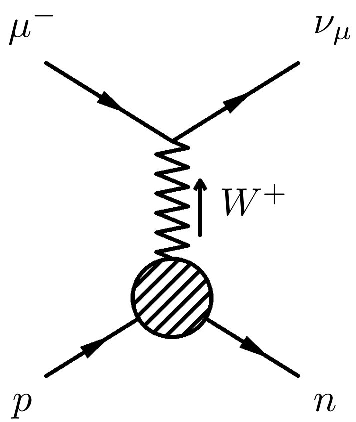




$$\sigma_{\nu n \to \mu p}(E_\nu = 1\,\mathrm{GeV}) = 10.1(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

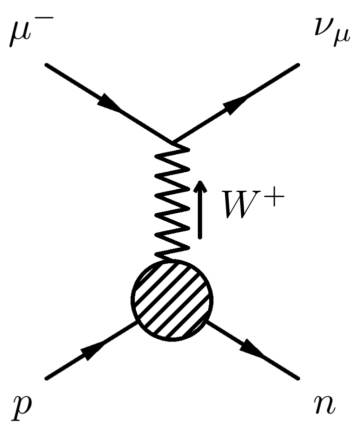
$$\sigma_{\nu n \to \mu p}(E_\nu = 3\,\mathrm{GeV}) = 9.6(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

muon capture



muon capture from ground state of muonic hydrogen:

- probes axial nucleon structure: FP, FA
- already competitive determination of rA
- potential for significant improvement



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$

$\big\downarrow$ perturbative matching

$$\mathcal{L} = -\frac{G_F V_{ud}}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu \, \bar{d}\gamma_\mu(1-\gamma_5)u + \mathrm{H.c.} + \ldots$$

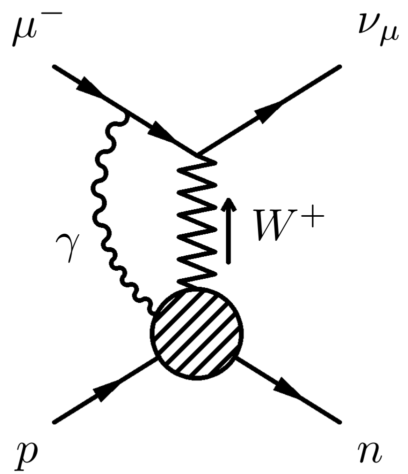
$\big\downarrow$ nonperturbative matching

$$H = \frac{p^2}{2m_r} - \frac{\alpha}{r} + \delta V_{\mathrm{VP}} - i\frac{G_F^2 |V_{ud}|^2}{2}\left[c_0 + c_1(\boldsymbol{s}_\mu + \boldsymbol{s}_p)^2\right]\delta^3(\boldsymbol{r})$$

$$\Lambda = \underbrace{G_F^2 |V_{ud}|^2}_{\text{weak}} \times \underbrace{[c_0 + c_1 F(F+1)]}_{\text{hadronic}} \times \underbrace{|\psi_{1\mathrm{S}}(0)|^2}_{\text{atomic}} + \ldots$$

factorization: weak hadronic atomic

$$c_0 = \frac{E_\nu^2}{2\pi M^2}(M-m_n)^2\left[\frac{2M-m_n}{M-m_n}F_1(q_0^2) + \frac{2M+m_n}{M-m_n}F_A(q_0^2) - \frac{m_\mu}{2m_N}F_P(q_0^2) + (2M+2m_n-3m_\mu)\frac{F_2(q_0^2)}{4m_N}\right]^2$$



expansion in small quantities:

$$\epsilon \sim \alpha \sim \frac{m_\mu^2}{m_\rho^2} \sim \frac{m_u - m_d}{m_\rho} \lesssim 10^{-2}$$

- axial radius enters at first order in epsilon, so need all other first order corrections (to ~10%, for a 10% measurement of rA2)

- will see that other corrections are at first-and-a-half order; need to ensure against numerical enhancements (need these to ~100%)

momentum expansion:

sensitivity to momentum dependence in the capture process

$$q_0^2 \equiv m_\mu^2 - 2m_\mu E_\nu = -0.8768\, m_\mu^2 \quad \sim \epsilon$$

in our power counting, rA2 competes with gP, and other well-determined quantities (g≡normalization, r2≡slope)

$$1 + \Big[g_1,\ g_A\Big] + \sqrt{\epsilon}\Big[g_2\Big] + \epsilon\Big[r_1^2,\ r_A^2,\ g_P\Big] + \ldots$$

gA: neutron lifetime ✔

g1,g2,r12: e-p, e-n scattering + H, muH ✔ ✔

$$F_P(q_0^2) = \frac{2m_N g_{\pi NN} f_\pi}{m_\pi^2 - q_0^2} - \frac{1}{3}\, g_A\, m_N^2\, r_A^2 + \ldots$$

gpiNN: pion-nucleon scattering, and NN scattering ✔

α expansion:

$$\mathrm{RC} = \underbrace{\mathrm{RC(electroweak)}}_{\text{matching, running in 4 Fermi theory}} + \overbrace{\mathrm{RC(finite\ size)}}^{\text{hadronic matrix element}} + \underbrace{\mathrm{RC(electron\ VP)}}_{\text{computed within QM}}$$

$$\mathrm{RC(electroweak)} = \frac{\alpha}{2\pi}\left[\overbrace{4\log\frac{m_Z}{m_p}}^{\text{large log}} \underbrace{- 0.595 + 2C}_{\text{finite terms (estimate with OPE)}} + \overbrace{g(m_\mu, \beta_\mu = 0)}^{\text{Sirlin g function (IR subtraction)}}\right] + \cdots = +0.0237(10) \quad ✔$$

$$\mathrm{RC(finite\ size)} = -0.005(1)$$

(should be done better: computed in large nucleus ansatz rE>>rA) ✔

$$\mathrm{RC(electron\ VP)} = +0.0040(2).$$ ✔

isospin violation:

vector form factors: CC from isovector NC

deviations in F1(0): second order in IV (definition of CVC)

deviations in F1(q2): first order in IV plus first order in q2

deviations in F2(0): first order in IV plus 0.5 order in kinematic prefactor (numerical estimate: 3.2e-4 << %)

2nd class currents:

$$\langle n|(V^\mu - A^\mu)|p\rangle = \bar{u}_n \Big[F_1(q^2)\gamma^\mu + \frac{iF_2(q^2)}{2m_N}\sigma^{\mu\nu}q_\nu - F_A(q^2)\gamma^\mu\gamma^5 - \frac{F_P(q^2)}{m_N}q^\mu\gamma^5 + \frac{F_S(q^2)}{m_N}q^\mu - \frac{iF_T(q^2)}{2m_N}\sigma^{\mu\nu}q_\nu\gamma^5 \Big] u_p + \dots ,$$

contribution of FS,FT: first order in IV plus 0.5 order in kinematic prefactor results:

$$\bar{g}_P{}^{\text{MuCap}}\big|_{r_A^2=0.46(22)\,\text{fm}^2} = 8.19\ (48)_{\text{exp}}\ (69)_{\bar{g}_A}\ (6)_{\text{RC}} = 8.19(84)$$

$$\bar{g}_P{}^{\text{theory}} = 8.25(25)$$

$$g_{\pi NN}^{\text{MuCap}} = 13.04\ (72)_{\text{exp}}\ (8)_{g_A}\ (67)_{r_A^2}\ (10)_{\text{RC}} = 13.04(99)$$

$$g_{\pi NN}^{\text{external}} = 13.12(10)$$

turning the tables, take QCD for granted and extract rA2:

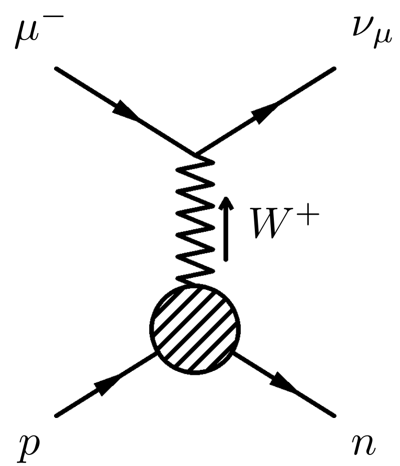
$$r_A^2(\text{MuCap}) = 0.43\ (24)_{\text{exp}}\ (3)_{g_A}\ (3)_{g_{\pi NN}}\ (3)_{\text{RC}} = 0.43(24)\ \text{fm}^2$$

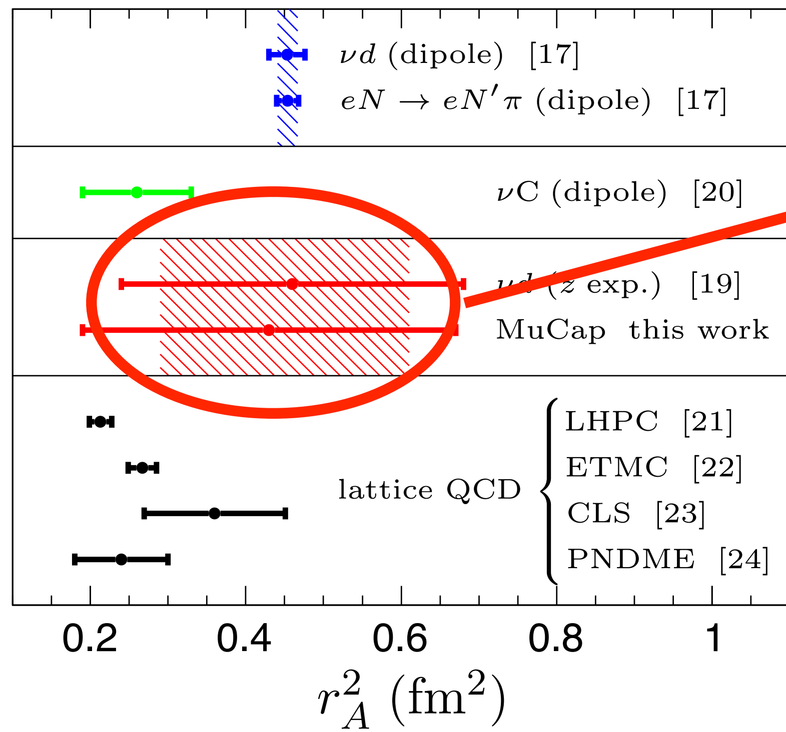
competitive with other methods with existing data, and potential for improvement

$$\delta r_A^2(\text{future exp.}) = (0.08)_{\text{exp}}\ (0.03)_{g_A}\ (0.03)_{g_{\pi NN}}\ (0.03)_{\text{RC}} = 0.10\ \text{fm}^2$$

factor 3 improvement

muon capture constraints





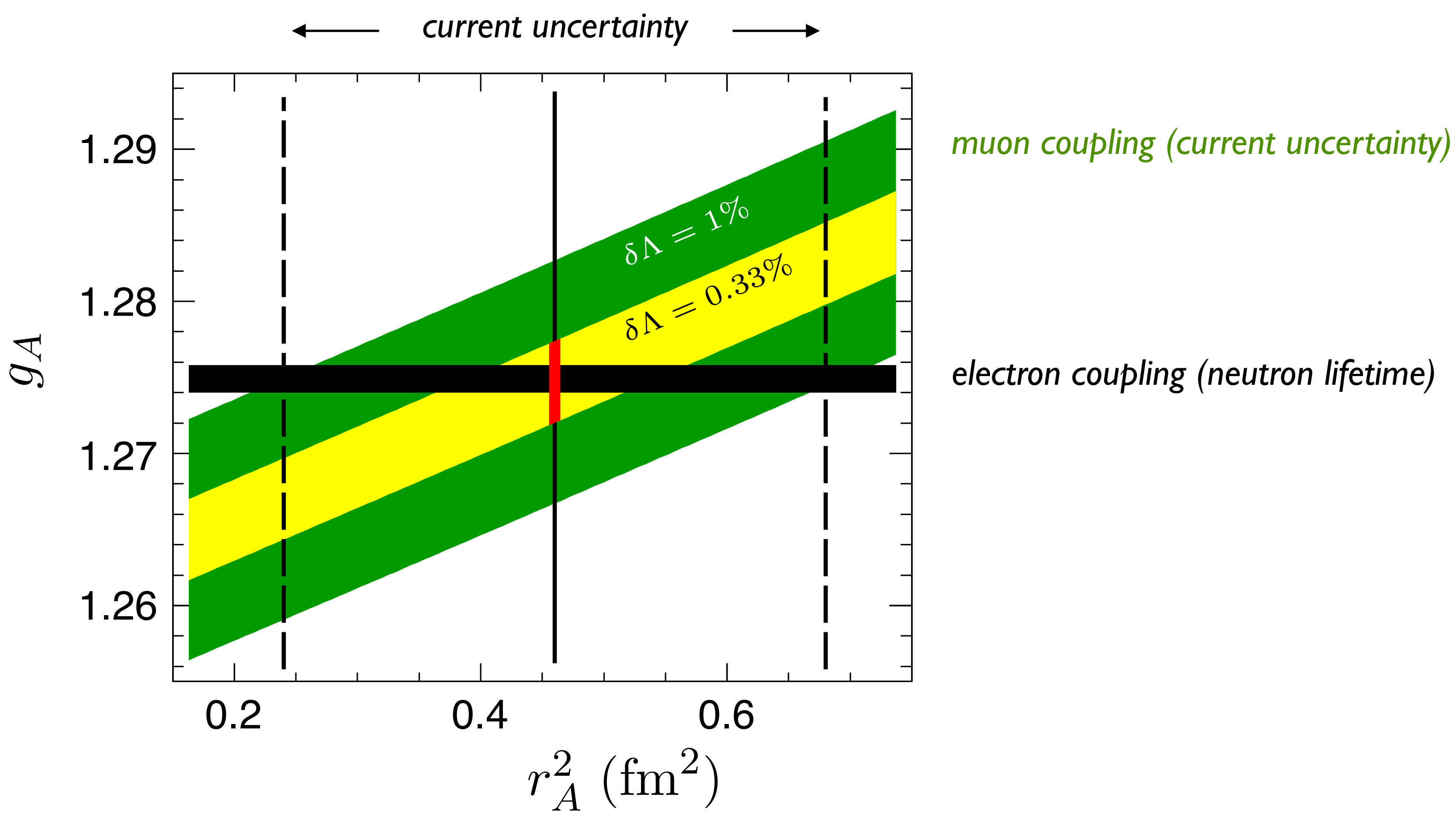
complete error budgets

RJH, Kammel, Marciano, Sirlin 1708.08462

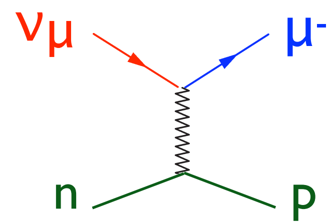
lattice average: see also Yao, Alvarez-Ruso, Vicente-Vacas 1708.08776 [rA2=0.26(4)]

- potential factor ~3 improvement from next generation muon capture experiment implications

test of electron-muon universality

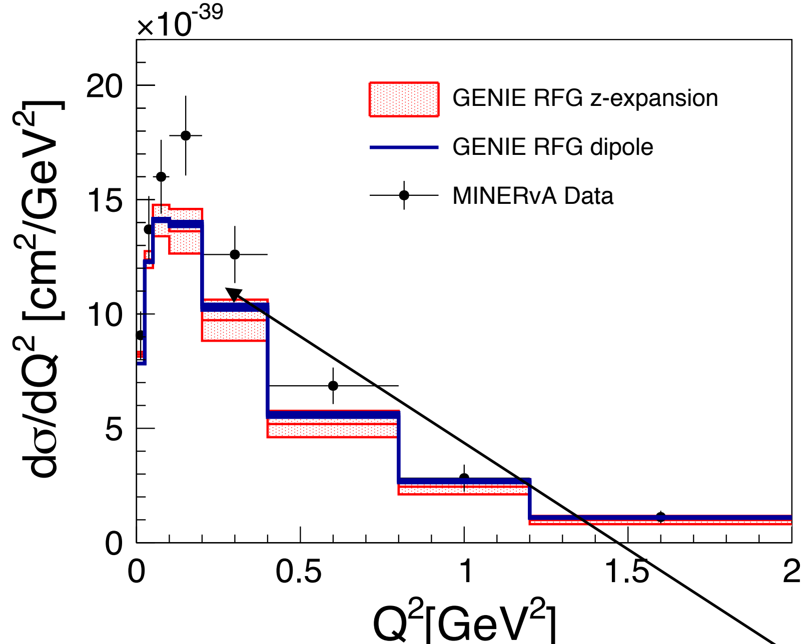


discriminating nuclear models



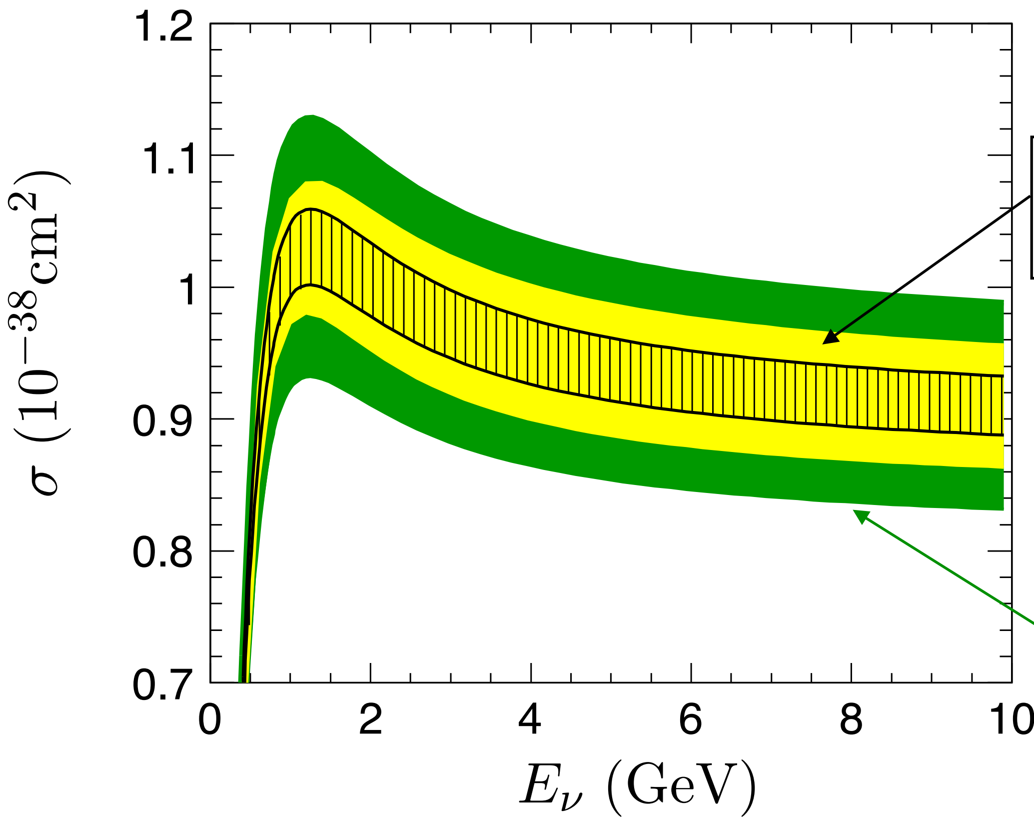
$$\sigma(\nu n \to \mu p) = |\cdots F_A(q^2) \cdots|^2$$

poorly known axial form factor



want to extract nuclear and flux effects from this comparison: but large nucleon level form factor uncertainty

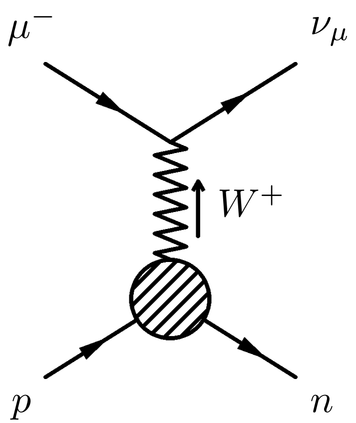
implications for quasielastic neutrino cross sections



with radius constraint: (hatched: external radius error $\delta r_A{}^2$=20%)

existing error (no external radius constraint)

Summary



Nucleon properties
capture rate on proton = measurement of nucleon structure

Nuclear properties
capture rate = constraint on nuclear model

Standard candle: experiment
e.g. muon capture as a test source

Standard candle: theory
e.g. muon capture as a test source

muon capture from ground state of muonic hydrogen:

- probes axial nucleon structure: FP, FA

- already competitive determination of rA

- potential for significant improvement

THANK YOU