# How much does matter matter for DUNE?

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January 25th, 2018

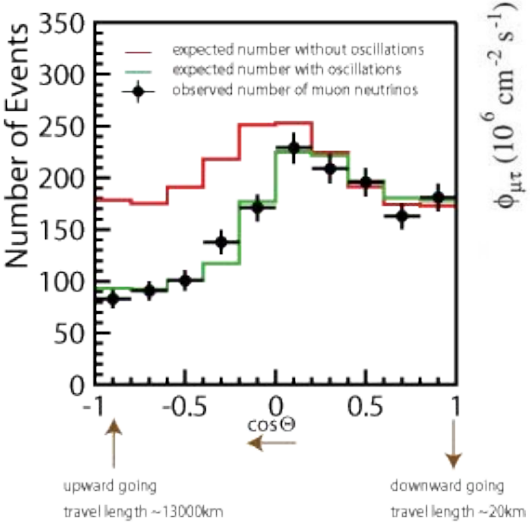
Based on 1802.XXXXX with Stephen J. Parke (FNAL)

# Outline

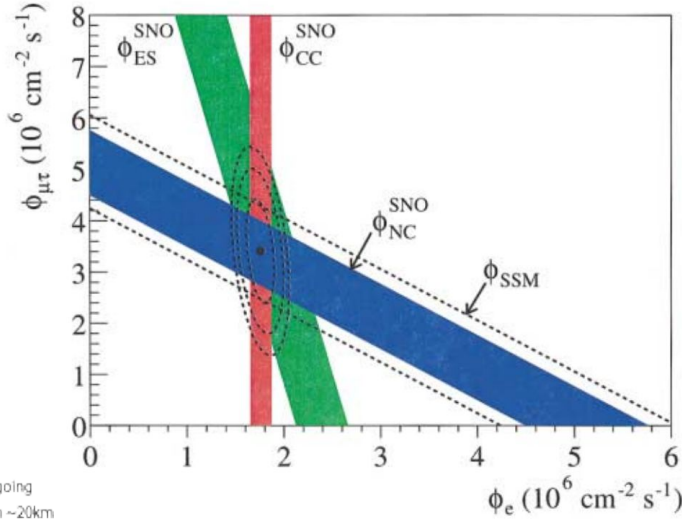
- Neutrino oscillations and matter effects
- Current knowledge of Earth's density profile
- DUNE sensitivity
  - Oscillation parameters
  - Matter density profile
  - Perturbative approaches

# Neutrino Oscillations

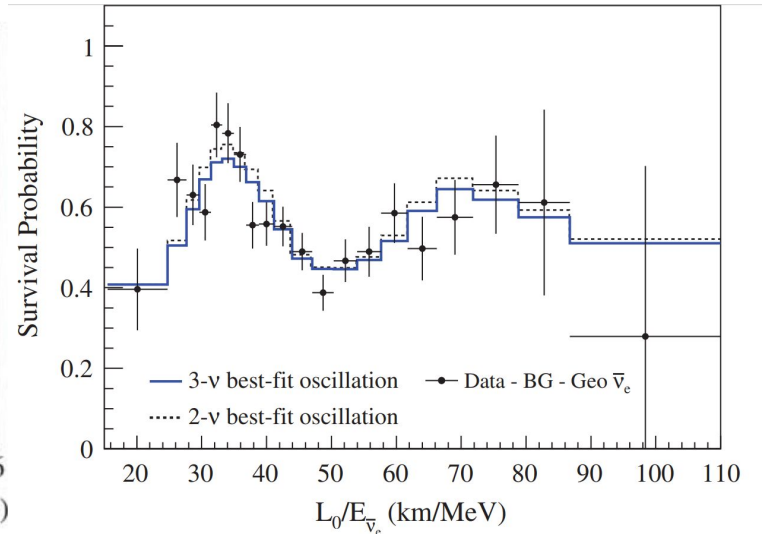
# Evidence for Oscillations




Atmospheric




Solar




Reactor

# Best explanation for these observations:

- Three neutrinos with different masses
- Mixing between mass eigenstates and flavor (interaction) eigenstates
- Large mixing angles and hierarchical mass splittings -- distinct “sectors” for oscillations.

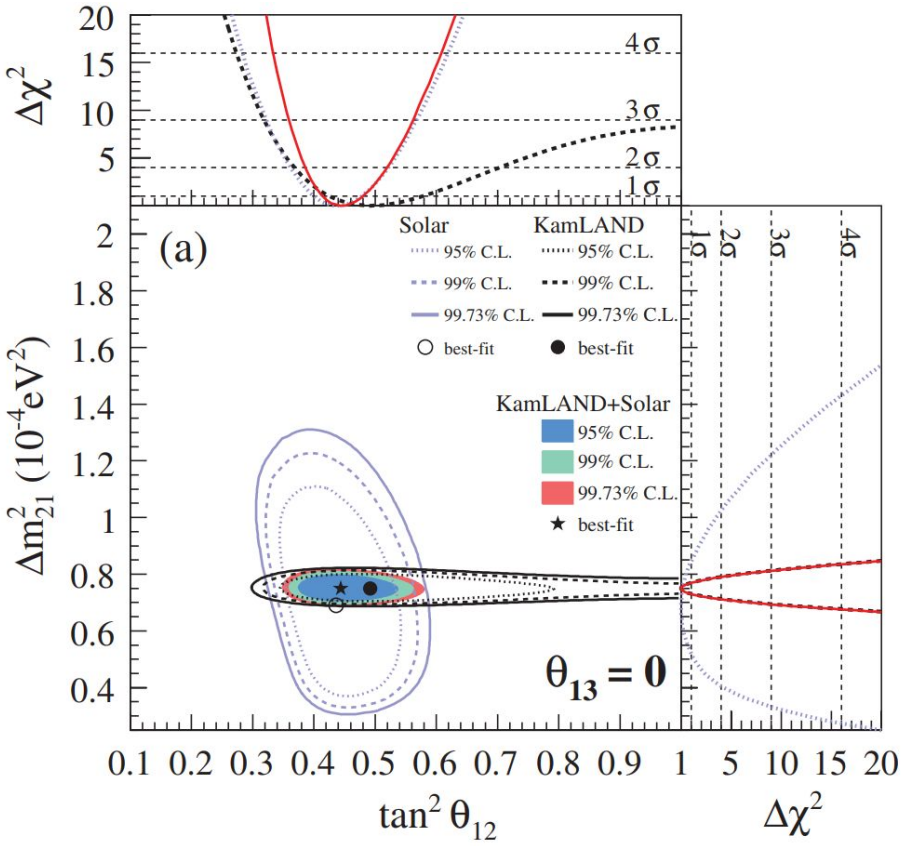
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$H_{ij} = \frac{1}{2E_\nu}\mathrm{diag}\left\{0, \Delta m^2_{21}, \Delta m^2_{31}\right\}$$

$$P_{\alpha\beta} = \left|\langle\nu_\beta| U e^{-iH_{ij}L} U^\dagger |\nu_\alpha\rangle\right|^2$$

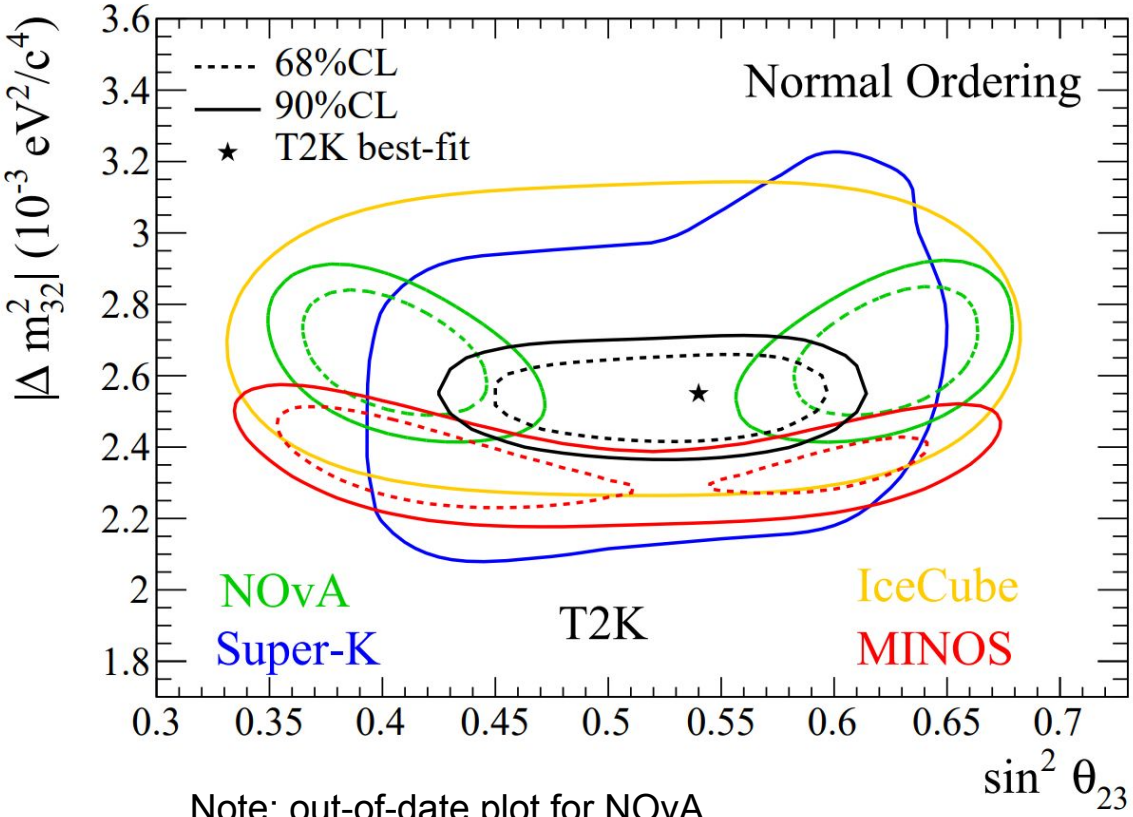
# Entering the age of Precision Neutrino Physics

Solar sector



# Entering the age of Precision Neutrino Physics

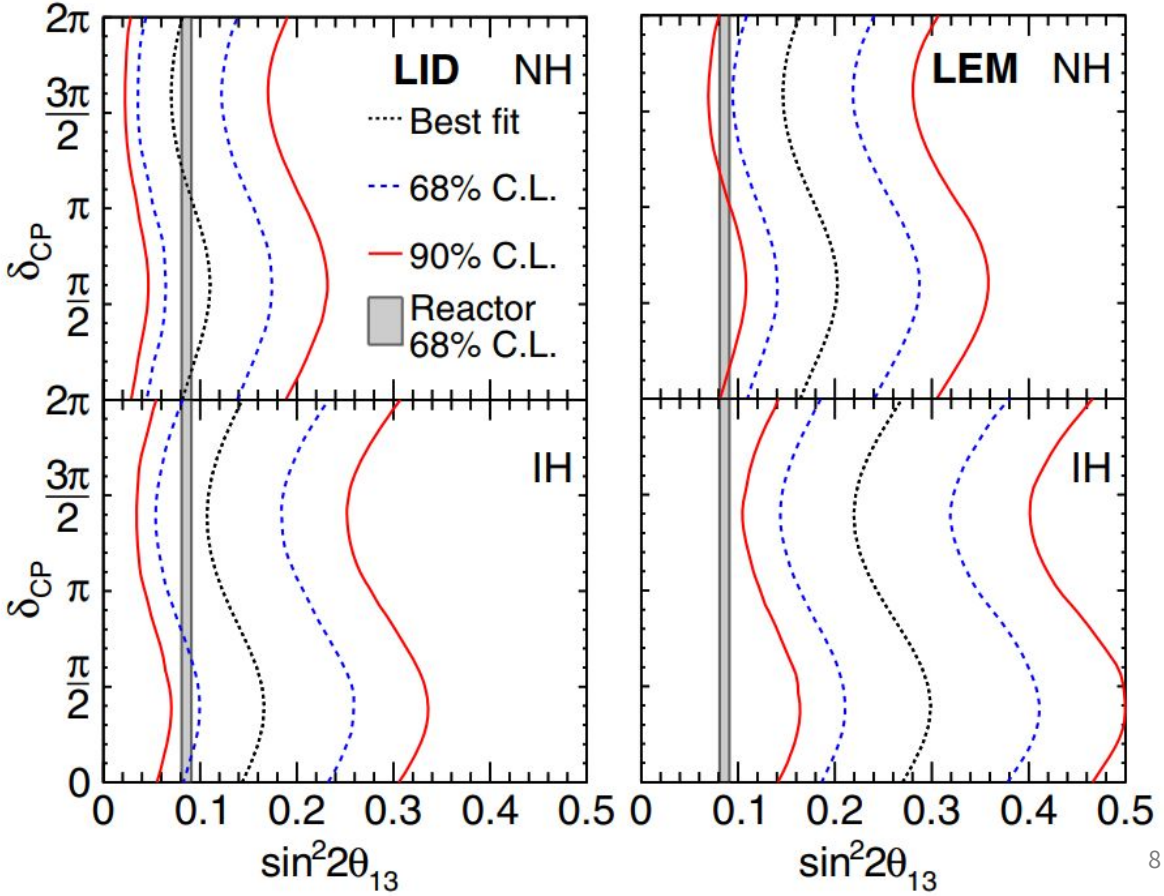
Atmospheric sector




Note: out-of-date plot for NOvA

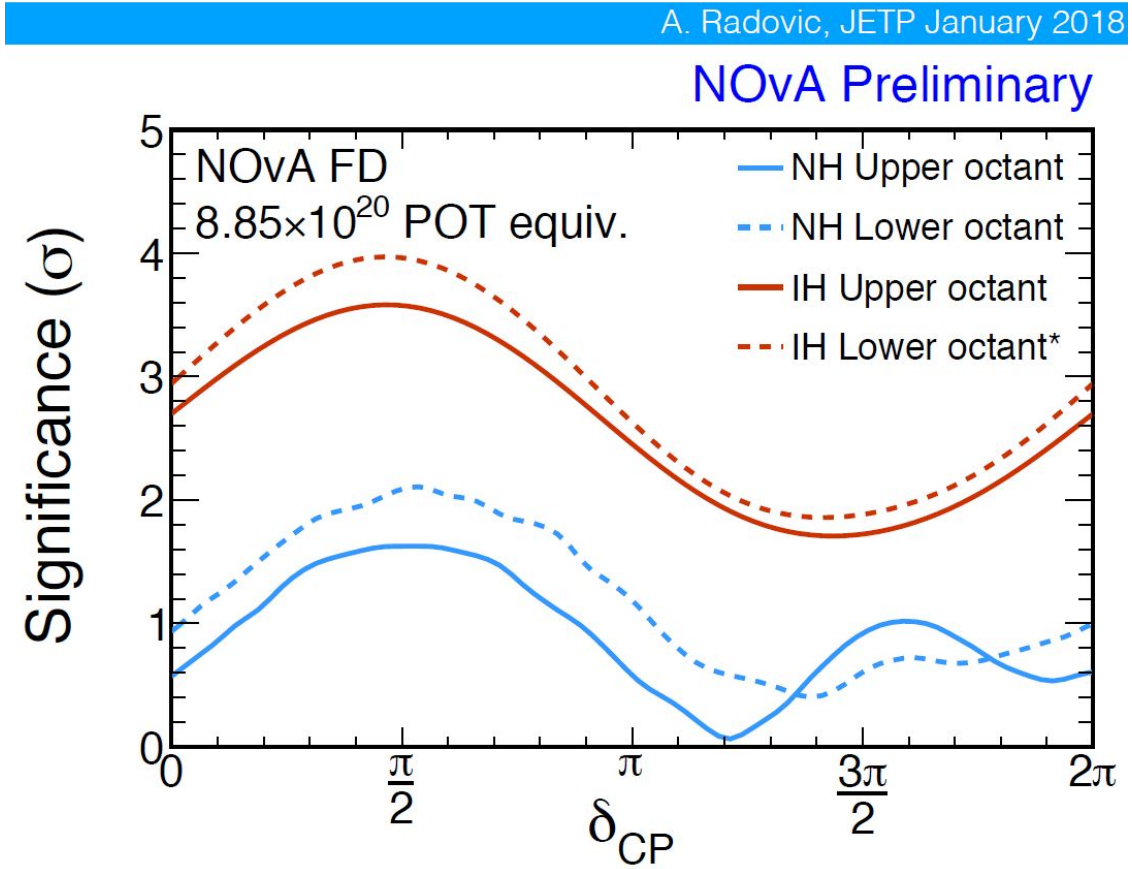
# Entering the age of Precision Neutrino Physics

Reactor sector



# Entering the age of Precision Neutrino Physics

Reactor sector



# Not so fast, however...

$$H_{ij} = \frac{1}{2E_\nu} \mathrm{diag}\left\{0, \Delta m^2_{21}, \Delta m^2_{31}\right\}$$ ← Only valid in vacuum

$$P_{\alpha\beta} = \left|\langle \nu_\beta | \, U e^{-iH_{ij}L} U^\dagger \, | \nu_\alpha \rangle\right|^2$$ ← Only valid with constant H.

Full Schrödinger Equation:

$$i\frac{\partial}{\partial x} |\nu\rangle = H |\nu\rangle \,, \qquad |\nu\rangle = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

# Not so fast, however...

$$H_{ij} = \frac{1}{2E_\nu}\mathrm{diag}\left\{0, \Delta m^2_{21}, \Delta m^2_{31}\right\}$$ ← Only valid in vacuum

$$P_{\alpha\beta} = \left|\langle\nu_\beta| U e^{-iH_{ij}L} U^\dagger |\nu_\alpha\rangle\right|^2$$ ← Only valid with constant H.

Or, piecewise-constant Hamiltoninan:

$$P_{\alpha\beta} = \left|\langle\nu_\beta| U \left(\prod_{n=1}^{N} e^{-iH^{(n)}_{ij}L_n}\right) U^\dagger |\nu_\alpha\rangle\right|^2$$

# Two-flavor Solution

Constant matter density probabilities are exactly solvable for two-neutrino oscillations:

$$P^{\text{vac.}} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

# Two-flavor Solution

Constant matter density probabilities are exactly solvable for two-neutrino oscillations:

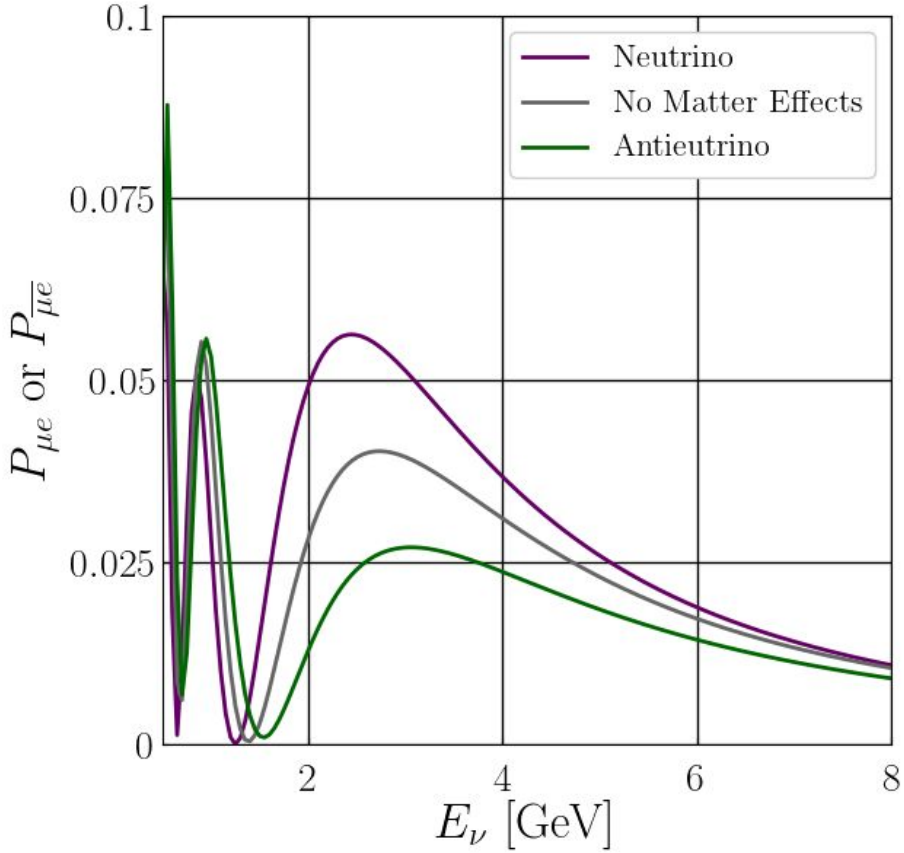
$$P^{\text{mat.}} = \frac{\sin^2(2\theta)}{(1 - 2\eta\cos(2\theta) + \eta^2)} \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\sqrt{1 - 2\eta\cos(2\theta) + \eta^2}\right)$$

$$\eta = \frac{2AE_\nu}{\Delta m^2}, \quad A = \sqrt{2}G_F n_e \simeq 10^{-4}\frac{\text{eV}^2}{\text{GeV}}$$

Modified mass-splitting

Modified Mixing Angle

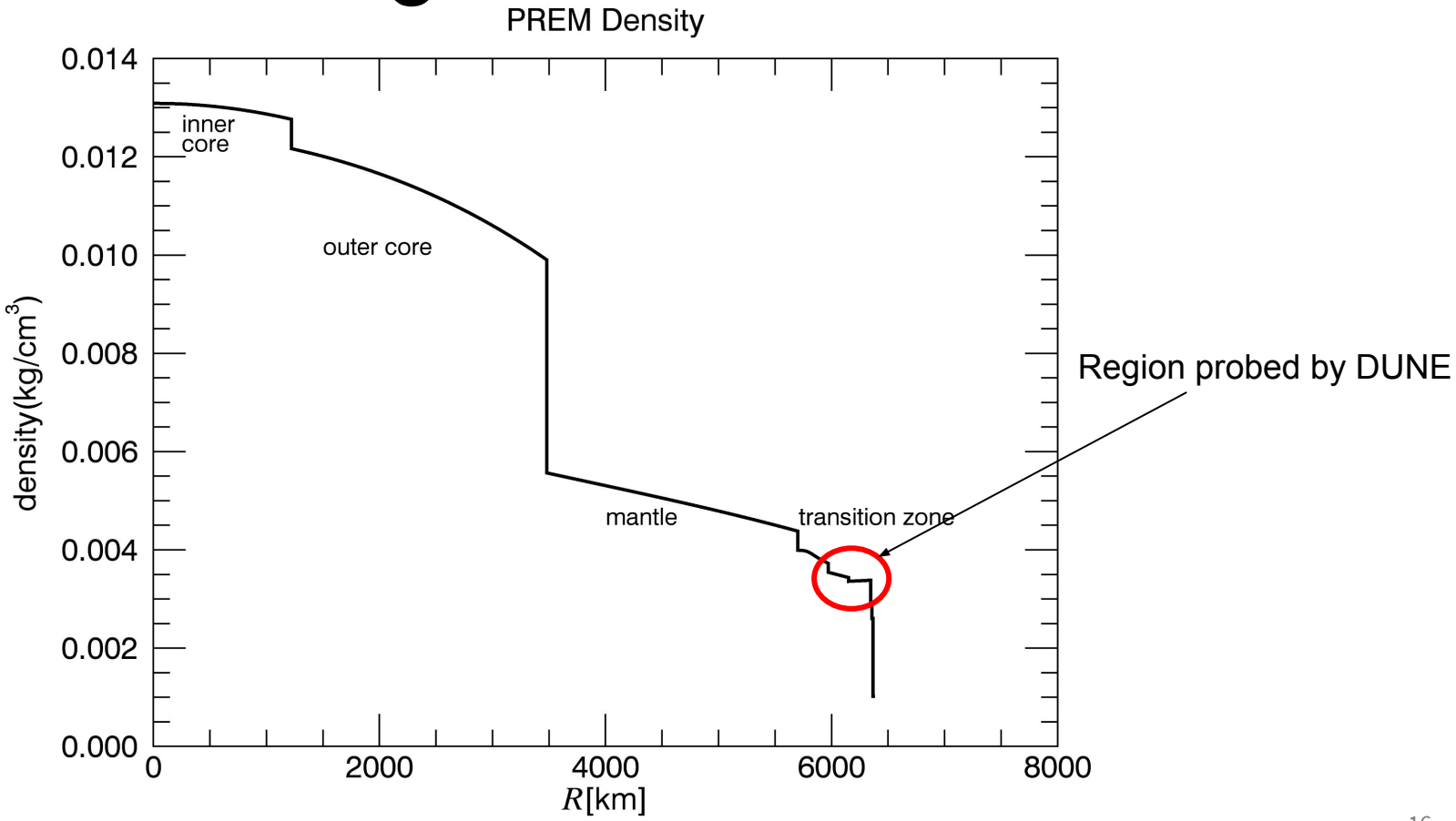
# Full, three-flavor probabilities



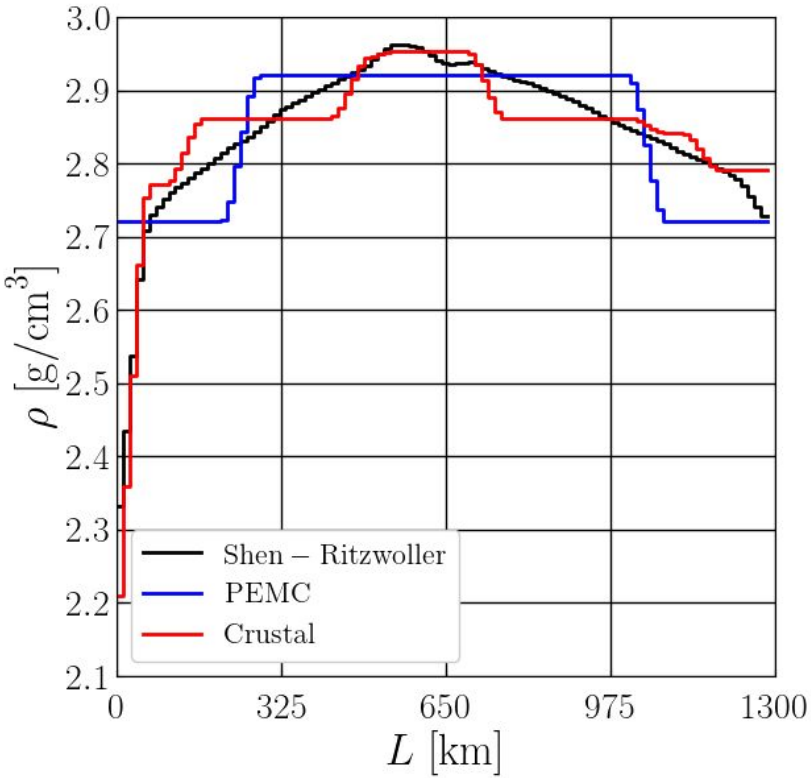

CP-violating effect even when the CP-violating phase is zero.

# The Earth’s Density Profile

# Current Knowledge



# Density as a function of propagation length




Three maps considered in B. Roe: [Phys.Rev. D95 (2017) no.11, 113004], all normalized to an average density of 2.845 g/cm$^3$.

Deviations from flat (e.g. PEMC -- Parametric Earth Model Continental) are on the order of 10% of the average density.

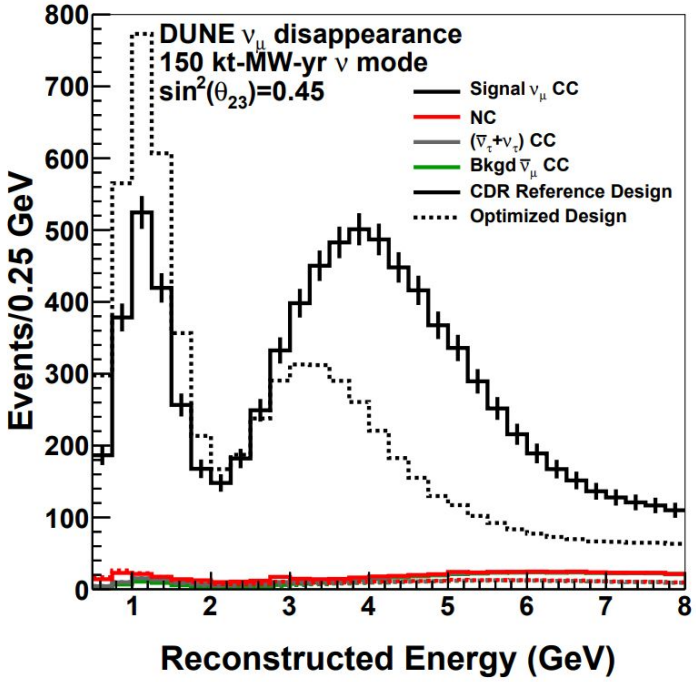
Big questions:

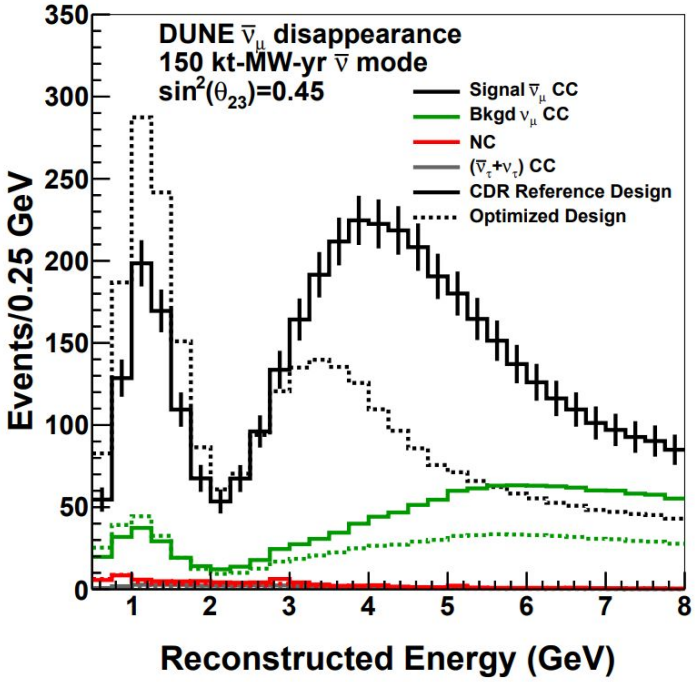
- Do differences between matter density profiles lead to measurable differences in oscillation probabilities at DUNE?
- Can uncertainty about density profile lead to issues measuring neutrino parameters?

Approx. Depth: 0 ---------- 30km ----------1.5 km

# DUNE Capabilities

# Zeroth-order Estimate of Sensitivity



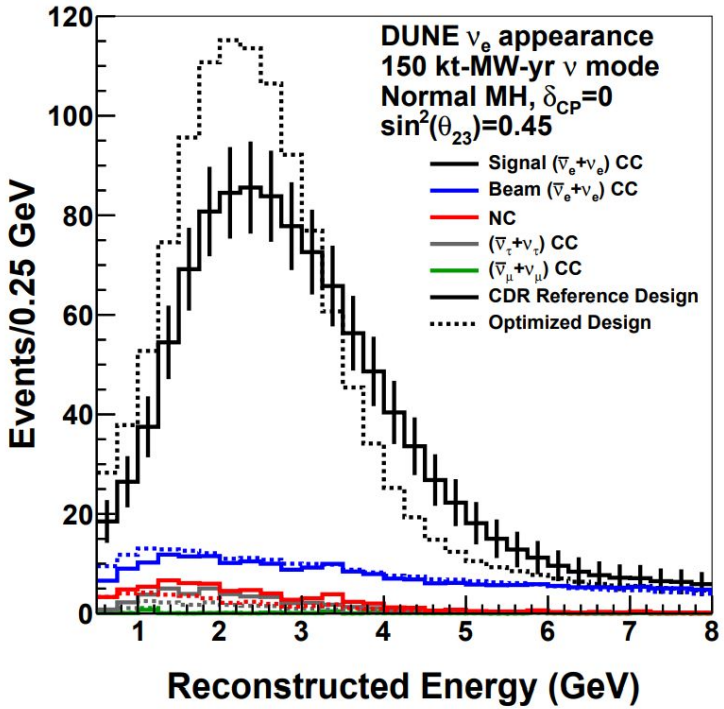



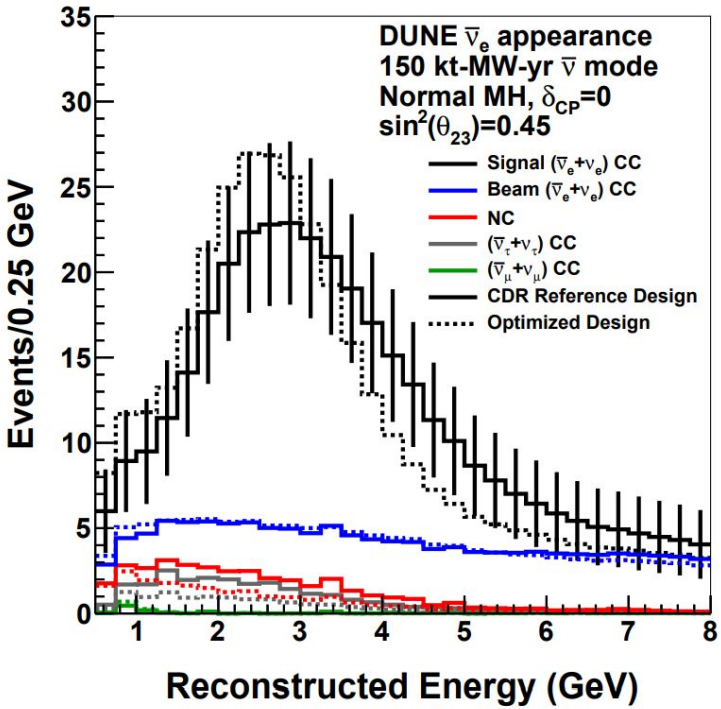

30 bins/channel

Number of events (in absence of oscillations) in a bin: ~500 (neutrino), ~250 (antineutrino)

$$N = S(E_\nu; \cdots) \times P_{\alpha\beta}$$

# Zeroth-order Estimate of Sensitivity




30 bins/channel

$$N = S(E_\nu; \cdots) \times P_{\alpha\beta}$$

# Precision on Probability Measurement

S: number of unoscillated events in a bin

$$P = \frac{N}{S}$$

| $P_{\alpha\beta}$ | $N$ | $1/\sqrt{N} \simeq \lvert\Delta P\rvert/P$ | $\Delta P$ |
|---|---|---|---|
| $1$ | $10^3$ | $3\%$ | $3\%$ |
| $10^{-1}$ | $10^2$ | $10\%$ | $1\%$ |
| $10^{-2}$ | $10$ | $30\%$ | $0.3\%$ |
| $10^{-3}$ | $1$ | $100\%$ | $0.1\%$ |

$$\Delta P = \frac{\sigma_N}{S} = \frac{\sqrt{N}}{S} = \sqrt{\frac{P}{S}}$$

S = 1000

$$\frac{\Delta P}{P} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{SP}}$$

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| $P_{\alpha\beta}$ | $N$ | $1/\sqrt{N} \simeq \lvert\Delta P\rvert/P$ | $\Delta P$ |
|---|---|---|---|
| $1$ | $10^5$ | $0.3\%$ | $0.3\%$ |
| $10^{-1}$ | $10^4$ | $1\%$ | $0.1\%$ |
| $10^{-2}$ | $10^3$ | $3\%$ | $0.03\%$ |
| $10^{-3}$ | $10^2$ | $10\%$ | $0.01\%$ |

$$\Delta P = \frac{\sigma_N}{S} = \frac{\sqrt{N}}{S} = \sqrt{\frac{P}{S}}$$

S = 100,000

$$\frac{\Delta P}{P} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{SP}}$$

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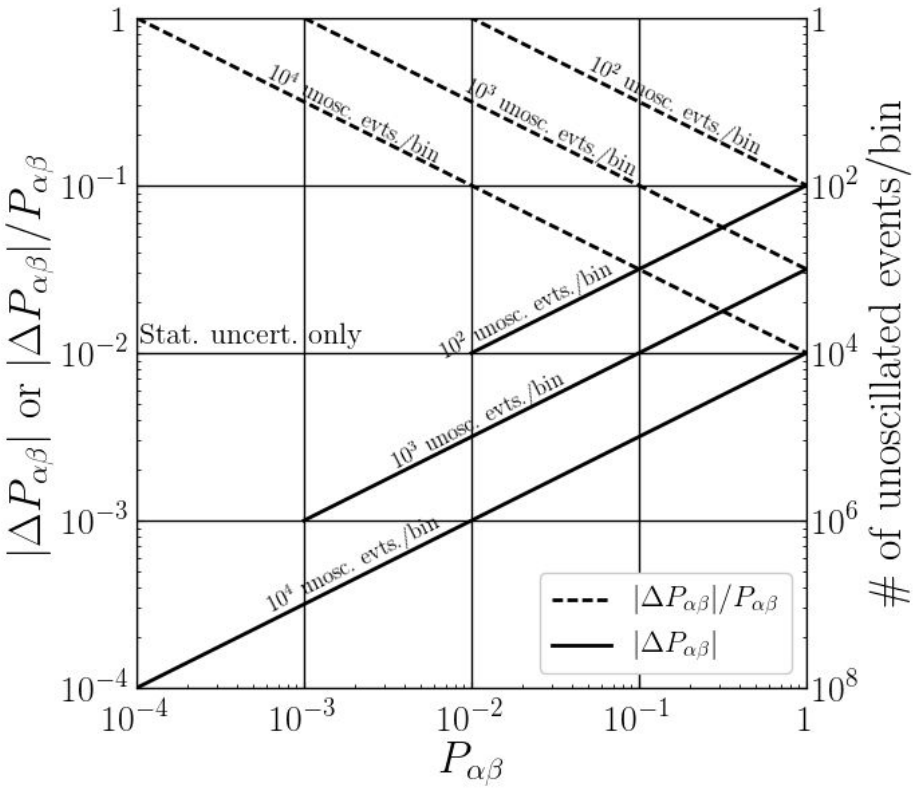
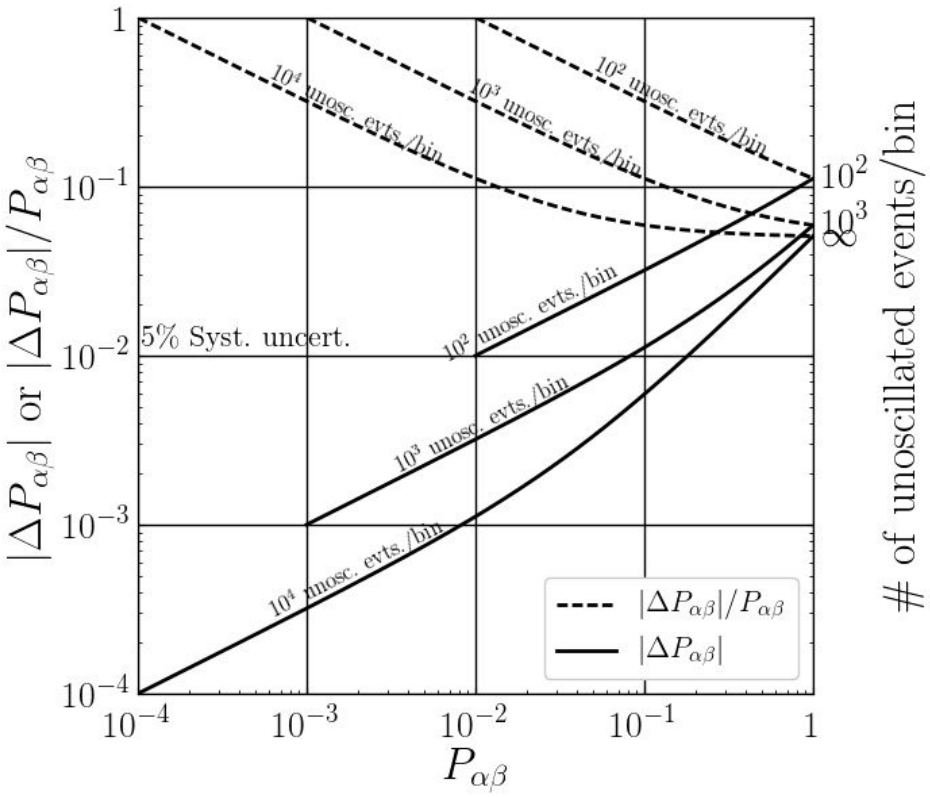


# Uncertainty on Flux, etc.?

$$P = \frac{N}{S}$$

$$\frac{\Delta P}{P} = \sqrt{\left(\frac{\sigma_N}{N}\right)^2 + \left(\frac{\sigma_S}{S}\right)^2}$$

$$\frac{\Delta P}{P} = \sqrt{\frac{1}{SP} + s^2}$$

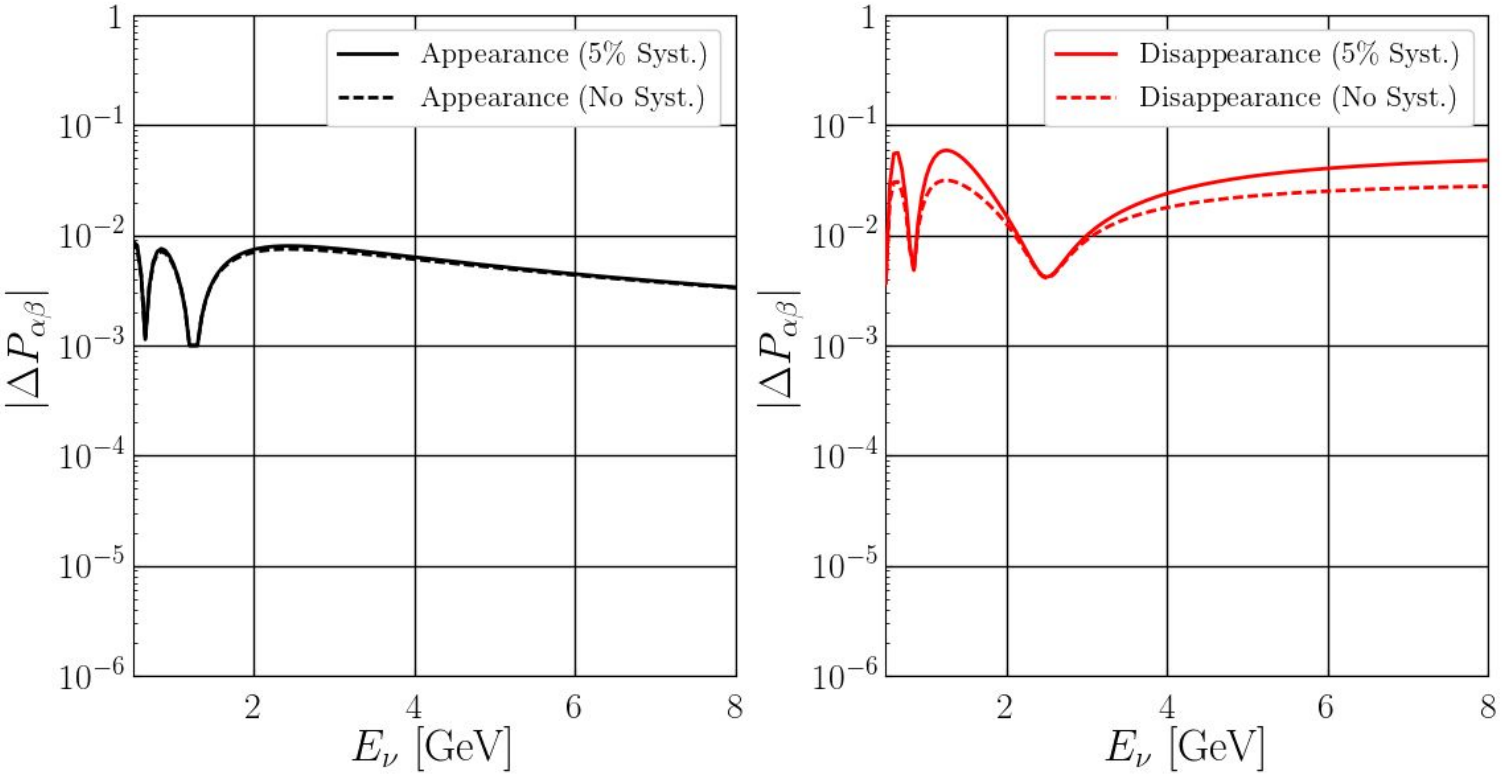



Systematic uncertainties only make things worse!

# Naïve Estimate of Probability Sensitivity

Caveats: Independent measurements, 1000 events (unoscillated in *each* bin)

Neutrino Probabilities: Antineutrinos qualitatively similar



# Actual Capability of DUNE - Parameter Measurements

| Parameter | Physical Value | $1\sigma$ Range |
|---|---|---|
| $\sin^2\theta_{23}$ | $0.450$ | $[0.442, 0.458]$ |
| $\delta$ | $0$ | $[-0.2, 0.2]$ |
| | $\pi/2$ | $[1.37, 1.77]$ |
| $\sin^2(2\theta_{13})$ | $0.085$ | $[0.080, 0.090]$ |
| $\Delta m^2_{31}$ | $2.457 \times 10^{-3}\ \mathrm{eV}^2$ | $[2.447, 2.467] \times 10^{-3}\ \mathrm{eV}^2$ |

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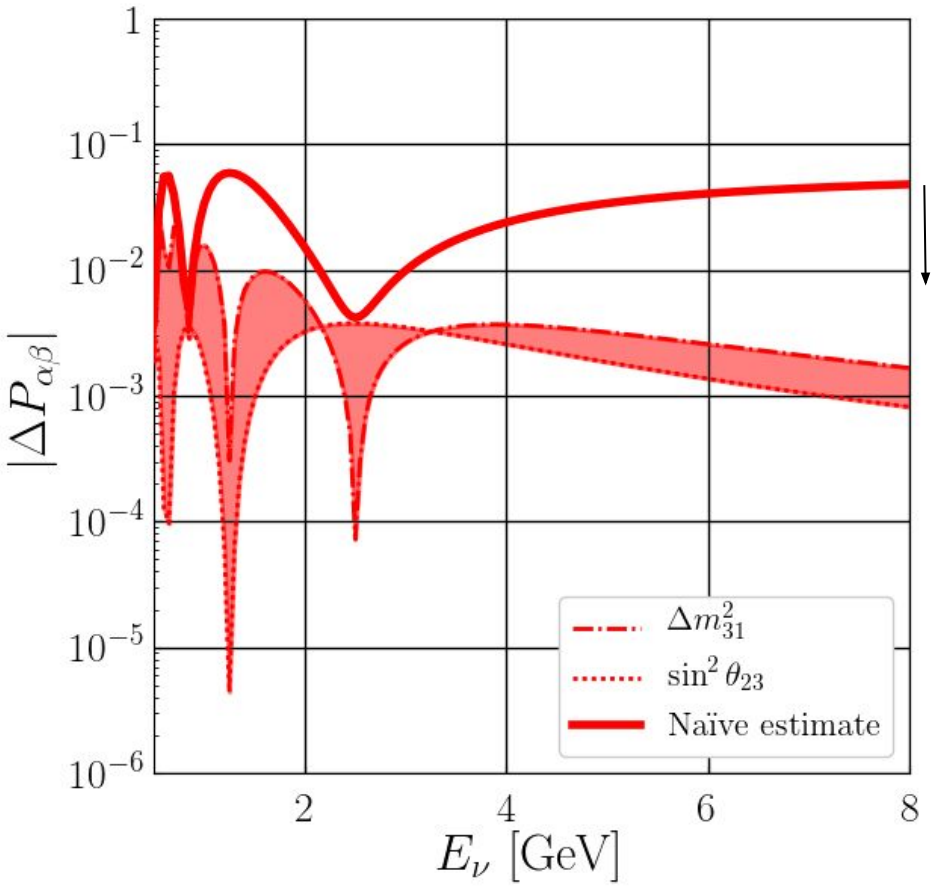
| $\Delta m^2_{31}$ | $2.457 \times 10^{-3}\ \mathrm{eV}^2$ | $[2.447, 2.467] \times 10^{-3}\ \mathrm{eV}^2$ |

Disappearance channels (primarily) measure atmospheric angle, mass-splitting.

# Disappearance Channel Capability




Naïve estimate should improve by factor of ~3: bin-to-bin comparisons, etc.

Neutrino Mode

# Actual Capability of DUNE - Parameter Measurements

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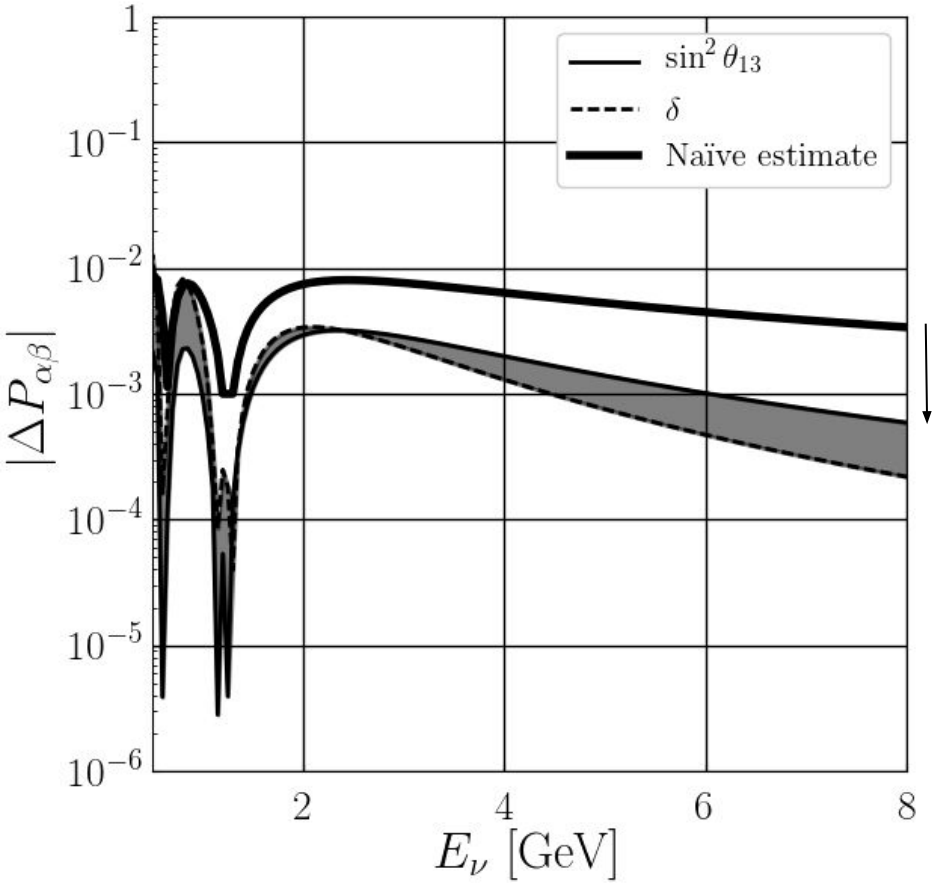
| $\delta$ | $0$ | $[-0.2, 0.2]$ |
| | $\pi/2$ | $[1.37, 1.77]$ |
| $\sin^2(2\theta_{13})$ | $0.085$ | $[0.080, 0.090]$ |
| | | |

Appearance channels (primarily) measure reactor angle, CP-violating phase.

# Appearance Channel Capability



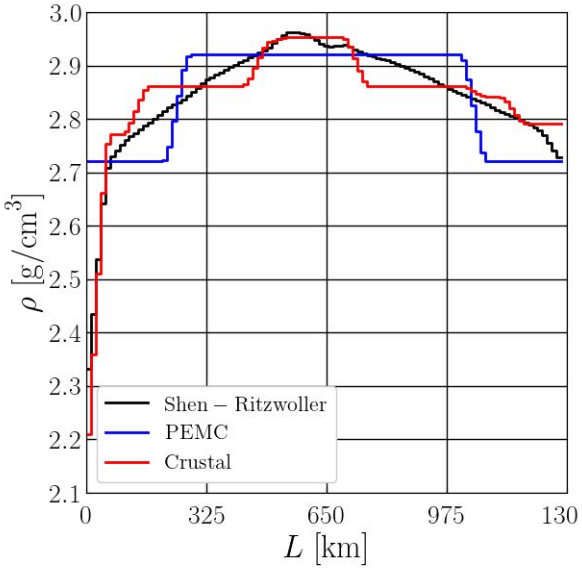

Naïve estimate should improve by factor of ~3: bin-to-bin comparisons, etc.

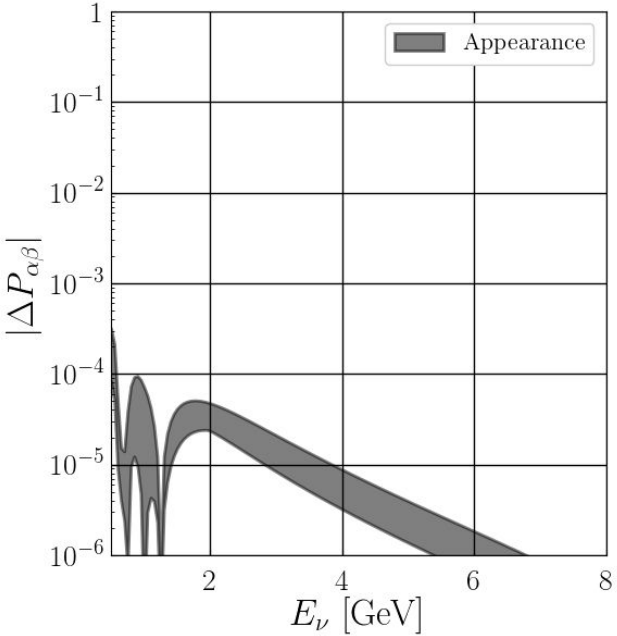
Neutrino Mode

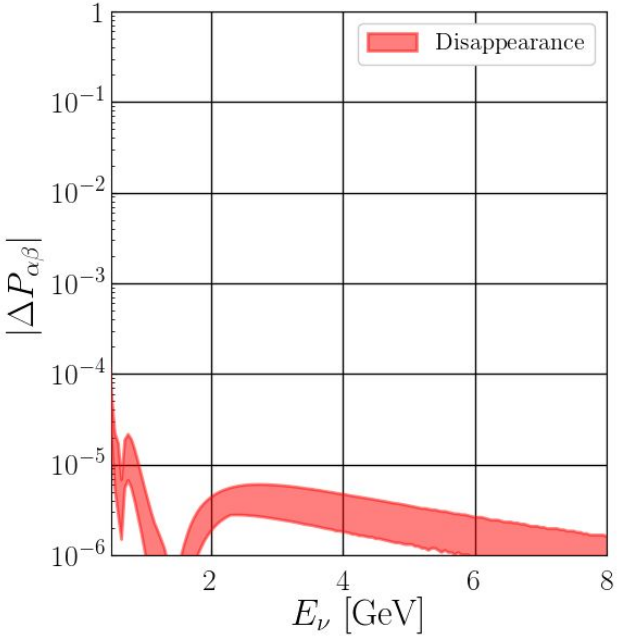
# Matter Density Profile Effects on Probability

# Change in Probability -- Comparing Profiles

Comparing different profiles: keeping average density fixed



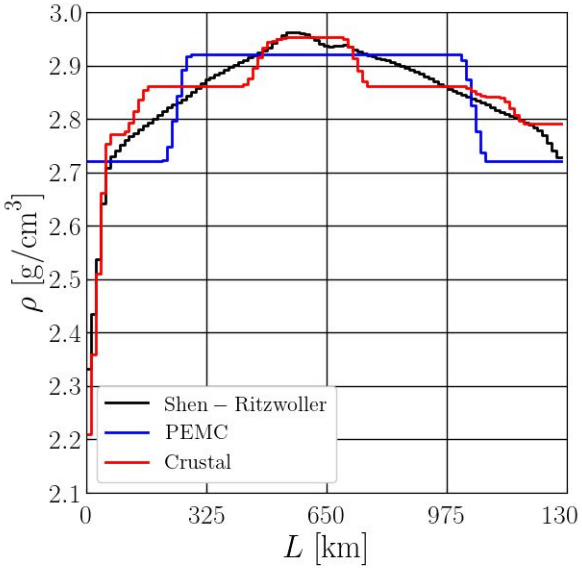



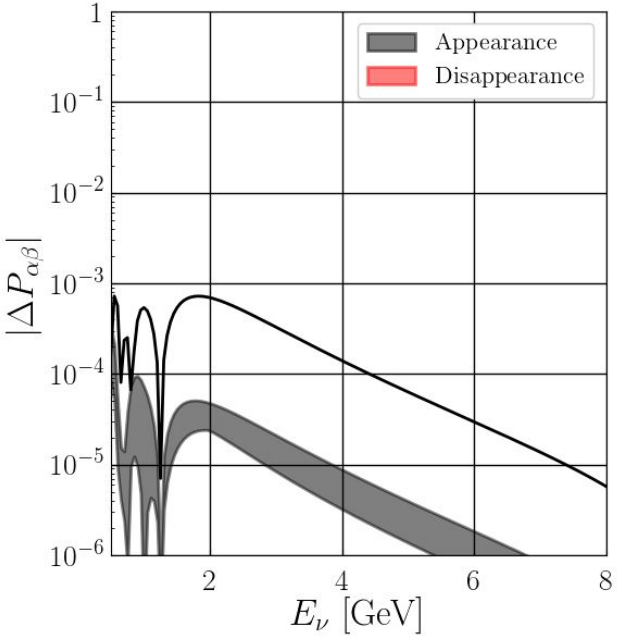



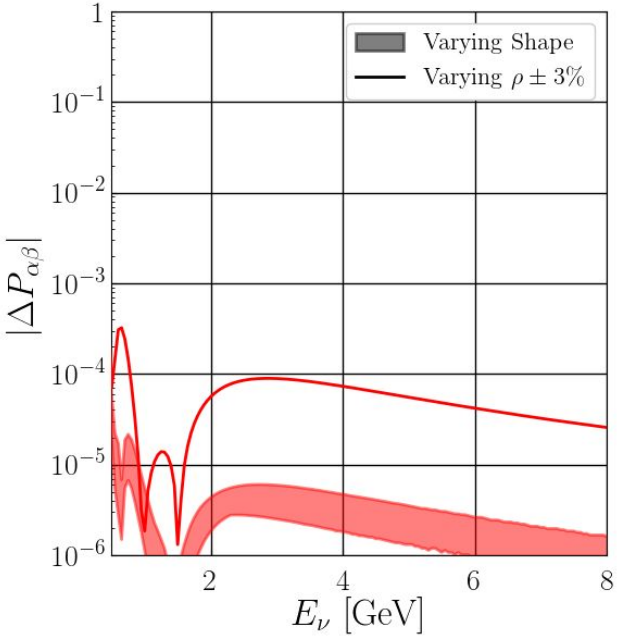

Neutrino mode

# Change in Probability -- Changing Average

Added lines: flat profile, changing average density



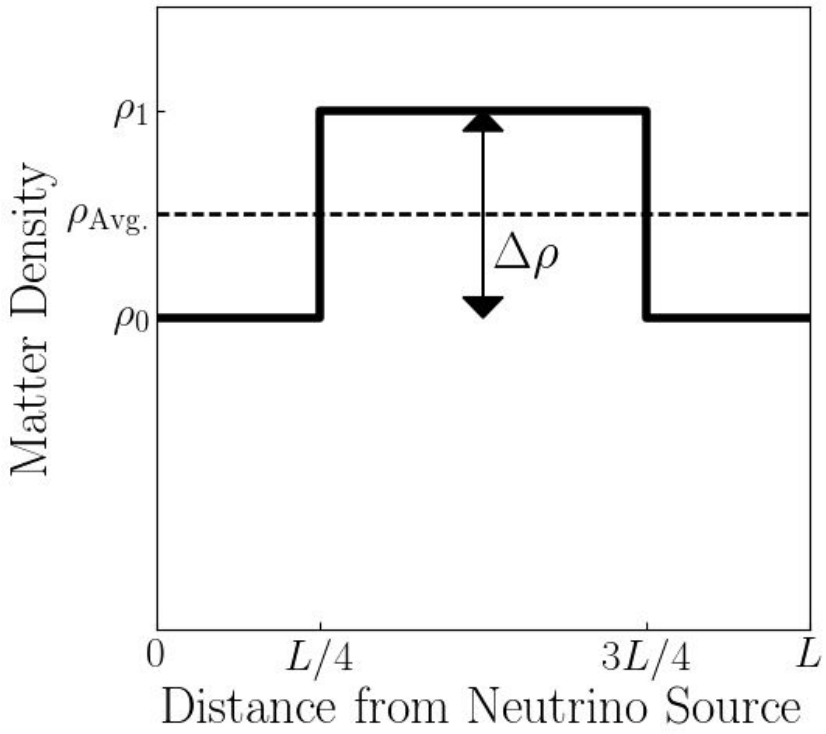



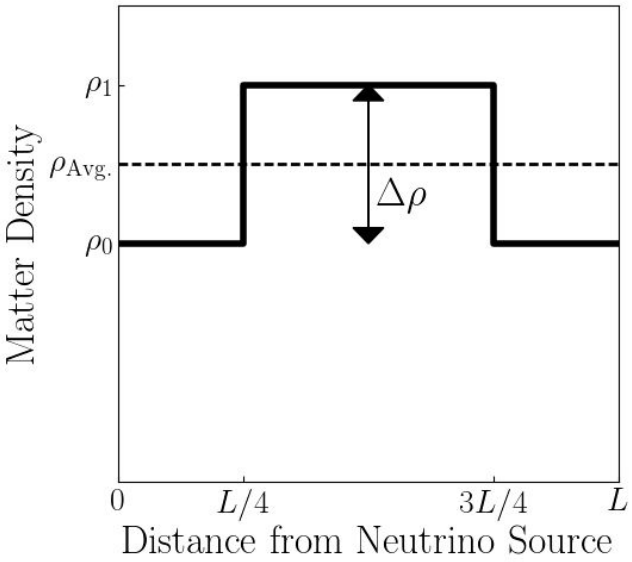

Changes of ~3% to average density induce an order of magnitude larger changes to probability than the differences in shape!

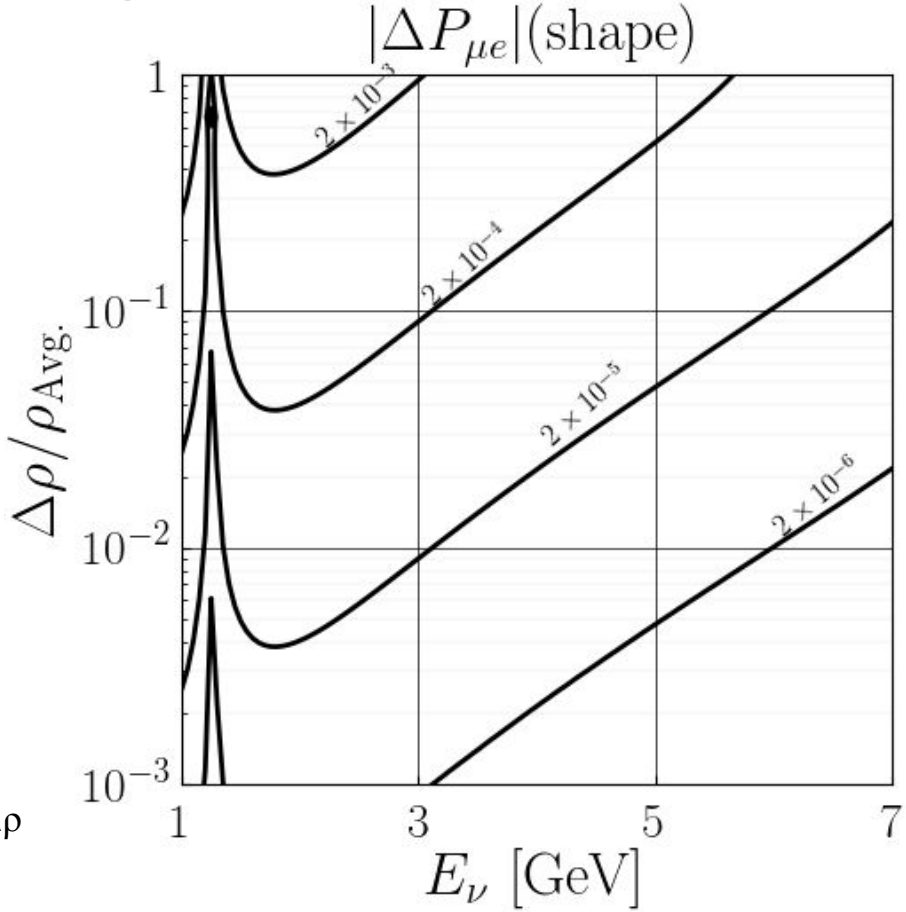
# Is this effect (Average Density dwarfing Shape) generic?

Let's consider this simple, symmetric matter density...



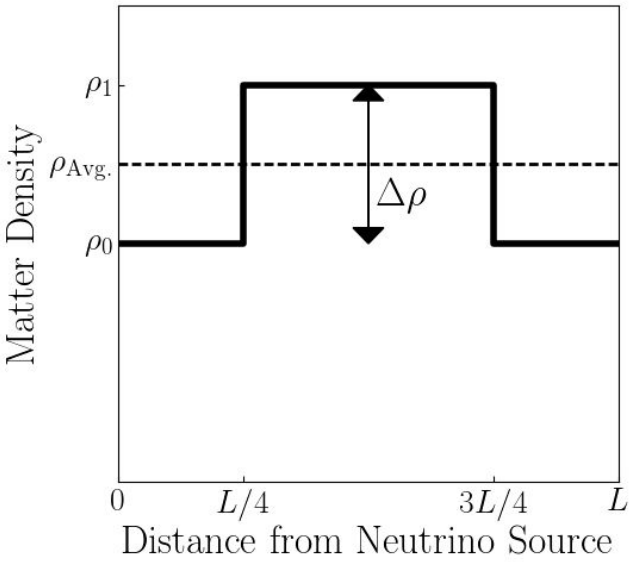
# Shape vs. Average Changes

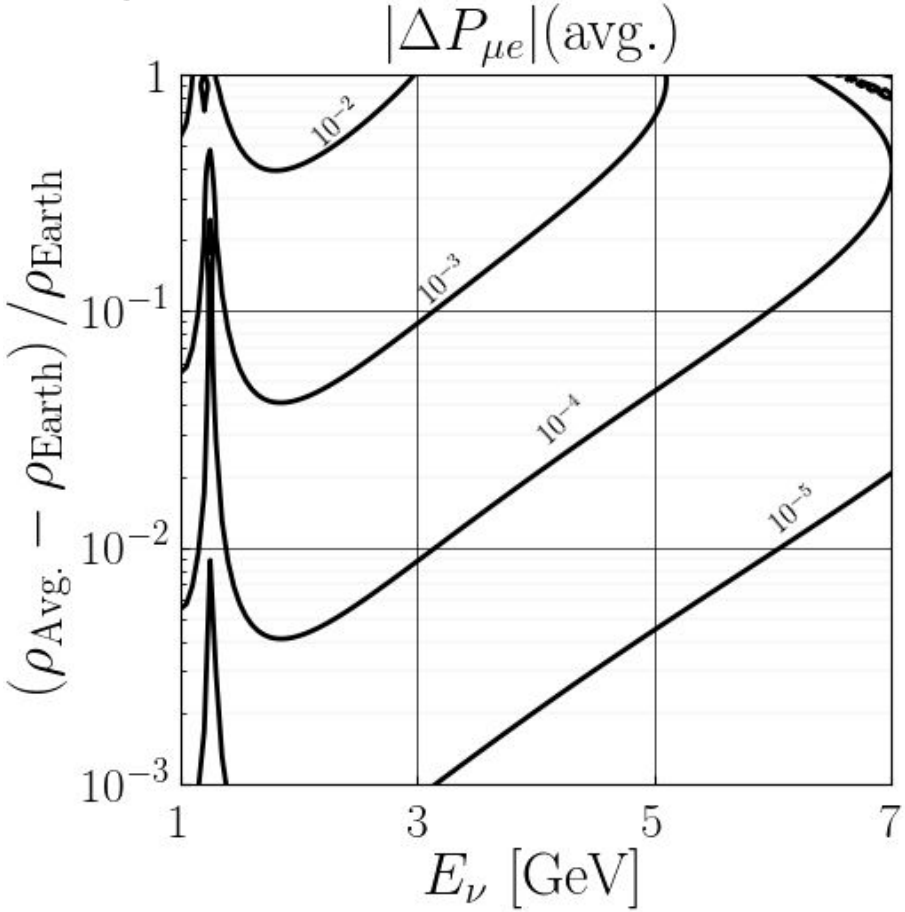



Keep average density at 2.845 g/cm$^3$, vary $\Delta\rho$

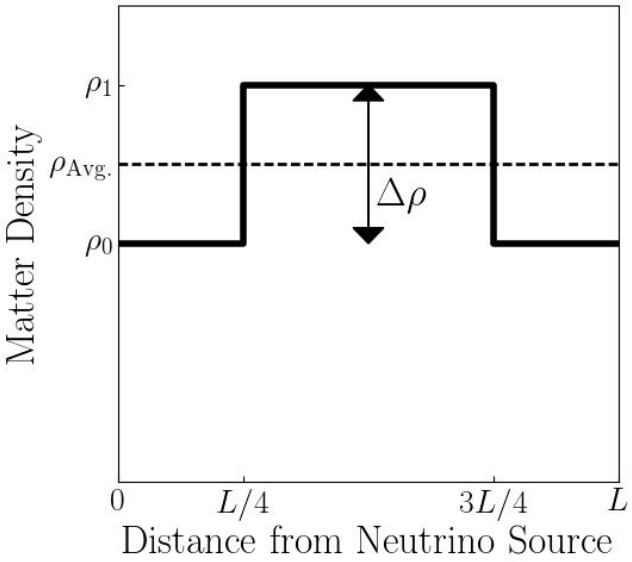
# Shape vs. Average Changes

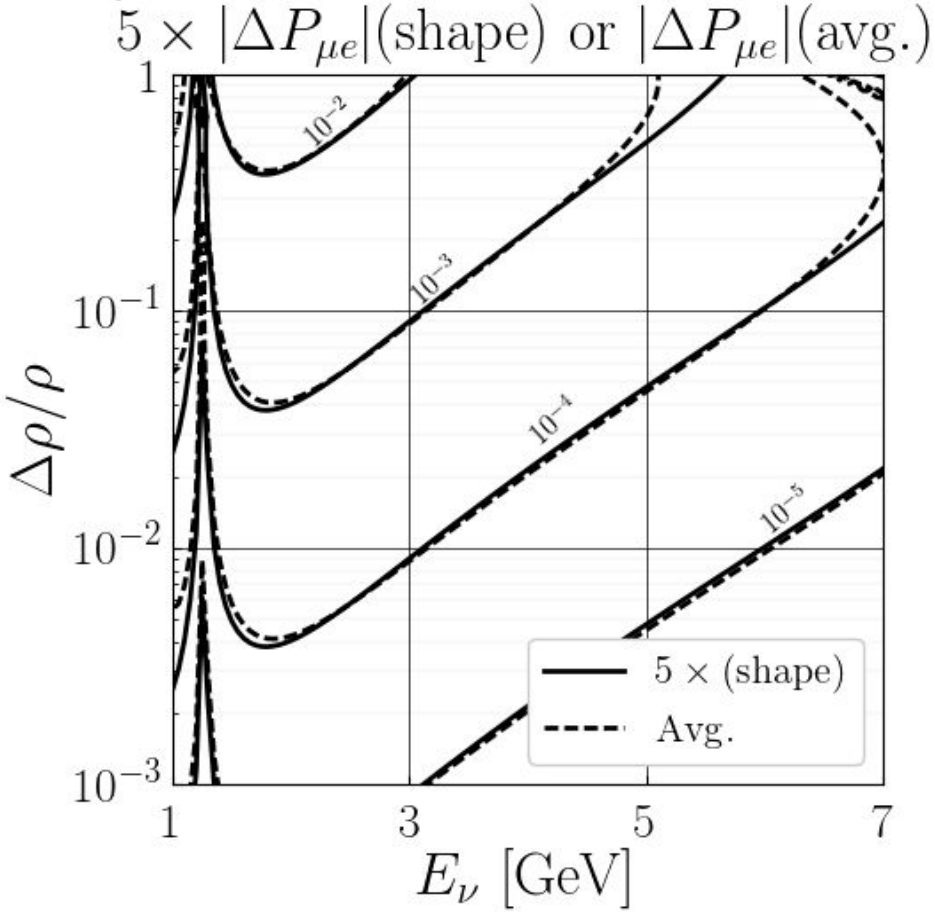



Fix $\Delta\rho = 0$, change average density.

# Shape vs. Average Changes



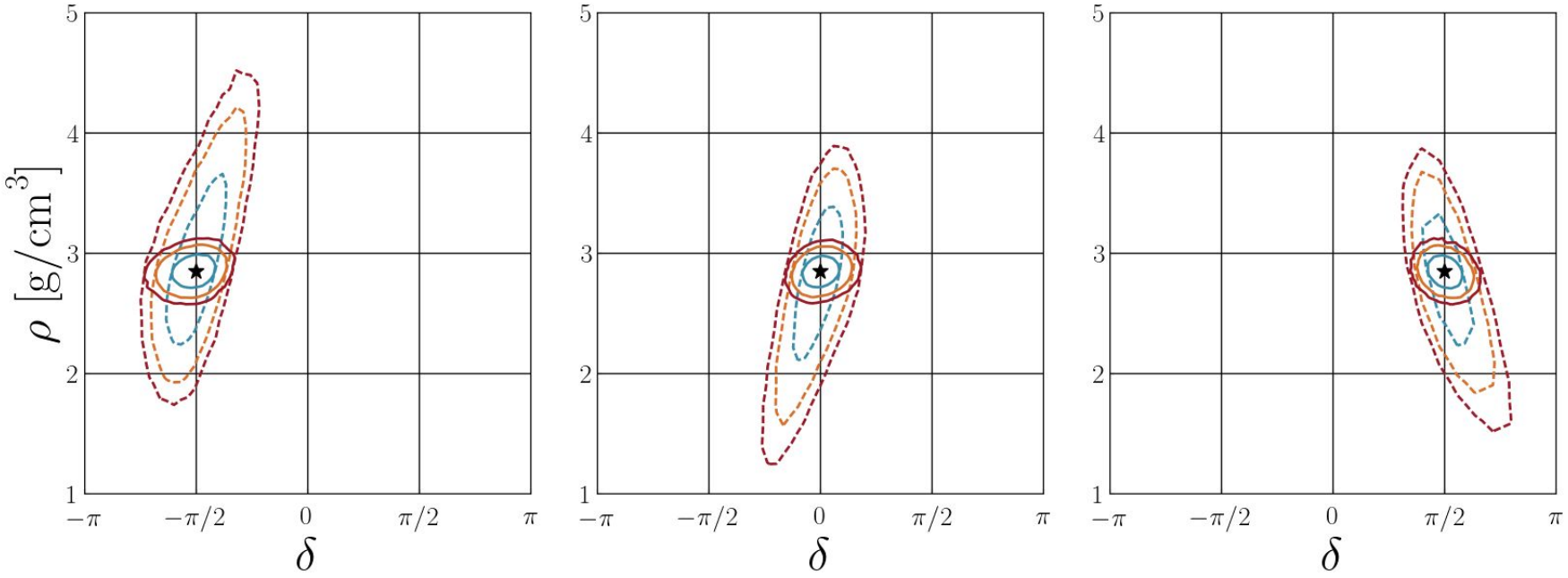




Compare the two, factor of 5 for shape changes.

# Practical Example: Measuring Density

# How well can DUNE measure Average Density?

- Free parameters: six oscillation parameters, average matter density (assumed flat profile).




Three different physical values of □.
Solid lines: assumed 3% prior on average density, dotted lines: no prior.
Blue: 68.3%, orange: 95%, red: 99% credible regions.

# Perturbative Approaches

# Denton, Minakata, Parke (DMP) Method

[1604.08167], [1801.06514]

At zeroth order, two mixing angles, two mass-splittings are modified:

$$\cos 2\widetilde{\theta}_{13} = \frac{\left(\cos 2\theta_{13} - a/\Delta m^2_{ee}\right)}{\sqrt{\left(\cos 2\theta_{13} - a/\Delta m^2_{ee}\right)^2 + \sin^2 2\theta_{13}}}$$

$$\cos 2\widetilde{\theta}_{12} = \frac{\left(\cos 2\theta_{12} - a'/\Delta m^2_{21}\right)}{\sqrt{\left(\cos 2\theta_{12} - a'/\Delta m^2_{21}\right)^2 + \sin^2 2\theta_{12} \cos^2\left(\widetilde{\theta}_{13} - \theta_{13}\right)}}$$

where

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

$$a' \equiv a\cos^2\widetilde{\theta}_{13} + \Delta m^2_{ee}\sin^2\left(\widetilde{\theta}_{13} - \theta_{13}\right)$$

$$\Delta\widetilde{m}^2_{21} = \Delta m^2_{21}\sqrt{\left(\cos 2\theta_{12} - a'/\Delta m^2_{21}\right)^2 + \sin^2 2\theta_{12}\cos^2\left(\widetilde{\theta}_{13} - \theta_{13}\right)}$$

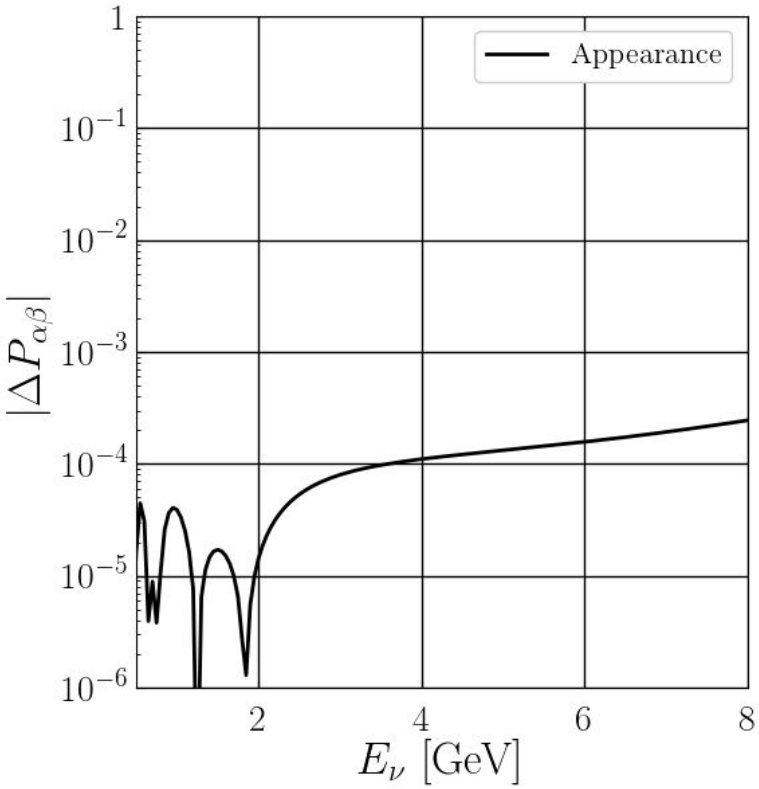
$$\Delta\widetilde{m}^2_{31} = \Delta m^2_{31} + \frac{1}{2}\left(2a - 3a' + \Delta\widetilde{m}^2_{21} - \Delta m^2_{21}\right).$$

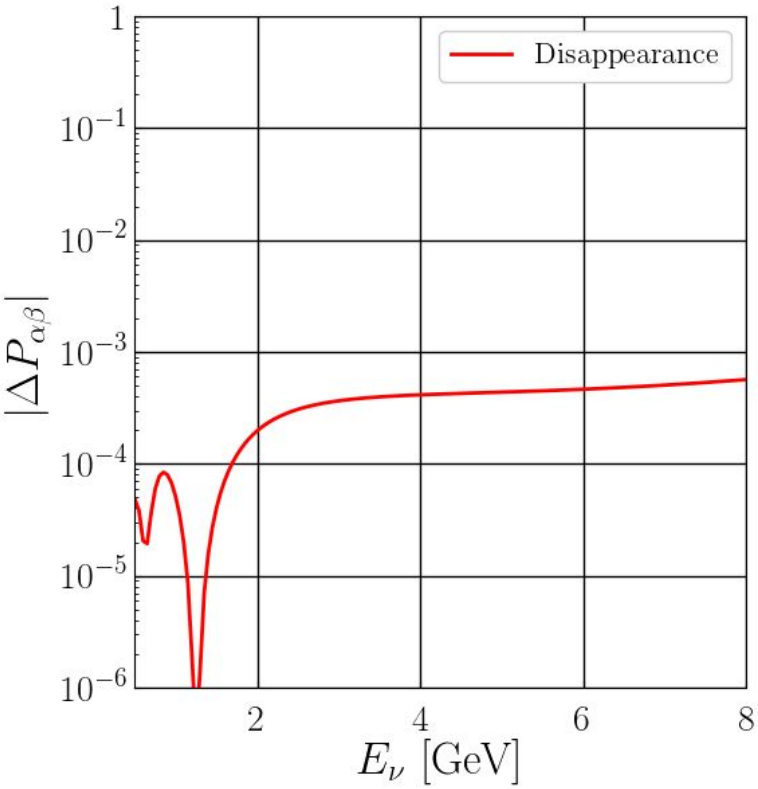
# Advantages to DMP

- Modified mixing angles/mass-splittings plugged into vacuum oscillation probabilities is *much* faster for calculations.
  - $O(10^{-7})$ s to calculate one probability with (C++ compiled) DMP code.
  - $O(10^{-5})$ s to calculate one probability for one layer of constant-density matter.
- If precision of zeroth order isn't enough, first- and second-order expansions exist (see [1604.08167], [1801.06514] for these).

But, is zeroth order precise enough?

# Precision of (zeroth order) DMP

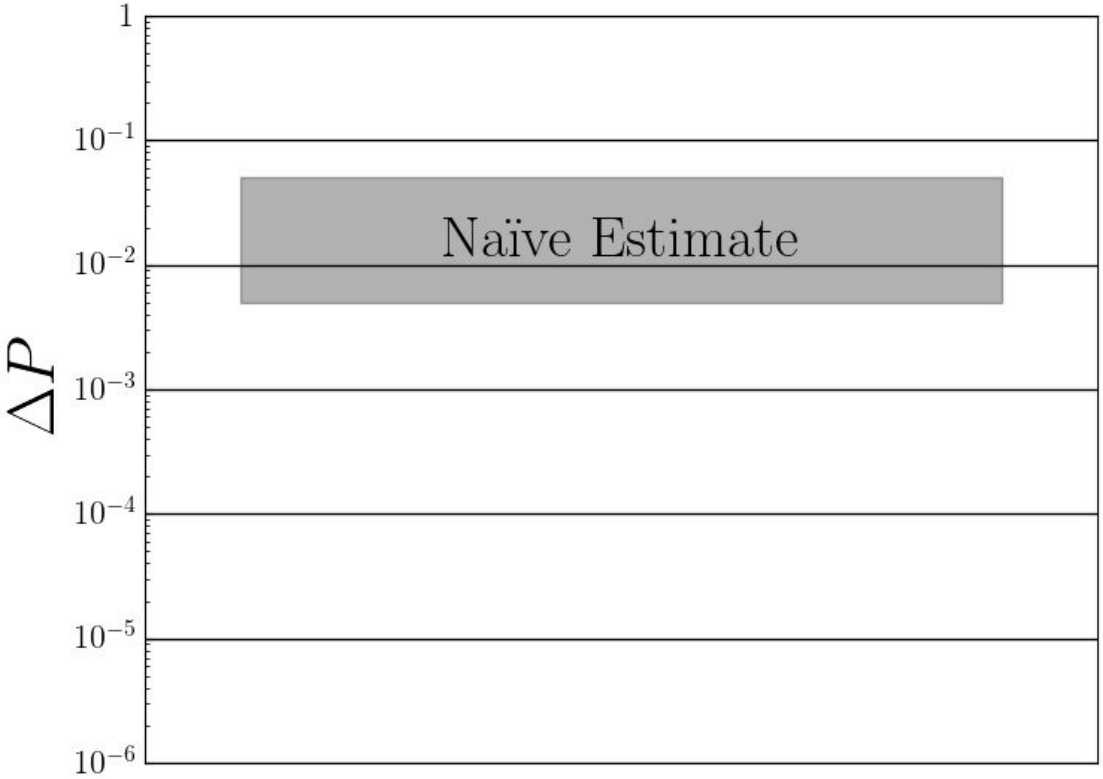



Zeroth order is more than precise enough for energies of interest at DUNE.

# Recap

# Scales of Sensitivity Discussed



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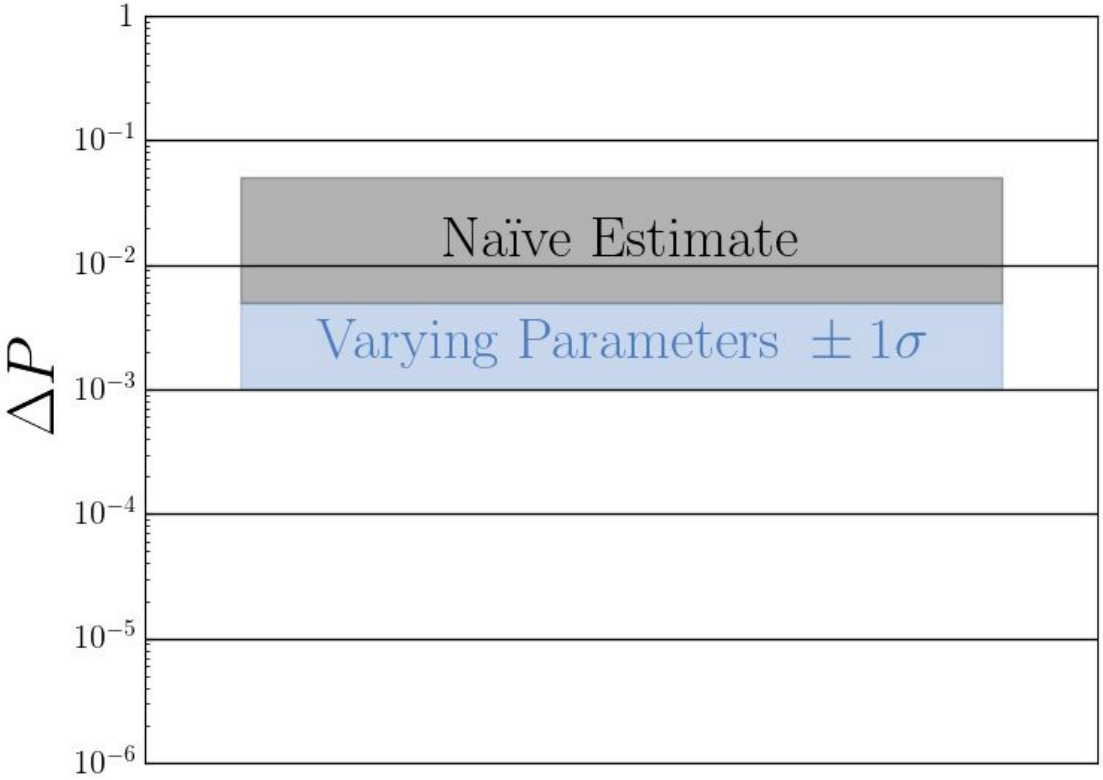

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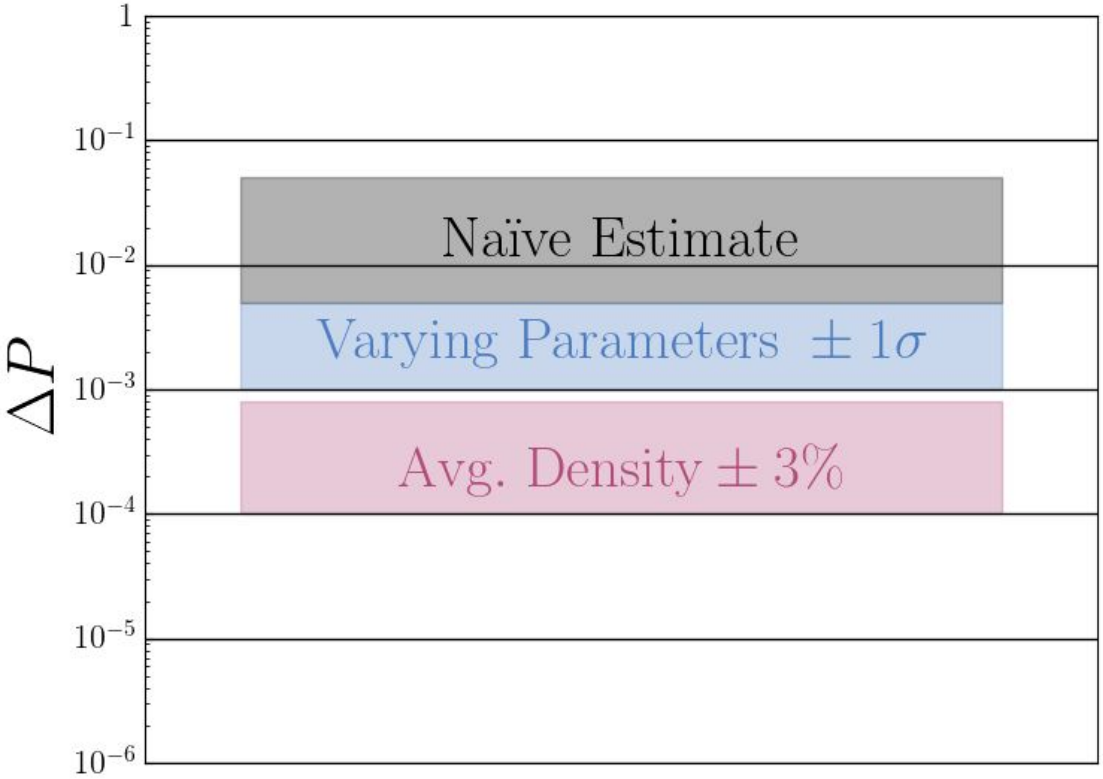

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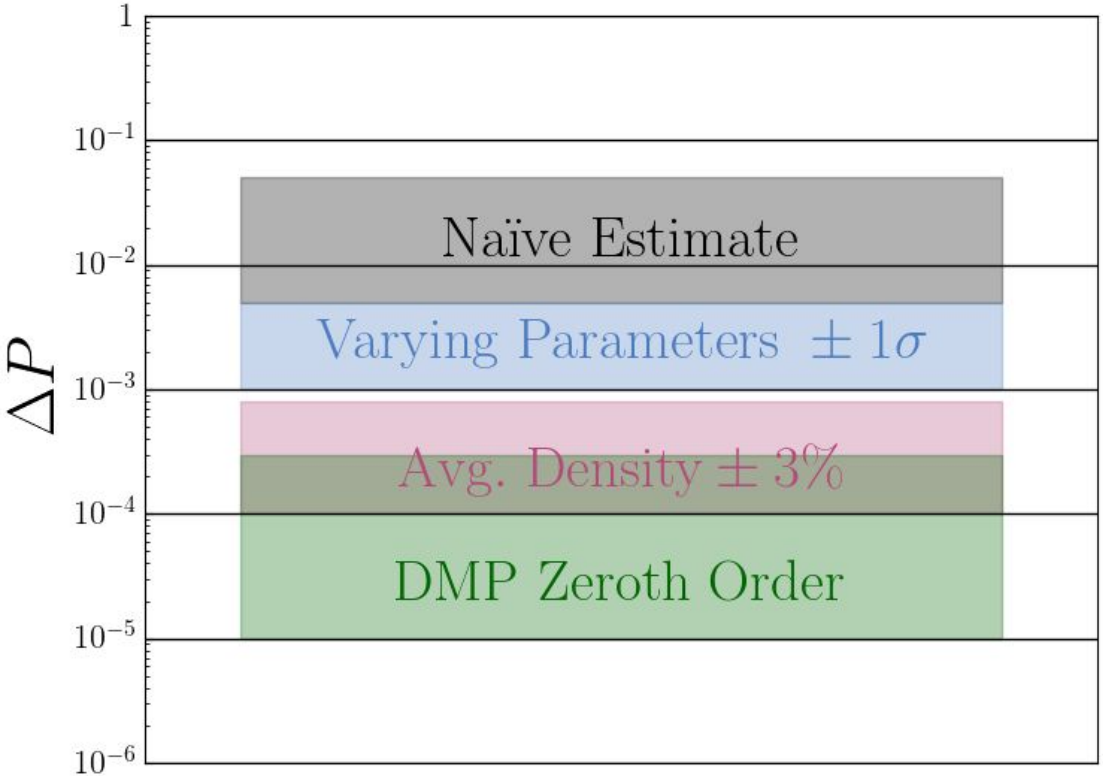

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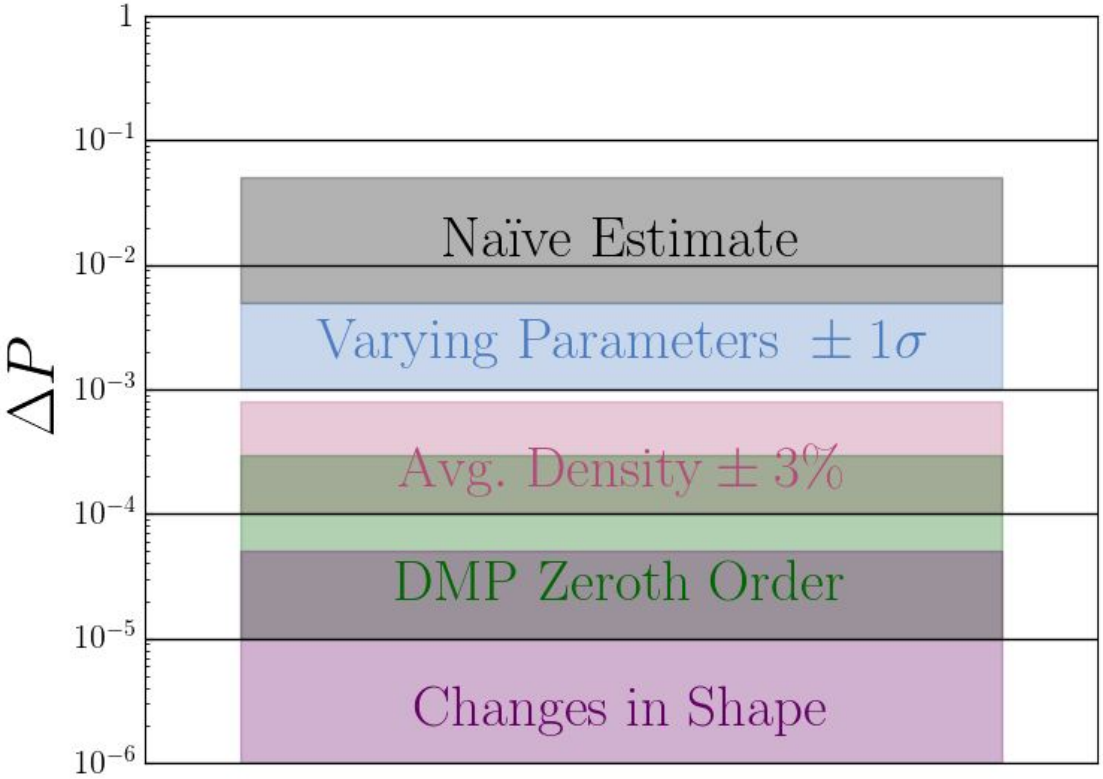

# Summary

- From event rates, we can estimate the sensitivity level of DUNE to changes in oscillation probability.
- Additionally, quoted parameter sensitivity at DUNE gives us a handle to the level of sensitivity for probabilities.
- Uncertainties in the matter density profile exist, however shape effects are *tiny* for the sake of DUNE.
- Large changes to the average density are necessary to be measured by DUNE -- current constraints on the average density are more than enough.
- Perturbative approaches exist for oscillations in matter that are more than precise enough to calculate probabilities at DUNE.