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# **Neutrino University: Neutrino Interactions**

Gabriel Perdue

Fermilab

July 19th, 2017

# The Goal of this Lecture

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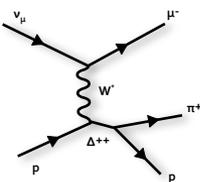
- Discuss Weak interactions and the interactions between neutrinos and other fundamental particles.
- Discuss neutrino-nucleus interactions.
- Discuss why neutrino-nucleus interactions are hard to understand.
- The focus will be holistic: how do you talk to your colleagues about these problems?... and how do you understand seminars at the lab? (MINERvA, MicroBooNE, DUNE, etc.)
- Some of the discussion will come in the context of an *event generator*.
  - Hopefully we both learn a bit about what is really happening in nature and make the generator you're using to do science (assuming neutrino research...) a bit less “black-boxy”.

# Who is your lecturer?

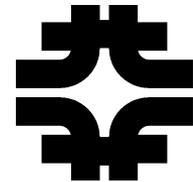
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- I am an associate scientist at Fermilab, working mostly on the MINERvA experiment, DUNE, and the GENIE neutrino event generator.
  - My graduate work was a fixed-target Kaon rare-decay search.
- Given my training and background, I will focus on accelerator-based neutrino scattering experiments in the  $\sim$ half to  $\sim$ few GeV region. Time constraints will keep us focused on the “nearly elastic” regime (we will largely avoid deep inelastic scattering).
  - For a more “complete” (accelerator-based experiment) understanding, it is useful to study high energy neutrino cross-section experiments (e.g. CCFR, NuTeV) and electron scattering experiments at a variety of energies (nuclear effects).

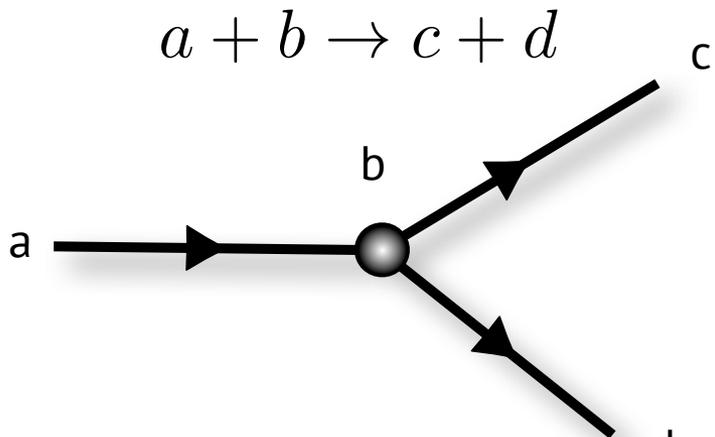




# Basic Formalism



$$a + b \rightarrow c + d$$



**Fermi's Second Golden Rule**

$$W = \frac{1}{h} |M_{if}|^2 \rho_f$$

$$W = \sigma \Phi = \sigma n_a v_{ab}$$

**M is the "Matrix Element"**

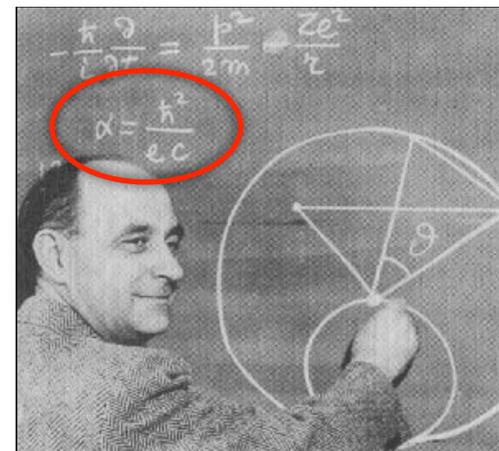
$$\text{Perturbation Theory: } M_{if} = \int \psi_f^* \mathcal{H} \psi_i d\tau$$

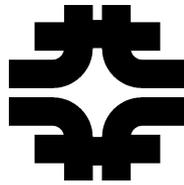
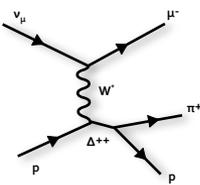
**$\rho_f$  is the density of states (phase space factor).**

$$\sigma(a + b \rightarrow c + d) \propto |M_{if}|^2 \rho_f$$



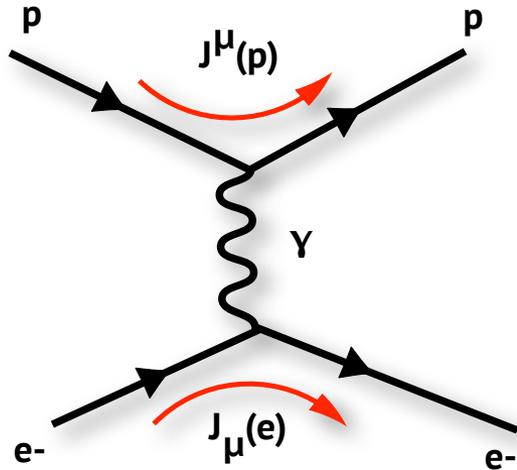
**Fermi makes the rules.**



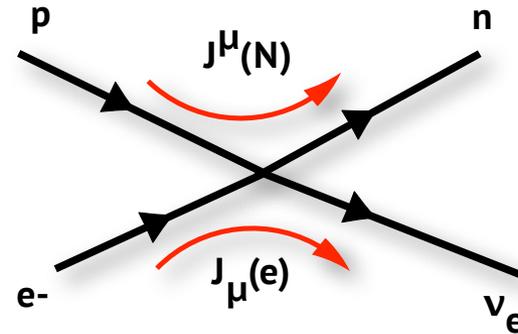


# First Attempt: Fermi, 1932

Current-Current description of EM.



Point interaction of four spin-1/2 fields.

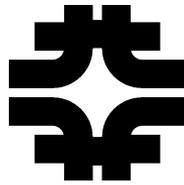


$$M_{em} = (e\bar{u}_p\gamma^\mu u_p) \left(\frac{-1}{q^2}\right) (-e\bar{u}_e\gamma^\mu u_e)$$

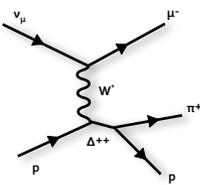
$$M_{weak-CC-Fermi} = G_F (\bar{u}_n\gamma^\mu u_p) (\bar{u}_\nu\gamma_\mu u_e)$$

$G_F$  is not dimensionless ( $\text{GeV}^{-2}$ ) : we need to measure it in  $\beta$  &  $\mu$  decays.

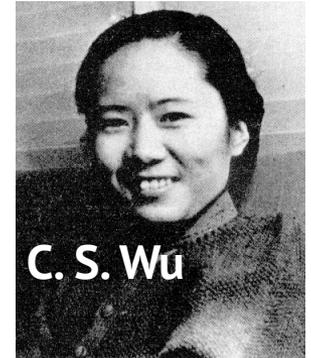
$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{\hbar}{\tau_\mu} \frac{192\pi^3}{(m_\mu c)^5}} \simeq 1.166 \times 10^{-5} \text{GeV}^{-2}$$



# First Attempt: Fermi, 1932

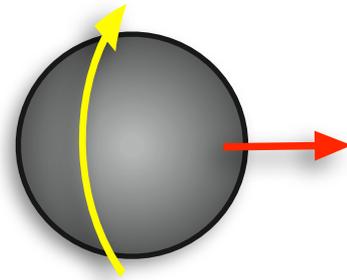


- Actually worked pretty well!
- Bethe-Peierls (1934) used it to compute the cross-section for inverse-beta decay for ~MeV neutrinos.
  - $\sigma \sim 5 \times 10^{-44} \text{ cm}^2$  for  $E \sim 2 \text{ MeV}$
- The calculation is correct to about a factor of two (to account for the then unknown phenomenon of *maximal parity violation* (discovered by Wu) in the weak interaction).

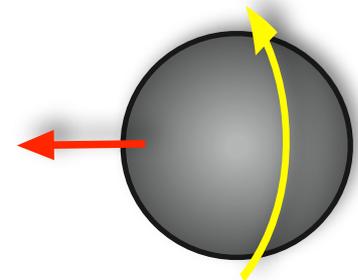


C. S. Wu

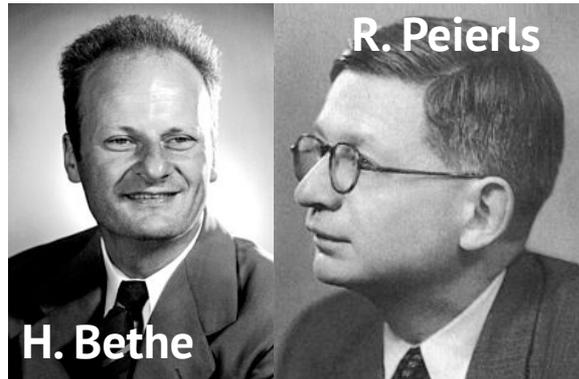
**Left Handed**



**Right Handed**



Fermilab



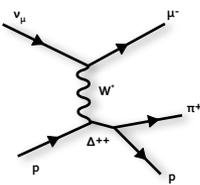
R. Peierls

H. Bethe

$$n \rightarrow e^{-} + p + \bar{\nu}_e$$

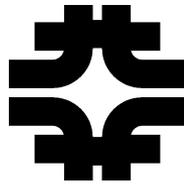
$$\nu_e + n \rightarrow e^{-} + p$$

$$\bar{\nu}_e + p \rightarrow e^{+} + n$$



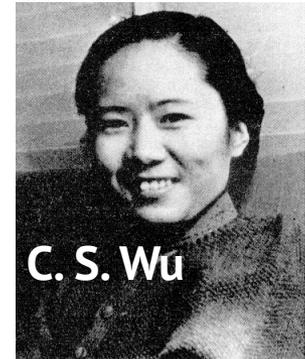
## Weak Interactions

# Parity Violation



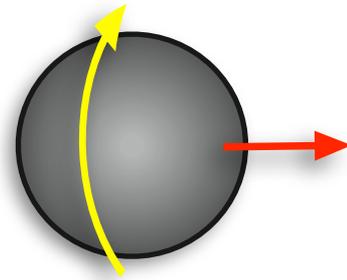
$$h = \vec{J} \cdot \hat{p}$$

- Handedness? We are typically talking about *helicity*.
- Helicity is the projection of a particle's spin onto the direction of the momentum. If the sign of "h" is negative, the particle is *left handed*, if it is positive, it is *right handed*.

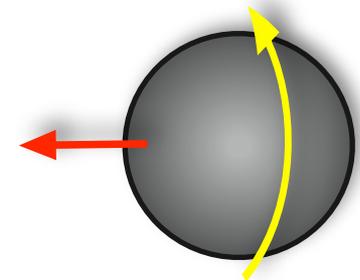


C. S. Wu

**Left Handed**

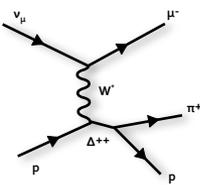


**Right Handed**

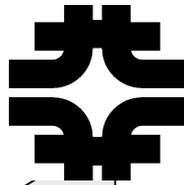


Use the "right" rule at the right time...



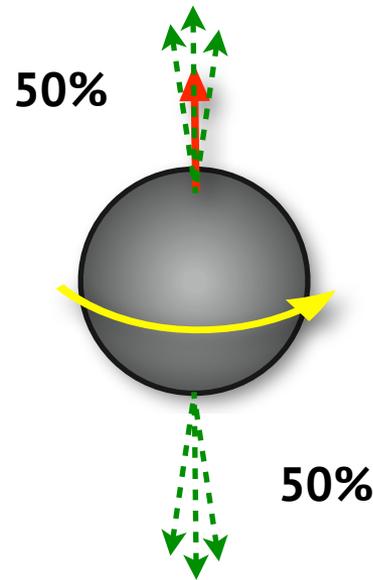


# Parity Violation

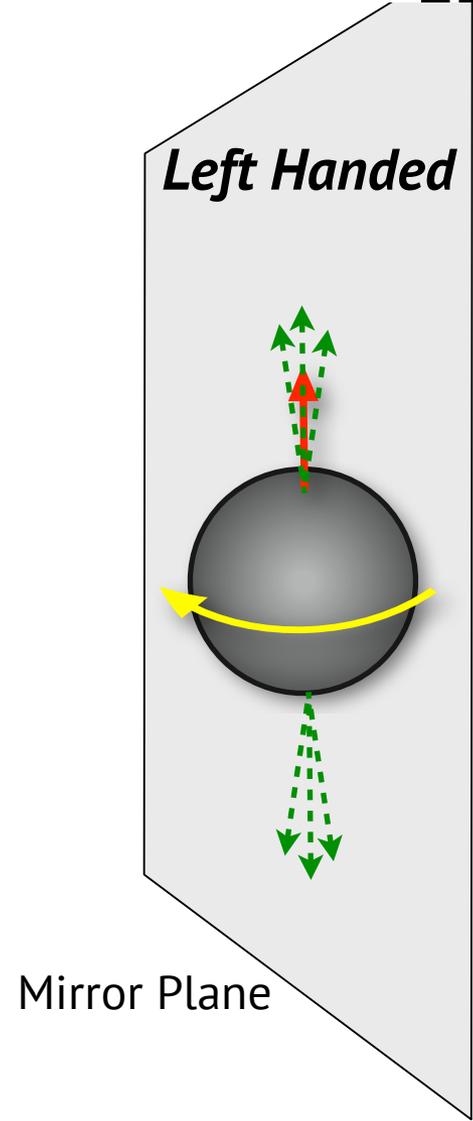


- Suppose we have an atom decaying into a lighter nuclei and emitting a daughter particle.
- If Parity were conserved, we would expect to see this...
- With a 50/50 chance for the direction of the emitted daughter to be aligned/anti-aligned with the parent spin, we can't use a mirror to check the physics...

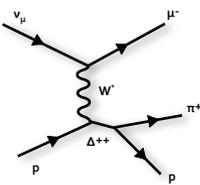
*Right Handed*



*Left Handed*



Mirror Plane

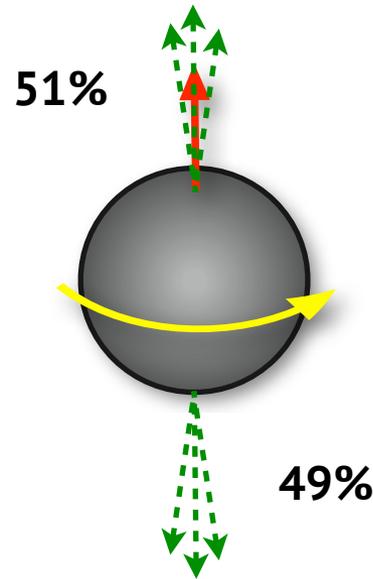


## Weak Interactions

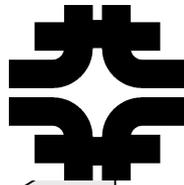
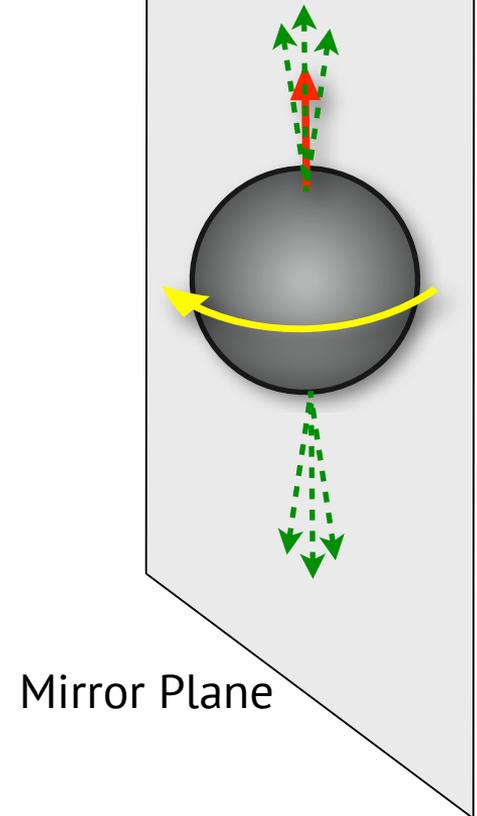
# Parity Violation

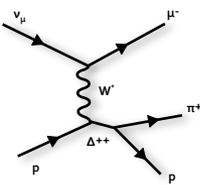
- As soon as we see this though, we know Parity is violated!
- There is a preference for a specific handedness in the decay.

*Right Handed*

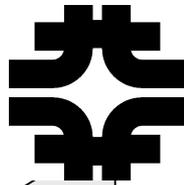


*Left Handed*



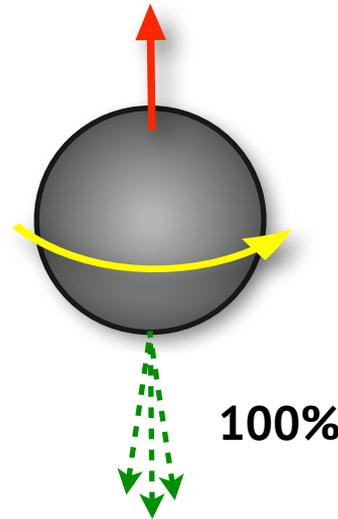


# Parity Violation

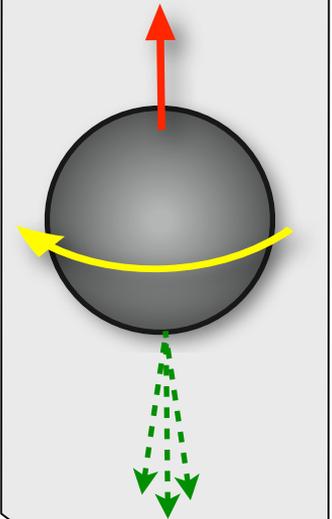


- Interestingly, the Weak force actually works like *this*...
- (Don't dwell on the specific cartoon drawn - the point is the handedness preference is *maximal*.)

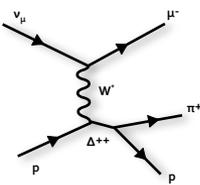
*Right Handed*



*Left Handed*

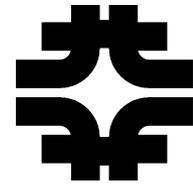


Mirror Plane



## Weak Interactions

# Parity Violation



Suppose the initial spin is 1 and we decay to spin-1/2 fermions A & B...  
(Black Arrow is momentum, Red is spin.)



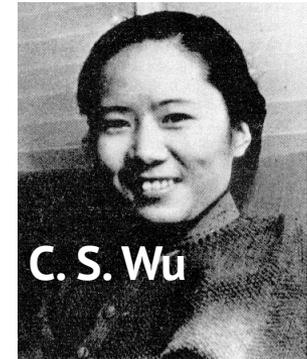
A is right handed, B is left handed.



A is left handed, B is right handed. Parity is not violated...

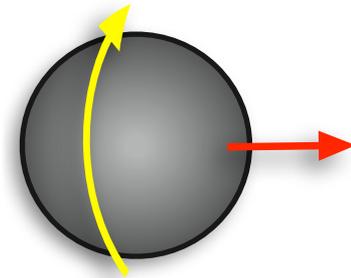


The neutrino is ALWAYS left handed!

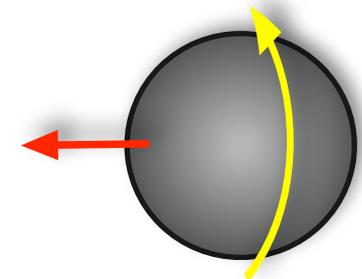


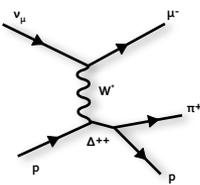
C. S. Wu

**Left Handed**

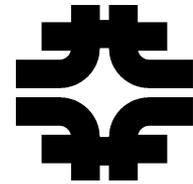


**Right Handed**

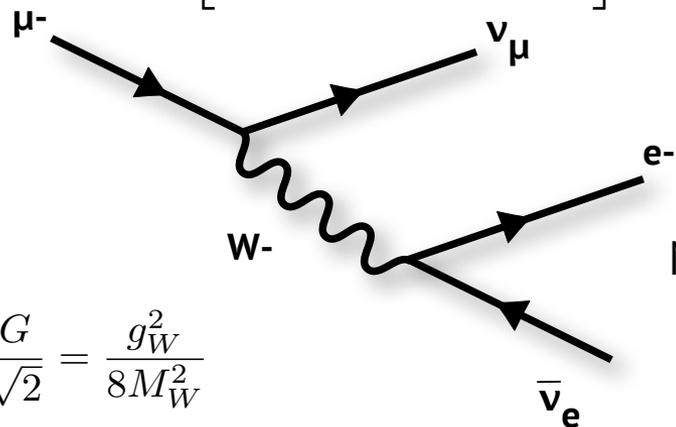




# Weak Interactions in the Standard Model



$$M_{\mu\text{-decay}} = \left[ \frac{g_W}{\sqrt{2}} \bar{u}_\nu \gamma^\sigma \frac{(1 - \gamma^5)}{2} u_\mu \right] \left( \frac{1}{M_W^2 - q^2} \right) \left[ \frac{g_W}{\sqrt{2}} \bar{u}_e \gamma^\sigma \frac{(1 - \gamma^5)}{2} u_{\bar{\nu}_e} \right]$$

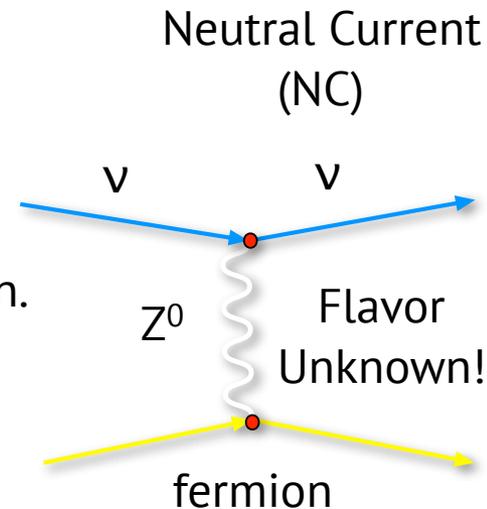


$$\frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

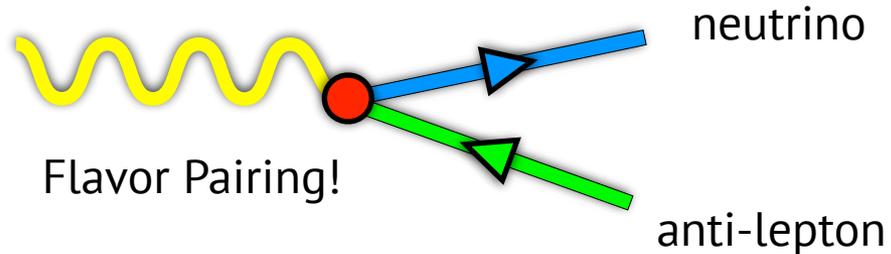
Lepton Number Conservation\*

Parti	e <sup>-</sup>	e <sup>+</sup>	ν <sub>e</sub>	anti-
l <sub>e</sub>	+1	-1	+1	-1

Massive Propagator!  
Parity Violation.

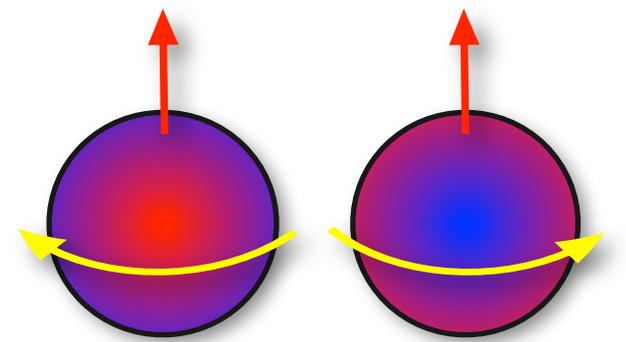


Charged Current (CC) W<sup>±</sup>



\*Actually, "hiding" behind Parity violation. Hmmmm...

# Helicity, Chirality, & Parody, oops, Parity!



Left-Helicity Right-Helicity

- *The Weak force is left-handed.*
  - $(1-\gamma^5)$  projects onto **left-handed states for massless fermions** and **right-handed states for massless anti-fermions.**

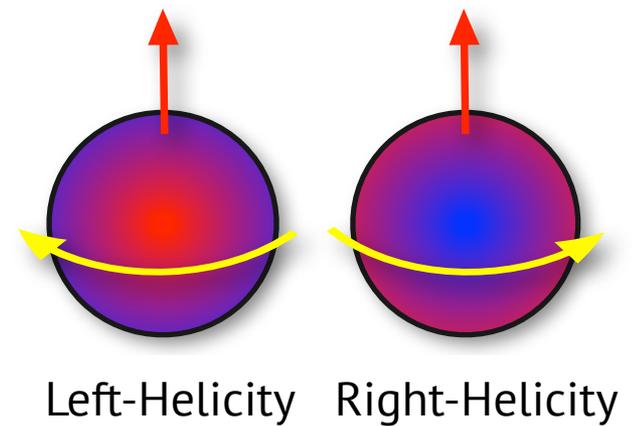
$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L$$

- **Helicity**
  - Projection of spin along a particle's momentum vector.
  - **Frame-dependent for massive particles.**

- **Chirality**

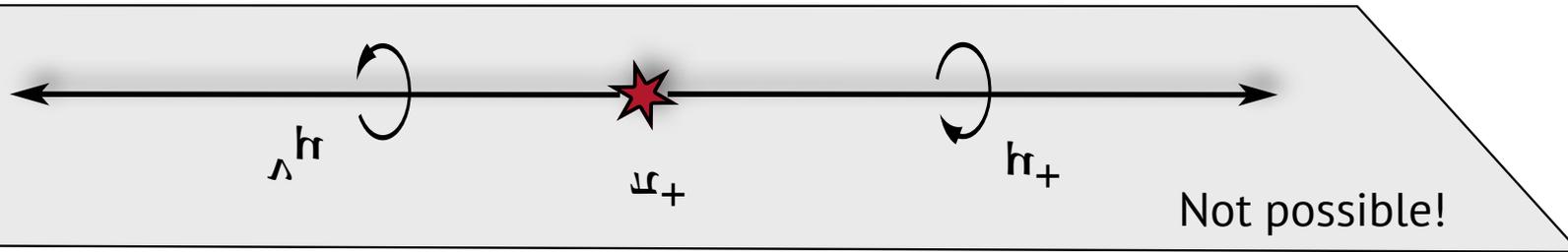
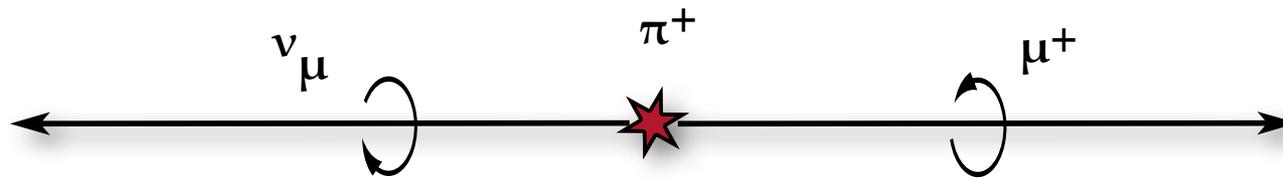
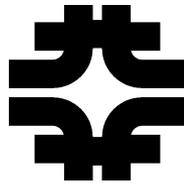
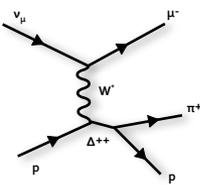
- Lorentz invariant version of helicity (= helicity for massless particles).
- It is determined by whether the particle transforms in a right or left-handed representation of the Poincaré group. Some representations (e.g. Dirac spinors) have right and left-handed components. We define projection operators that project out either the right or left hand components.

# Helicity, Chirality, & Parody, oops, Parity!



- *The Weak force is left-handed.*
  - More simply, the Weak force couples to *left-handed stuff* and *right-handed anti-stuff*.
  - Handedness is frame dependent for massive particles.
  - To the extent neutrinos are massless, the Weak force couples to left-handed neutrinos and right-handed anti-neutrinos only.

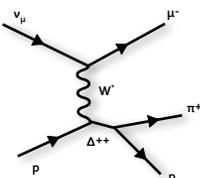
$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L$$



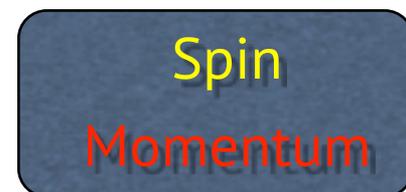
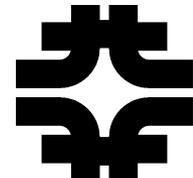
- The pion is spin zero, so daughters must have opposite spins (equal helicities).
- The neutrino is always left-handed, so anti-lepton must also be left-handed. But if the anti-lepton were truly massless, it would only exist as a right-handed particle and the decay would be impossible!

$$R_\pi = \frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 \sim 1.23 \times 10^{-4}$$

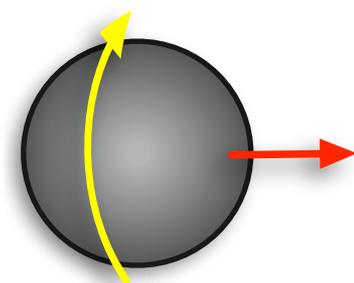
To the extent the electron is “massless,” pion decay to electrons is highly suppressed.



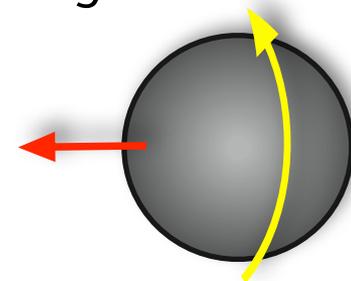
# What about CP?



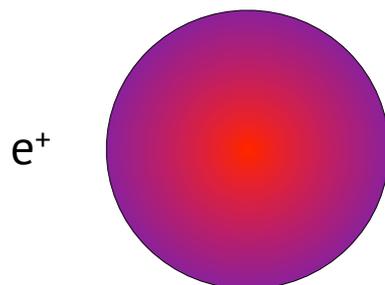
*Left Handed*



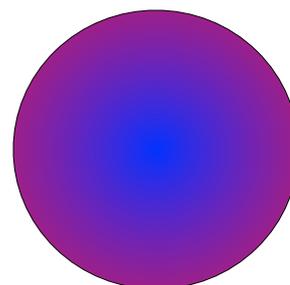
*Right Handed*



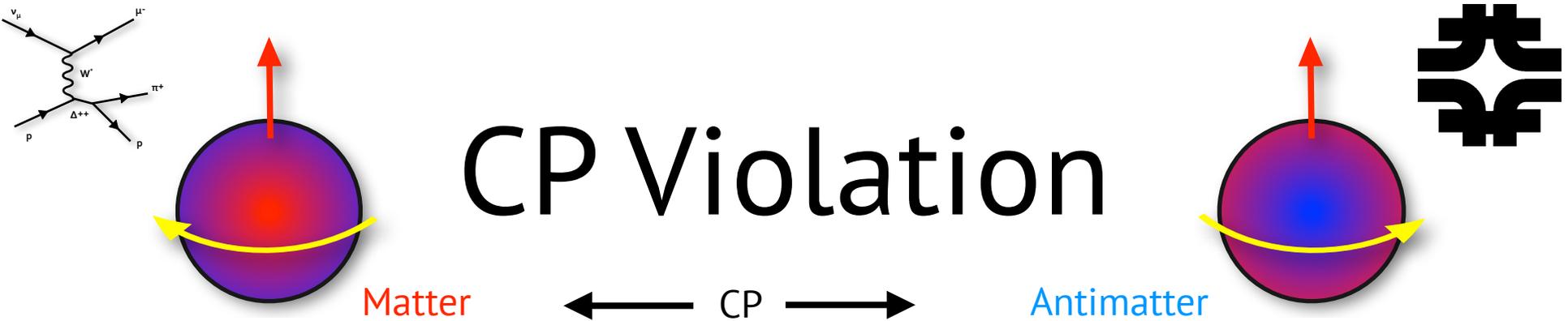
- Charge Conjugation Symmetry (C): Flips the sign of all internal quantum numbers (e.g., electric charge, lepton number, etc.). C does not affect mass or chirality (handedness).
- Parity Symmetry (P): Inverts space (sends a vector  $x$  to  $-x$ ). This inverts the handedness of a particle.



$e^+$

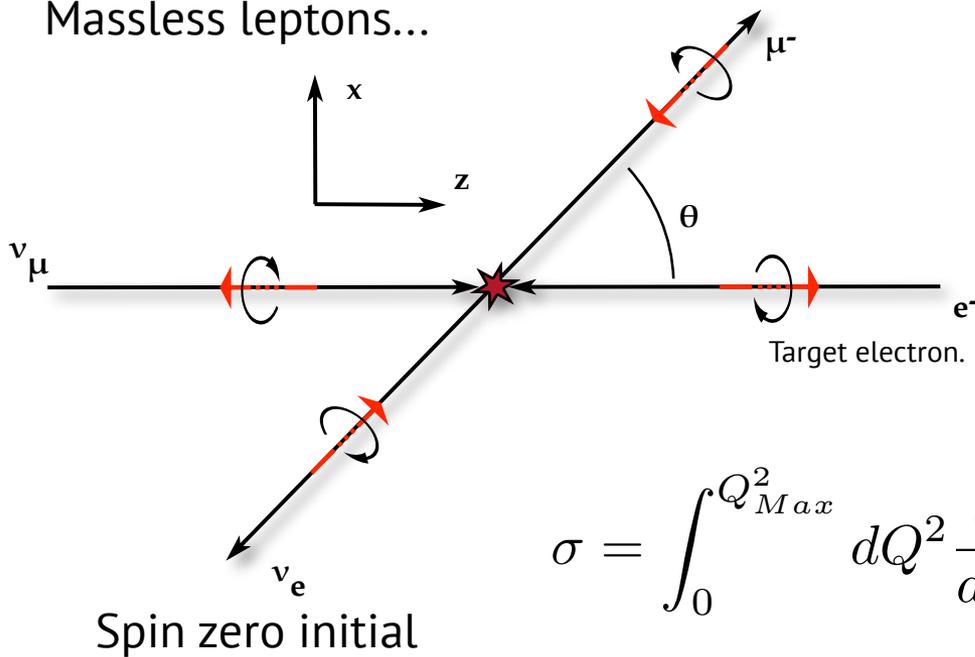


$e^-$



- It is required to explain the **baryon asymmetry** of the universe - **why we have more matter than antimatter**.
- CP violation emerges naturally, in a three generation quark model. But it is too small to explain the baryon asymmetry by itself.
- **It has not been observed in the lepton sector.**

Massless leptons...



Spin zero initial state!

$$\frac{d\sigma}{dq^2} = \frac{|\mathcal{M}|^2}{64\pi p_\nu^2 M_T^2} \propto \frac{1}{(q^2 - M_W^2)^2}$$

$$\sigma = \int_0^{Q_{Max}^2} dQ^2 \frac{d\sigma}{dQ^2} \propto \int_0^{Q_{Max}^2} dQ^2 \frac{1}{(Q^2 + M_W^2)^2} = \frac{Q_{Max}^2}{M_W^4} \text{ for } M_W^2 \gg Q^2$$

Constant of proportionality...

$$\frac{g_W^4}{32\pi} = M_W^4 \times \frac{G_F^2}{\pi}$$

Center of momentum frame...

$$Q^2 = 2E_\nu^{*2} (1 - \cos \theta^*)$$

$Q^2$  bounds...

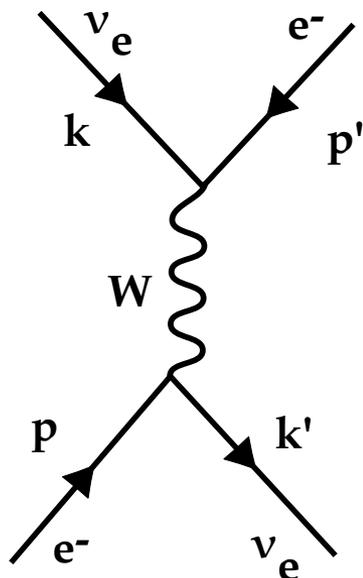
$$0 \leq Q^2 \leq 4E_\nu^{*2} = s$$

$$\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e) = \frac{G^2}{\pi} s$$

$$s = m_e^2 + 2m_e E_\nu$$

~Zero!

## Neutrino-Electron Scattering



Assume:  $m_e = 0$  &  $s = (k + p)^2 = 2k \cdot p = 2k' \cdot p'$

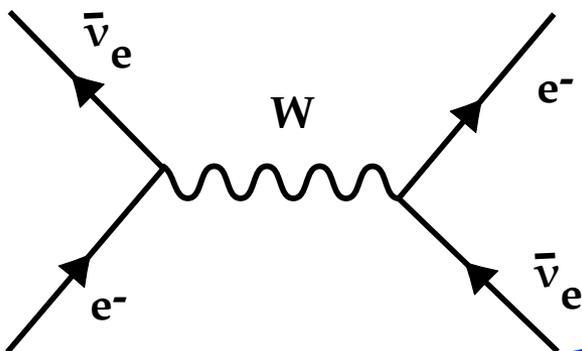
$$\frac{1}{2} \sum_{spins} |\mathcal{M}|^2 = 64G_F^2 (k \cdot p) (k' \cdot p')$$

$$= 16G_F^2 s^2$$

Skip a lot of steps! See: *Halzen & Martin Quarks & Leptons* or *Griffiths Intro. to Elementary Particles*.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\overline{\mathcal{M}}|^2 = \frac{G_F^2 s}{4\pi^2} \implies \sigma = \frac{G_F^2 s}{\pi}$$

## Anti-Neutrino-Electron Scattering



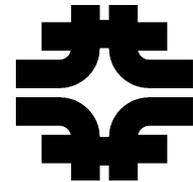
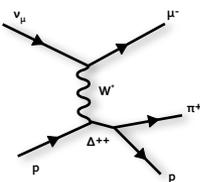
By crossing the neutrinos of previous diagram, we have the result for antineutrinos, replacing  $s$  with  $t$ :

$$\frac{1}{2} \sum_{spins} |\mathcal{M}|^2 = 16G_F^2 t^2$$

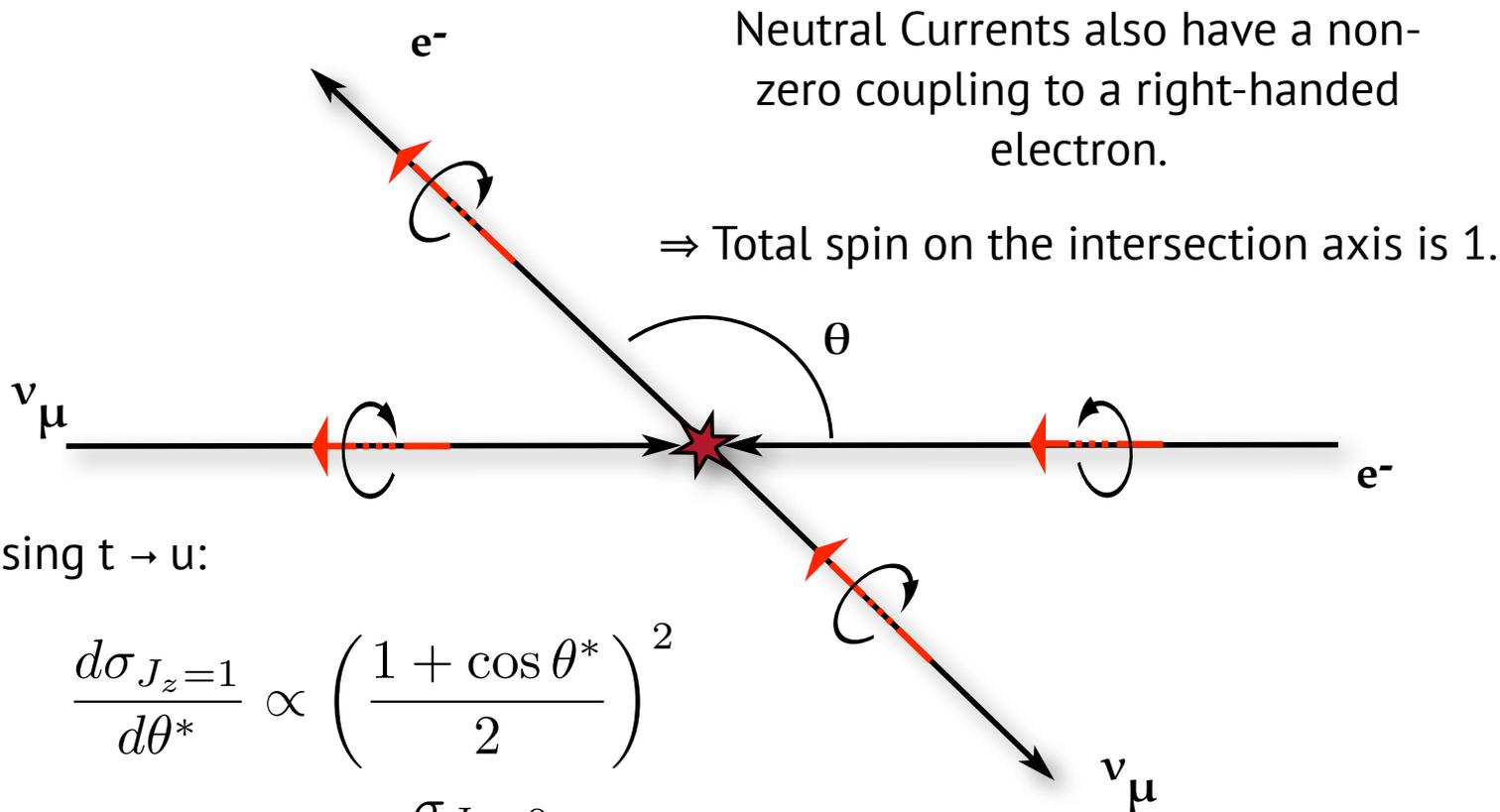
$$= 4G_F^2 s^2 (1 - \cos \theta)^2$$

Integrating over angles, we have:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{16\pi^2} (1 - \cos \theta)^2 \implies \sigma = \frac{G_F^2 s}{3\pi}$$



# Neutral Current Lepton Scattering

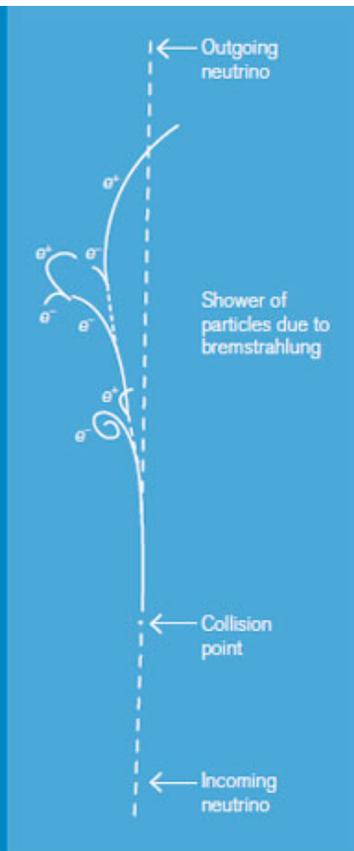


Now crossing  $t \rightarrow u$ :

$$\frac{d\sigma_{J_z=1}}{d\theta^*} \propto \left( \frac{1 + \cos \theta^*}{2} \right)^2$$

$$\sigma_{J_z=1} = \frac{\sigma_{J_z=0}}{3}$$

Non-forward scattering is suppressed.



# Neutral Current Couplings

<http://www.symmetrymagazine.org/cms/?pid=1000741>

	$g_L$	$g_R$
$e, \mu, \tau$	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
$\nu$	$1/2$	$0$
$u, c, t$	$1/2 - 2/3 \times \sin^2\theta_W$	$-2/3 \times \sin^2\theta_W$
$d, s, b$	$-1/2 + 1/3 \times \sin^2\theta_W$	$1/3 \times \sin^2\theta_W$

The couplings are linear terms in the matrix element and are therefore squared in the cross-section:

$$\sigma_{J_z=0} = \frac{G_F^2 s}{\pi} \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2$$

$$\sigma_{J_z=1} = \frac{1}{3} \frac{G_F^2 s}{\pi} (\sin^2 \theta_W)^2$$

$$\sigma_{Total} (\nu_\mu e^- \rightarrow \nu_\mu e^-) = \frac{G_F^2 s}{\pi} \left( \frac{1}{4} - \sin^2 \theta_W + \frac{4}{3} \sin^4 \theta_W \right)$$

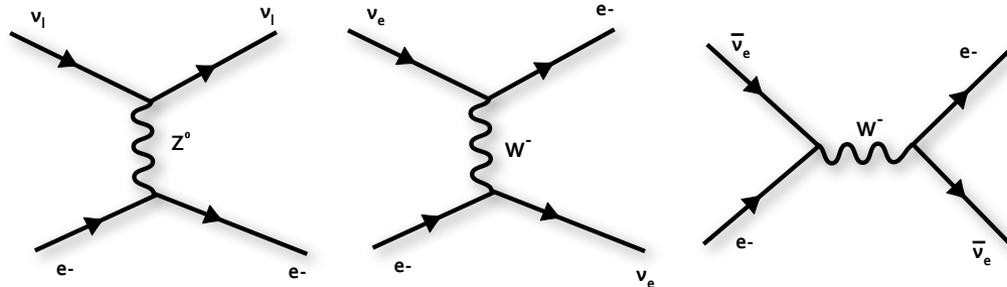
For  $\nu_e$ , CC interactions are of course available and NC and CC interfere.

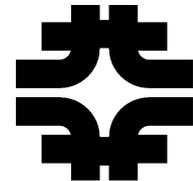
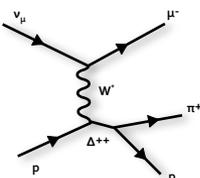
⇒ Add amplitudes, not cross-sections.

This provides an effective coupling:

$$-1/2 + g_L = -1 + \sin^2 \theta_W$$

$$\sigma_{Total} (\nu_e e^- \rightarrow \nu_e e^-) = \frac{G_F^2 s}{\pi} \left( 1 - 2 \sin^2 \theta_W + \frac{4}{3} \sin^4 \theta_W \right)$$





# Strength of the Weak

If  $M_W^2 \gg q^2 \dots$

$$\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5} \text{ GeV}^{-2} = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W c^2} \right)^2$$

$$M_W \sim 80 \text{ GeV}/c^2 \Rightarrow g_W \sim 0.7$$



$$\alpha_{EM} = \frac{g_e^2}{4\pi} = \frac{1}{137}$$

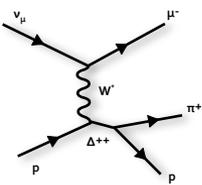
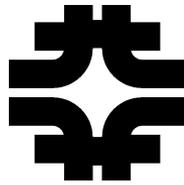
$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29}$$



In the limit of  $5 \approx 1$  (☺), these couplings are *equal*.

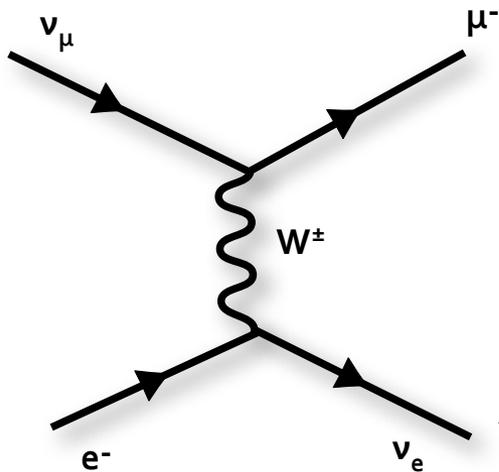
At sufficiently high center-of-mass energy, the interactions are of equal strength.

But what about energies well below  $M_W$ ? Why is the Weak interaction called weak?



# Strength of the Weak

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



$$\begin{aligned}
 s &\equiv (p_1 + p_2)^2 \\
 &= (E_\nu + m_e)^2 - (\mathbf{p}_\nu)^2 \\
 &= E_\nu^2 - p_\nu^2 + m_e^2 + 2E_\nu m_e \\
 &\simeq 2E_\nu m_e
 \end{aligned}$$

For a 100 GeV neutrino...

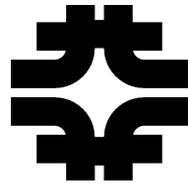
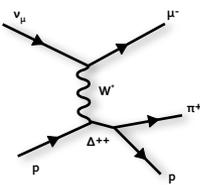
$$E_{CM} = s \simeq 2E_\nu m_e = 2 \times (100 \times 0.000511) \text{ GeV} = 0.1 \text{ GeV}$$

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(M^2 - q^2)^2} \quad \& \quad M_W \sim 80 \text{ GeV}/c^2 \Rightarrow \text{Must "borrow" energy}$$

$$80 \gg 0.1 \Rightarrow \Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow t \sim \frac{\hbar}{\Delta E} \sim 8 \times 10^{-27} \text{ s}$$

$$d = t \times c \sim 3 \times 10^{-18} \text{ m}$$

"Range" of the force.



# Comment...

- It has likely already become clear that neutrinos interact rarely.
  - R. Plunkett: “The neutrinos see a world of ghosts when they are traveling.”\*
- What is the mean free path for a neutrino in lead?

$$MFP_{lead} \sim \frac{1.66 \times 10^{-27} \text{ kg}}{(\sigma_{\nu-N} \text{ m}^2) (11400 \text{ kg/m}^3)}$$

$$\sim 10^{16} \text{ m}$$

*Over a light year!*

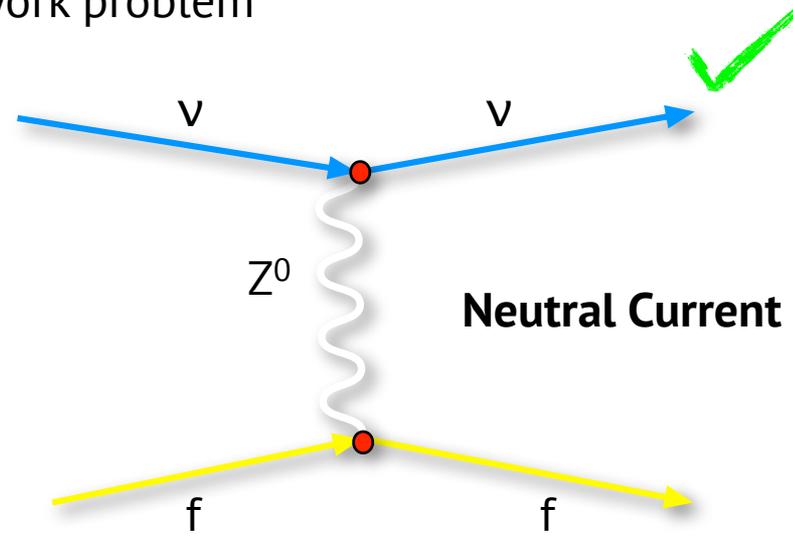
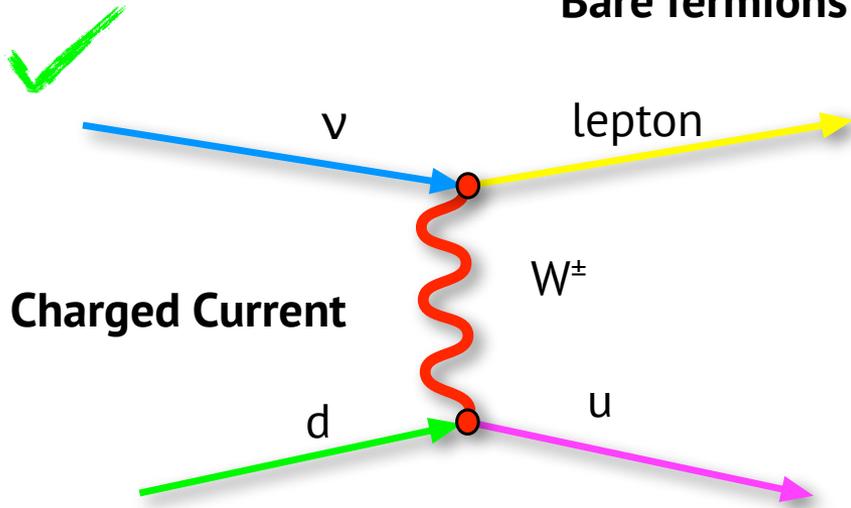
Accelerator (1-100 GeV): MFP  $\sim 10^{12}$  m (~billion miles).

Protons?  $\sigma \sim 10^{-25} \text{ cm}^2$ ; MFP  $\sim 10 \text{ cm}$

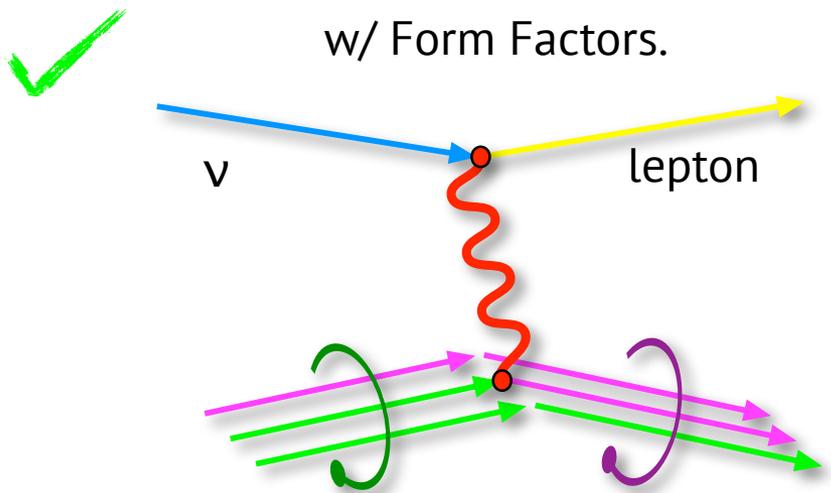
\*<http://chicagotonight.wttw.com/2011/09/28/faster-light-experiments>

- A bit old, but a good example of how to talk to the public about science.

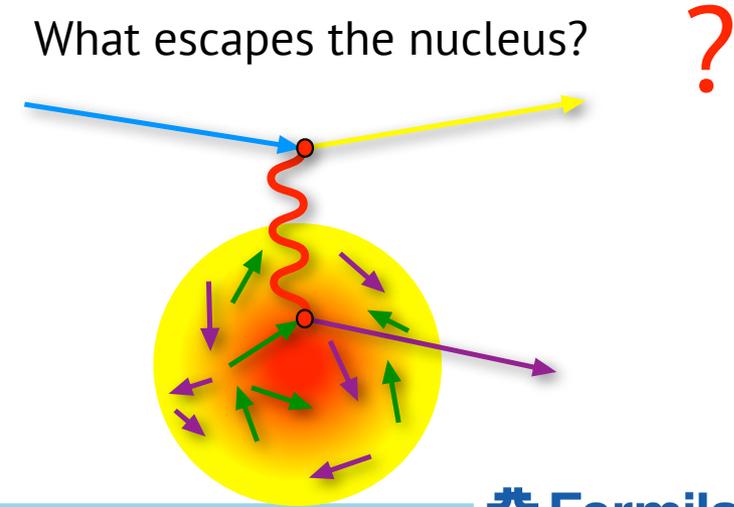
# Bare fermions: Homework problem



**Free Nucleon:**  
Parameterize  
w/ Form Factors.

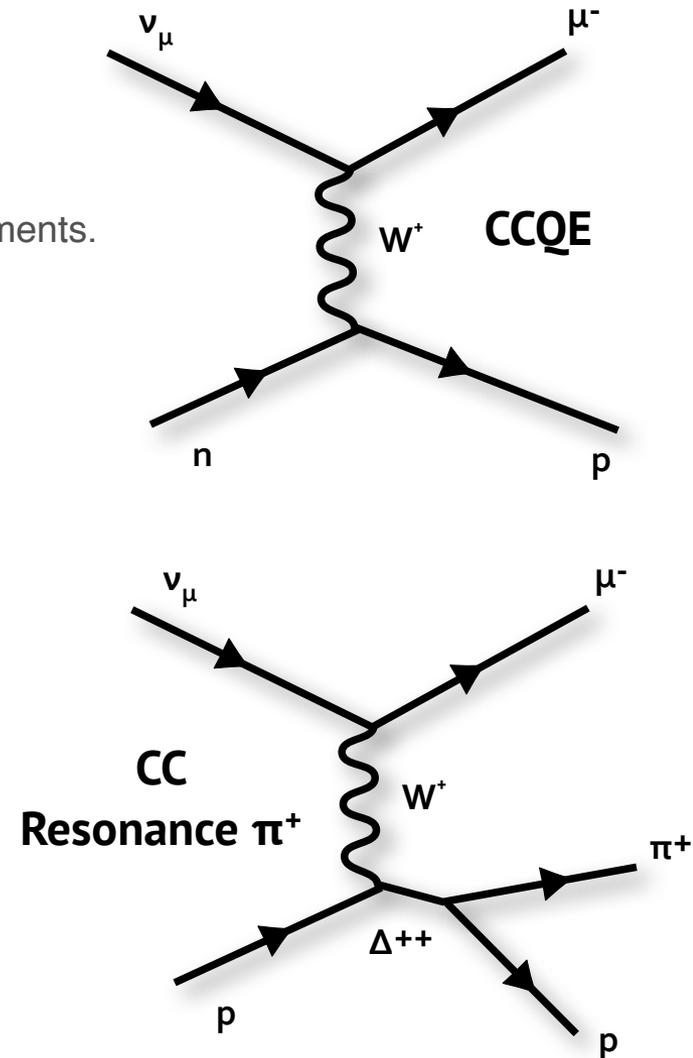
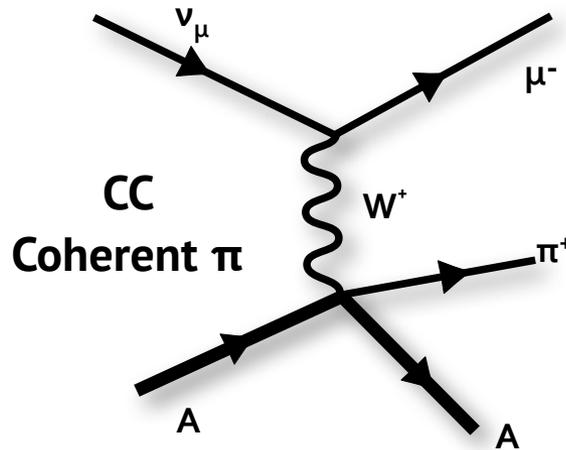
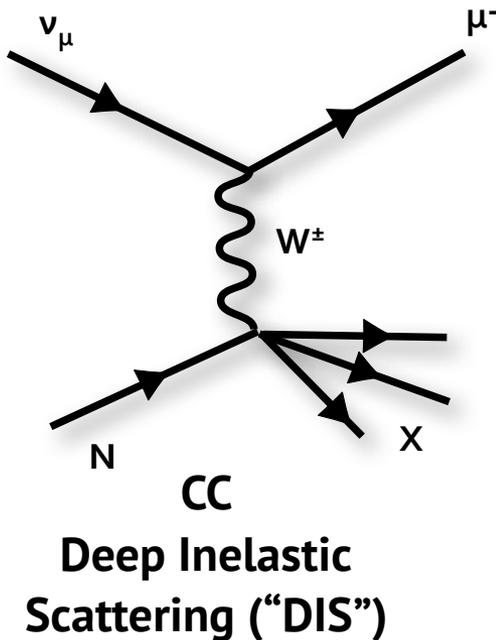


**Nucleus:**  
What is the initial state?  
What escapes the nucleus?



# Reaction Channel Menagerie

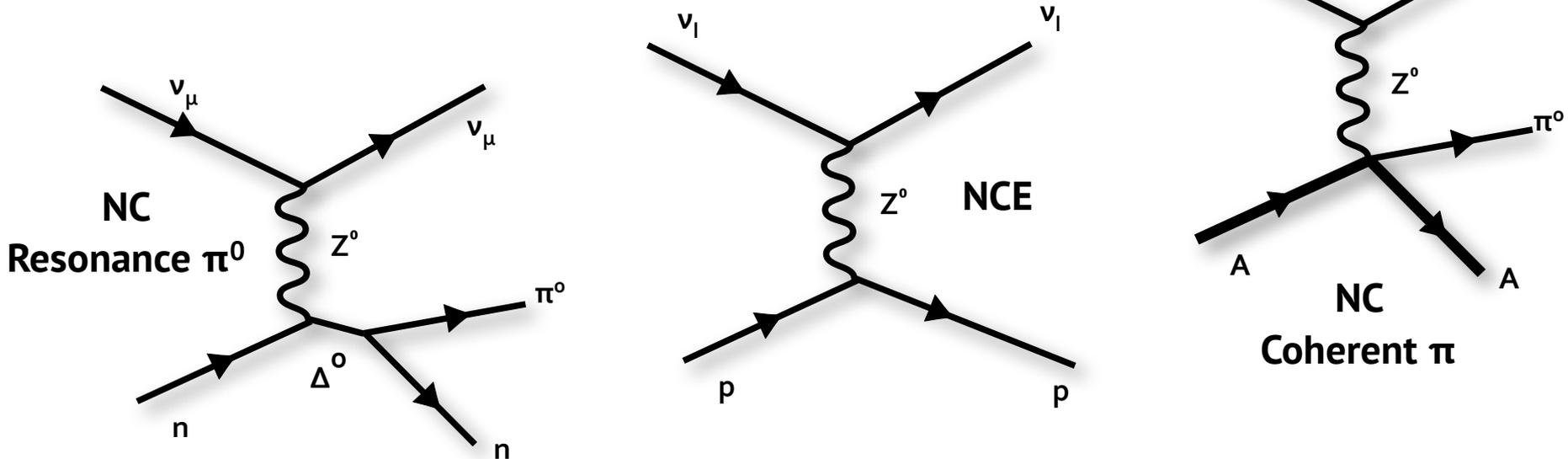
- Charged-Current: Exchange a W boson.
  - CCQE : Charged-Current Quasi-Elastic
  - CC  $\pi^\pm, \pi^0$ 
    - Coherent (no break-up) & Resonance Production
    - Background & Signal for the next-generation oscillation experiments.
  - DIS / Inelastic (scatter on a quark)
  - Inclusive



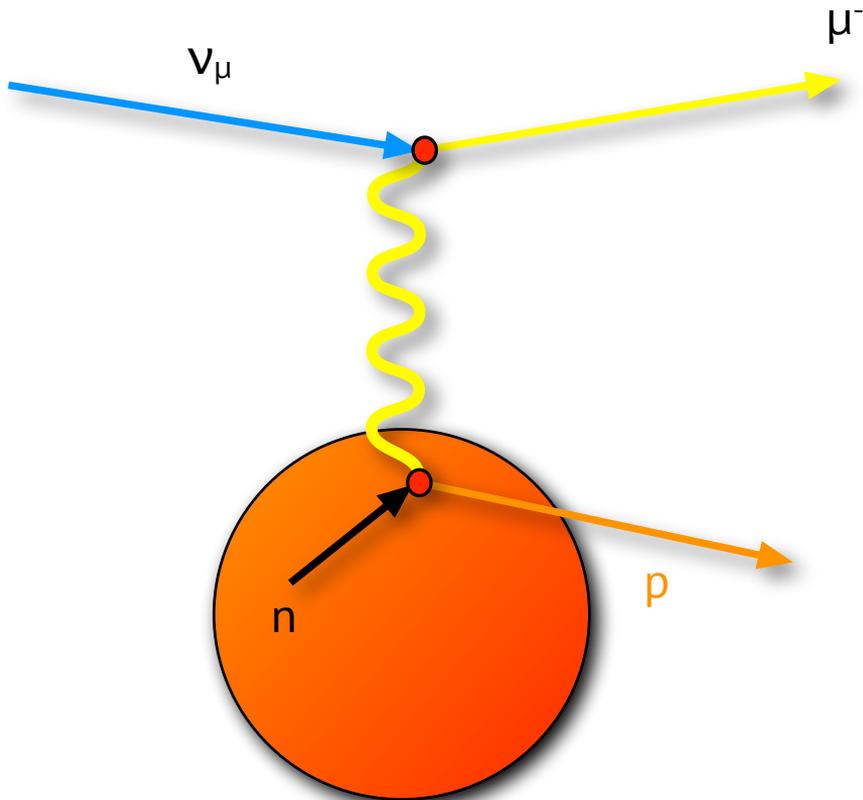
# Reaction Channel Menagerie

- Neutral Current: Exchange a Z boson.
  - NC Elastic
    - Predicted from CCQE except for NC contribution to the axial form-factor (via strange quarks).
  - NC  $\pi^0$ 
    - Important  $\delta_{CP}$  & Mass Hierarchy background.
  - Also have DIS, etc.

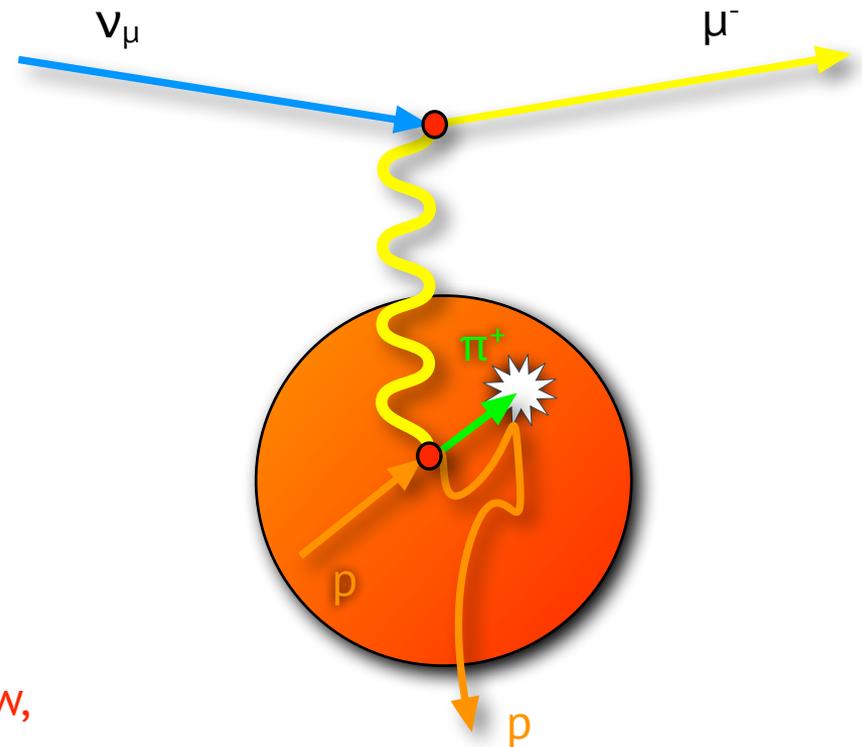
**Key Difference: Don't know neutrino flavor!**



# There is a catch...



Interactions take place in dense nuclear matter. (Otherwise, your experiment takes 100 years.)



Final State Interactions (FSI) are critical.

$$E_{\text{visible}} \neq E_\nu$$

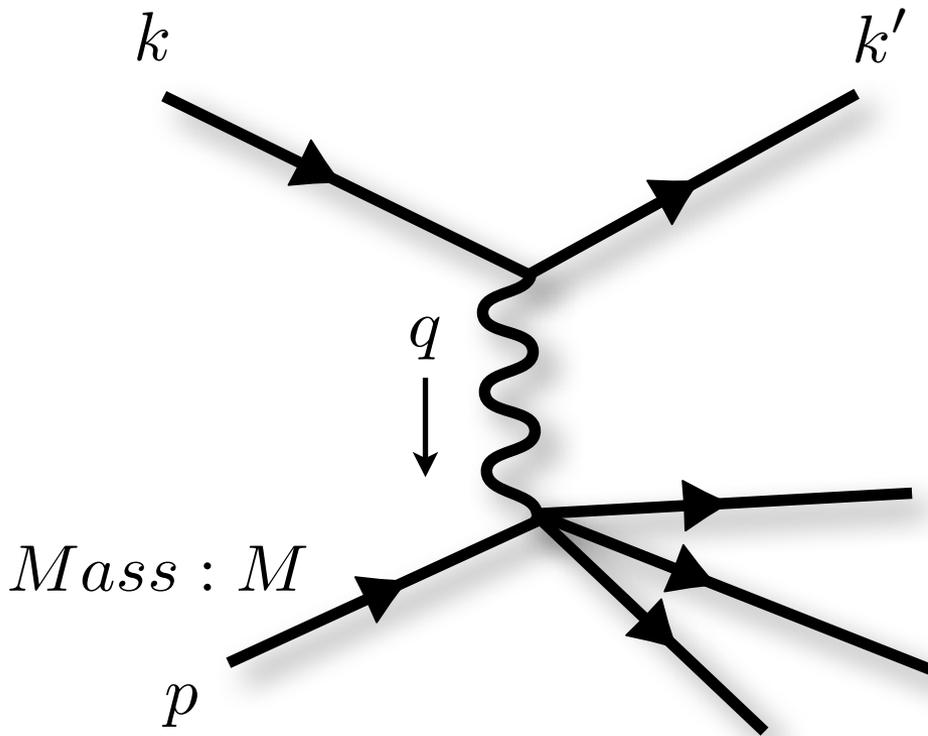
Not a calibration problem! You need to know,  
"what are the physics?"

# Reaction Channels

---

- Our breakdown above was a bit artificial.
  - We may only really be precise in channel definition when scattering from free nucleons.
  - Point of confusion: when people say "CCQE," what do they mean? It can mean something very strict when considering free nucleons, or just "any final state with no pions" when considering nuclear targets.
- In some senses, and especially for nuclear targets, the better way to think about final states is:
  - by current,
  - by number and type of baryon in the final state,
  - by number and type of meson in the final state.
- This is all we may observe. (Well, and the remnant.)

# Neutrino Kinematics Jargon : Technical



$$q^2 \equiv (p' - p)^2 = -Q^2$$

**(Momentum Transfer)<sup>2</sup>**

**Energy transfer.**

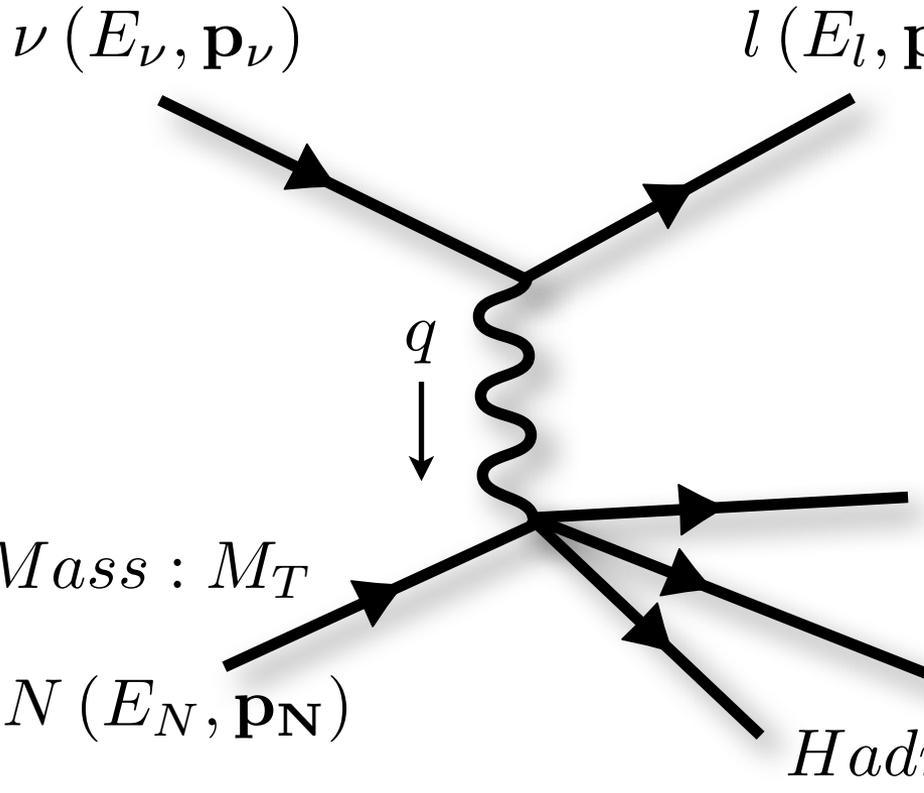
$$\nu \equiv \frac{p \cdot q}{M}$$

$$W^2 \equiv (p + q)^2 = E_H^2 - \mathbf{p}_H^2 \quad \text{(Hadronic Invariant Mass)<sup>2</sup>}$$

$$y = \frac{p \cdot q}{p \cdot k} \quad \text{Inelasticity}$$

$$x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu} \quad \text{Parton Momentum Fraction}$$

# Neutrino Kinematics Jargon : Practical



$$\begin{aligned}
 q^2 &\equiv (E_l - E_\nu)^2 - (\mathbf{p}_l - \mathbf{p}_\nu)^2 \\
 &= -Q^2 \\
 &= t \quad \text{(Momentum Transfer)}^2 \\
 Q^2 &= 4E_\nu E_\mu \sin^2(\theta/2)
 \end{aligned}$$

$$\begin{aligned}
 \nu &= E_\nu - E_l \\
 &\text{Energy transfer.}
 \end{aligned}$$

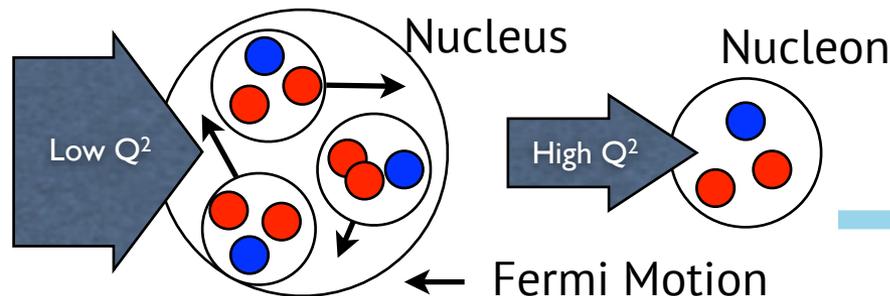
$$W^2 = M_T^2 + 2M_T E_H - Q^2 \quad \text{(Hadronic Invariant Mass)}^2$$

$$y = 1 - \frac{E_l}{E_\nu} \quad \text{Inelasticity}$$

$$x_{Bj} = \frac{Q^2}{2M_T \nu} \quad \text{Parton Momentum Fraction}$$

# How to think about neutrino interactions

- $W/Z$  boson exchange with target is complex.
- We slide around in  $Q^2$ ,  $W$ ,  $x$ , and  $y$  with reasonably hard divisions between coherent, elastic, and inelastic reactions, but the target is very messy - do we resolve partons? nucleons? correlated groups of nucleons? the whole nucleus? It depends on the kinematics.
- The produced particles can vary a lot for a given set of kinematics and the inverse is also true - many different kinematic configurations can produce the same set of produced particles.
- Then everything gets smeared by final state interactions.

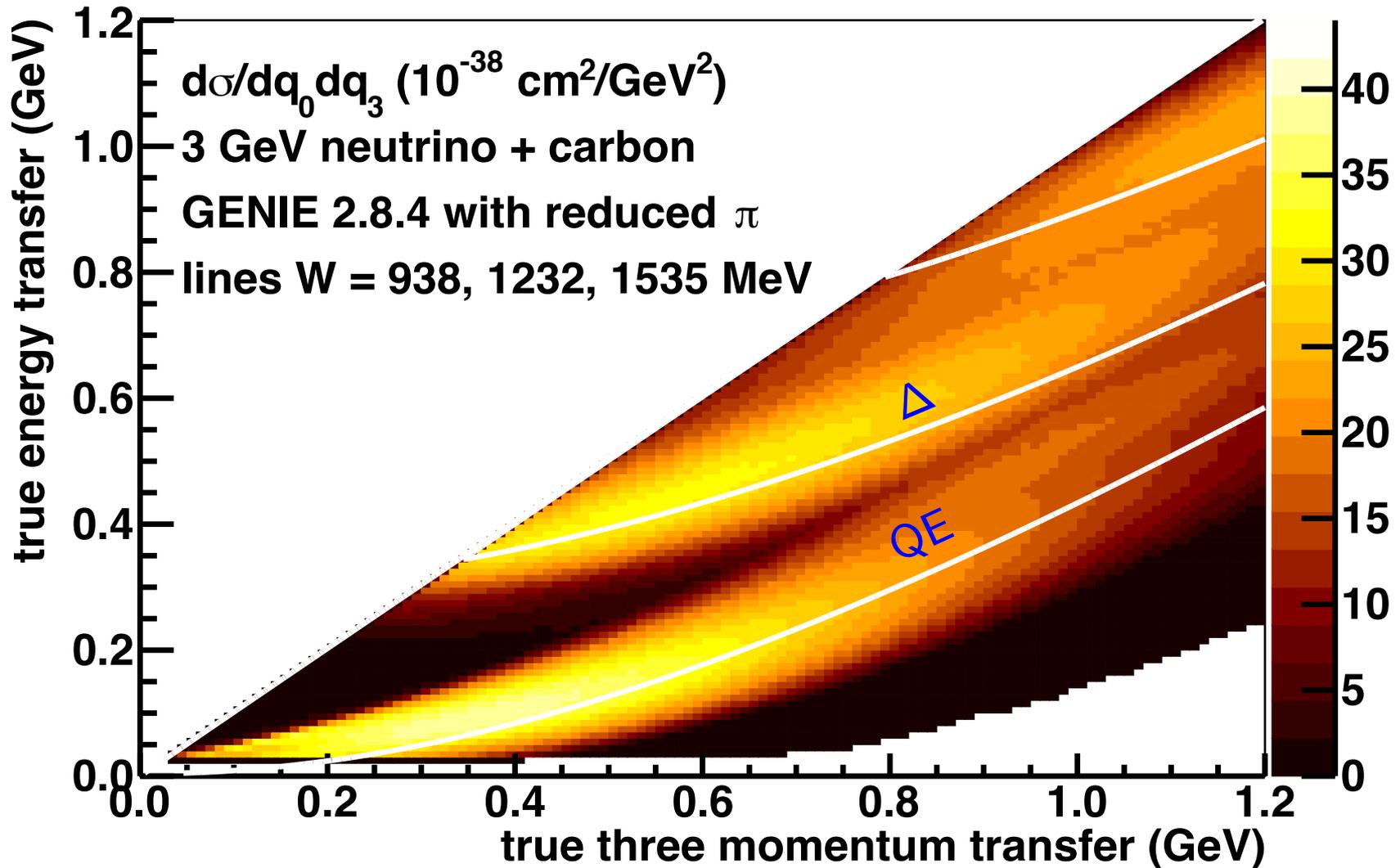


# How to think about neutrino interactions

---

- Generically though, at very low energy transfer to the nucleus ( $t$ ), reactions are coherent. The nucleus remains in the ground state and we produce a pion and a lepton.
- As we move up in energy transfer and four momentum transfer, we start to resolve more of the inside of the nucleus and scatter of nucleons and correlated groups of nucleons (all bound, of course). We may or may not produce resonances (or mesons through non-resonant processes). We generally call these reactions quasielastic, elastic, or “2p2h” depending on current and scattering target.
- In this smeary region of energy and momentum transfer we produce pions through resonances and other means - we are no longer “elastic”.
- We pass through a complex transition region until we begin resolving partons. This is the domain of deep inelastic scattering (high  $Q^2$ , high  $W$ , very messy and busy final states).

One example of this... (note - missing 2p2h in this GENIE)

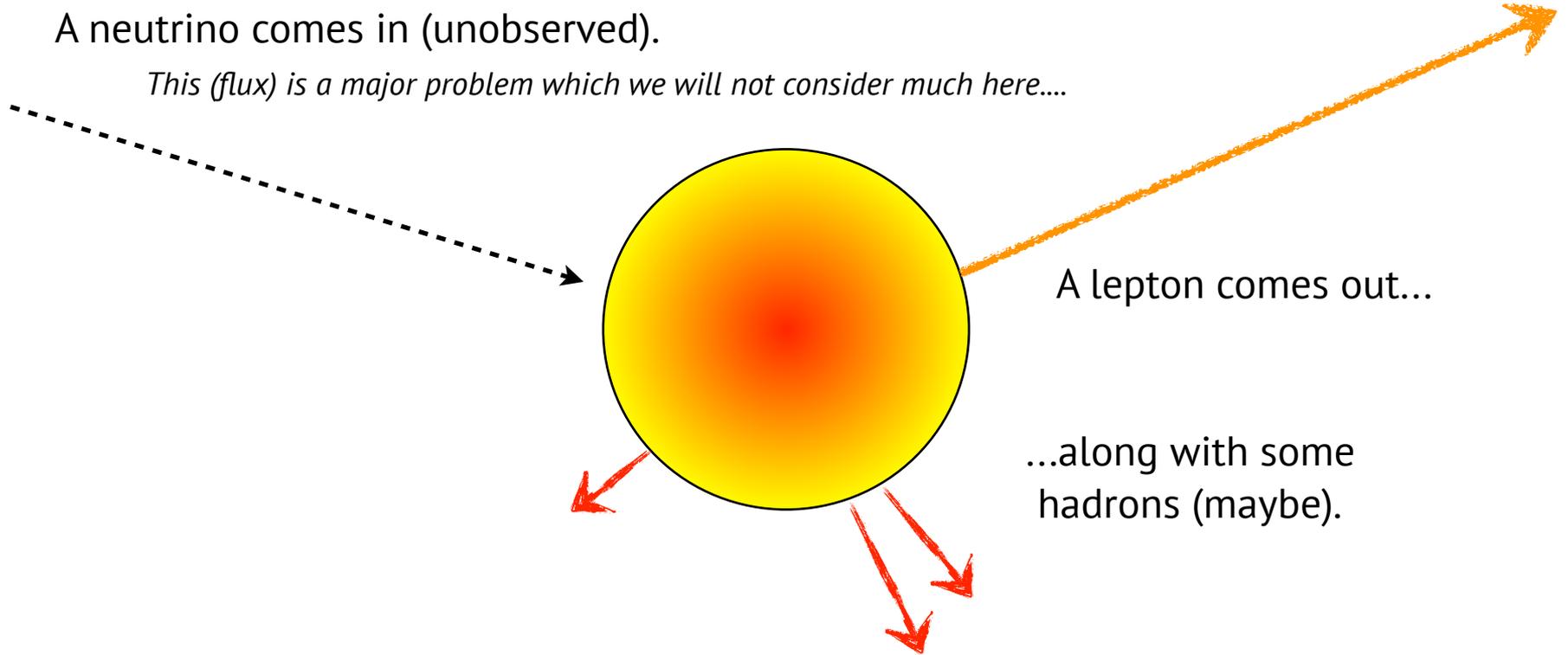


# The Basic Problem

---

A neutrino comes in (unobserved).

*This (flux) is a major problem which we will not consider much here....*



A lepton comes out...

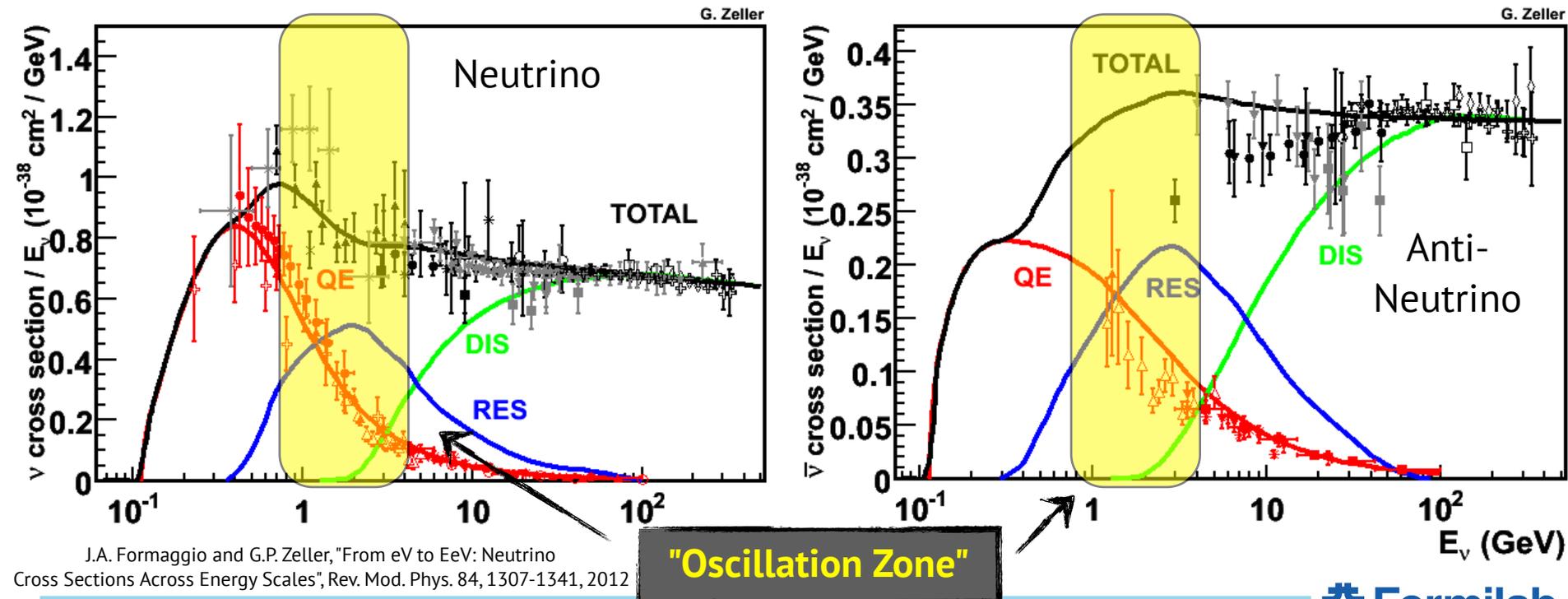
...along with some  
hadrons (maybe).

*What was the neutrino's energy?*

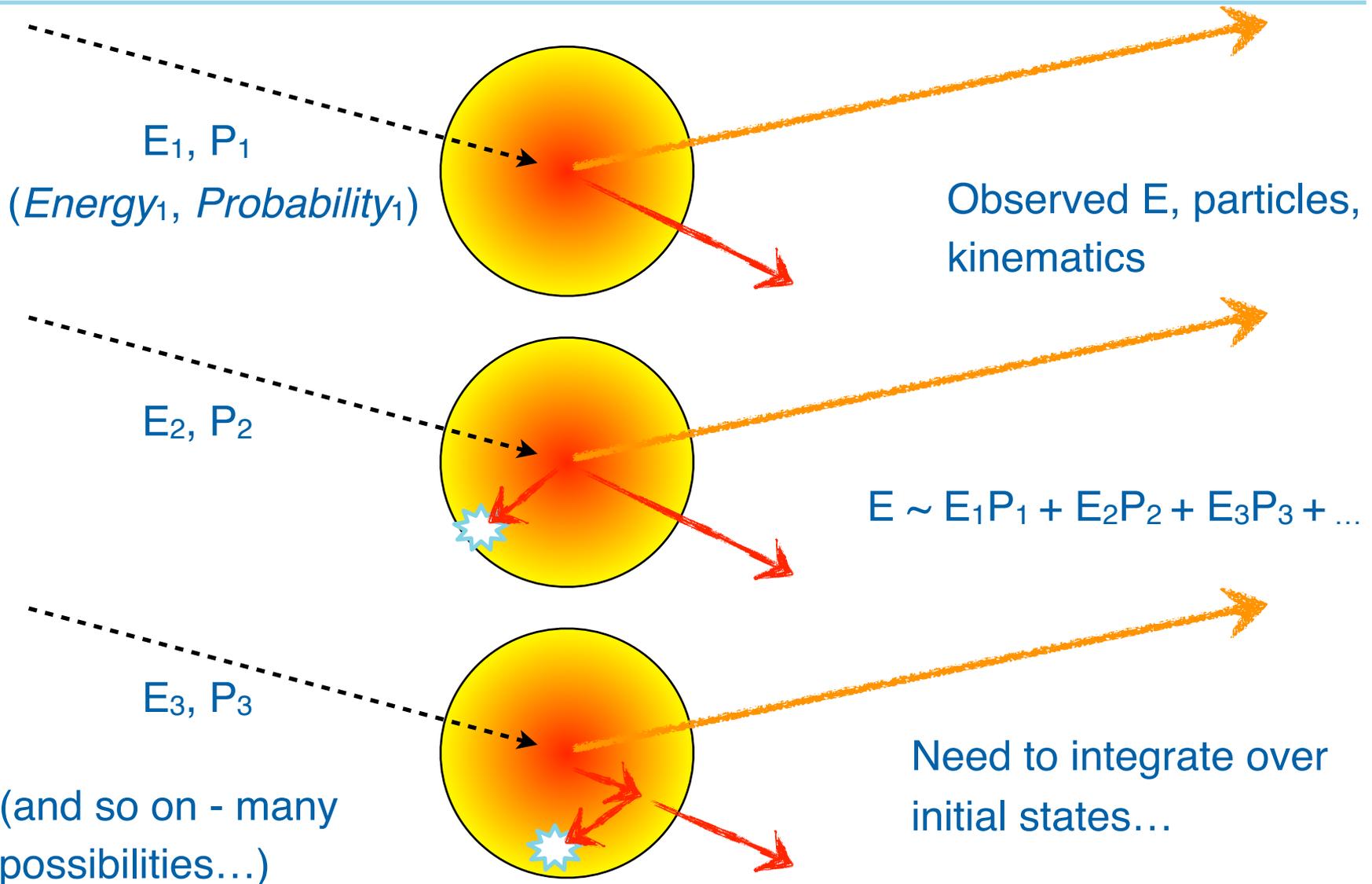
*We really want flavor too...*

# Embedded Assumptions

- There are a few facts that are often buried in the details of discussions of neutrino interactions:
  - Your knowledge of the flux is typically only good to 10-20% and you have *no information event-by-event*.
  - Kinematic distributions are always integrated over a *specific* (barely known) flux.
  - Measurements are always convolutions of flux, cross section, nuclear effects, and detector efficiencies.



# The Basic Problem: The BEST We Can Do



# The Basic Problem: The BEST We Can Do

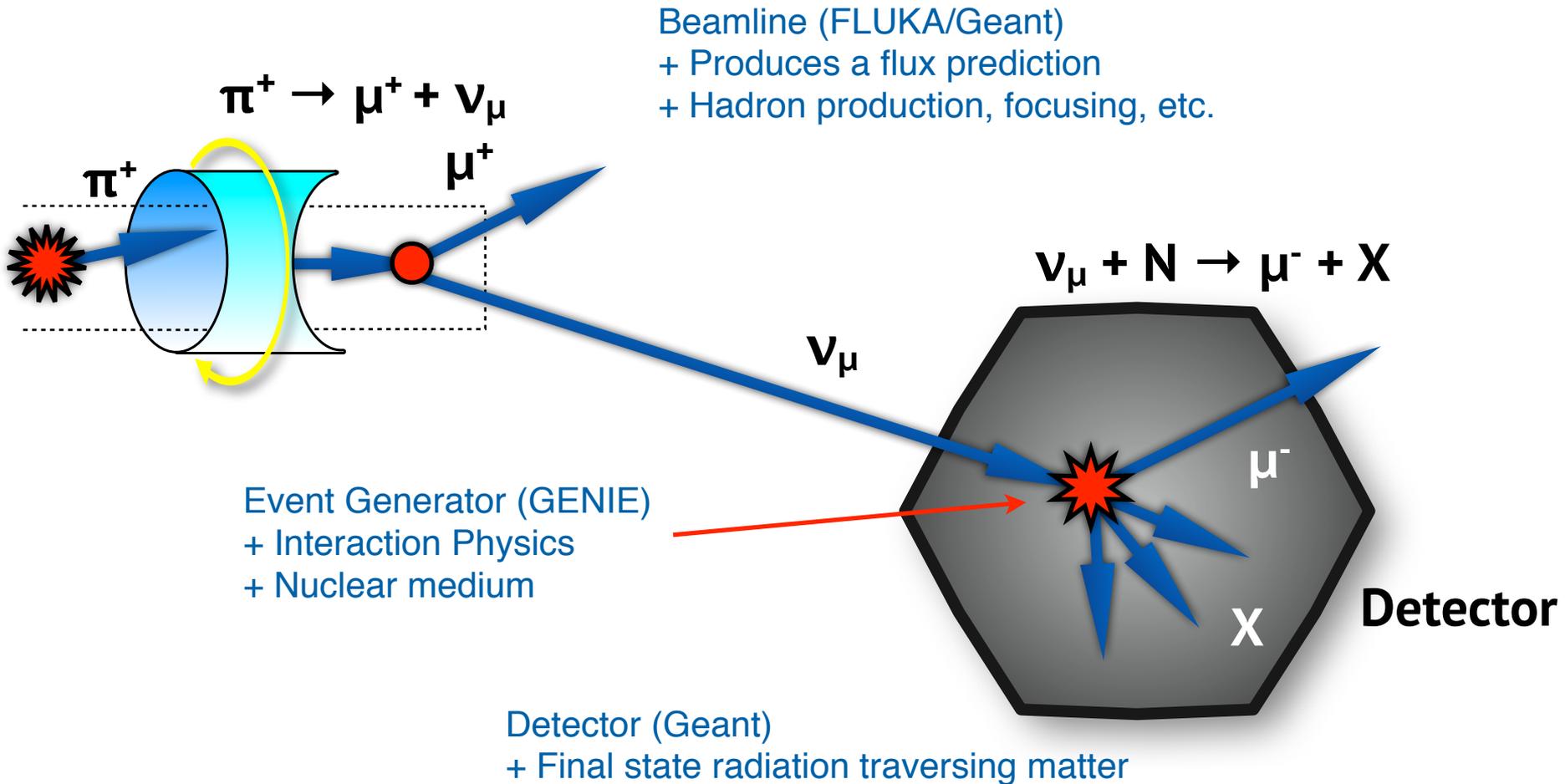
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- The best we can do is build a map, weighted by probability, that provides all the possible initial states for an observed final state.
- With this map and a sample of events, we may infer a neutrino energy distribution (or some other kinematic distribution).
- How do we make any progress without an initial energy to begin with?
- For measurements, we use an ***event generator*** to predict backgrounds and the efficiency.
  - We may constrain the background prediction with data.
  - We must impose systematic uncertainties on our efficiency based on model estimates.
    - The more measurements we have, the better we may constrain these uncertainties and the better is our probability map.

`std::map<observed_topolgy, std::list<std::pair<probability, physics>>> = ?`

The generator is *crucial* to do the physics!

# Neutrino Simulations: A Three-Part Software Stack



# Neutrino MC Event Generators

---



- The generator must simulate all the types and momenta of every particle that appears in the final state.
- Some generators (MadGraph, Pythia, etc.) are computation aids for theorists, but GENIE is not.
- This is because we lack a theoretical framework that is both *complete* and *consistent*.
- The ideal input theory would be internally consistent and provide fully-differential cross sections in the kinematics of every final state particle over all reaction mechanisms, energies, and targets.
- Modern theory typically provides final state kinematics for the lepton only, and only over limited ranges in energy or momentum transfer, and may be fully exclusive or fully inclusive with no guidance on how to merge the regimes.
  - But the experiments must go on! So we must *stitch together* an ensemble that is consistent with all the data.

## What else do neutrino event generators provide?

---



- Interfaces to geometry engines for modeling complex detectors.
- Flux drivers for computing exposure (atmospheric/solar sources) or normalizing responses to accelerator beams.
- Event re-weighting engines for studying systematic uncertainties and performing error propagation.
- Databases of electron, hadron, and neutrino scattering experiments with applications for comparing simulation and data.
  - Electron and hadron scattering event generator functionality.
- Nucleon decay generators.
- Libraries of pre-computed cross sections.

# GENIE

---



- <https://genie.hepforge.org>
- The software:
  - Created to be a “universal event generator”.
    - Additionally run in electron and hadron scattering modes.
  - Many tools for studying systematics, comparison to data, etc.
  - Event handling is decoupled from physics routines, easy to create arbitrary algorithm stacks.
- The collaboration:
  - International collaboration with about a dozen collaborators (essentially all experimentalists) and many more contributors.
    - Collaborators do service work (validation, distribution, user support, developer support, etc.)
    - Contributors (many theorists) offer individual models or pieces of validation software, sometimes consulting, etc.

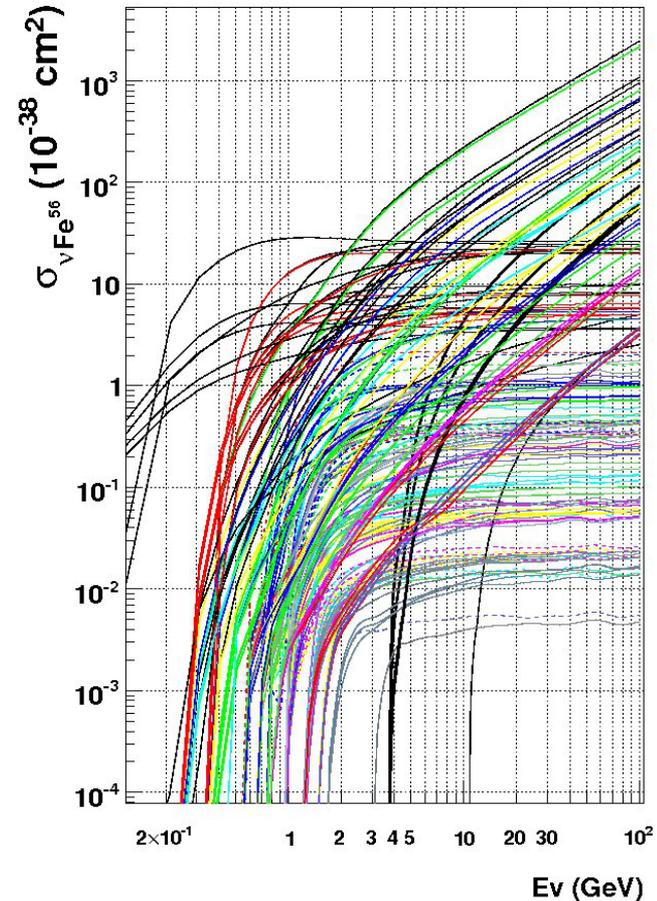
# How does GENIE work?

- The first step is to compute the total cross section for the input energy, flavor, helicity, and target isotope.
- Perform a sum over exclusive channels (square then sum, sigh).
- Numerical integration of the corresponding differential cross section expression:
  - Computationally intensive procedure (100's of millions of differential cross section evaluations), but only needs to be run once per release.

<https://www.hepforge.org/archive/genie/data/>



$\nu_\mu, \bar{\nu}_\mu + Fe$ , all processes



# How does GENIE work?

- Currently implemented GENIE physic models rely heavily on a factorization assumption.
- Some cases blend boxes together a bit (but for the most part they do not).



"Is that safe?"

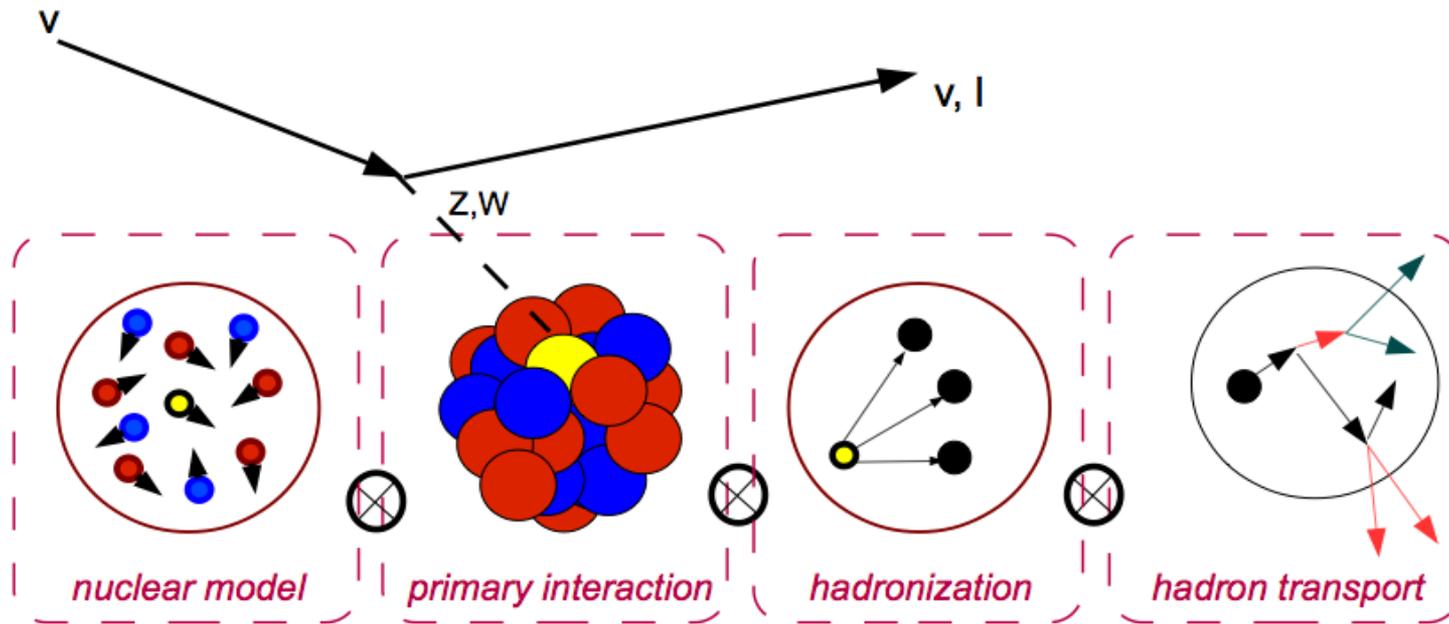


Figure by C. Andreopoulos

# Pieces (Usually)



- Vertex selection
  - Simple nuclear density model
- Initial state nuclear model
  - Removal energy and momentum
    - RFG with Bodek-Ritchie tails.
    - New: Local Fermi Gas
    - New: Effective Spectral Function
    - Almost there: "Benhar" spectral function
    - Just started: Correlated Fermi Gas (MIT)

GROUND STATE

- Hard scattering process
  - Differential cross section formula to get event kinematics (x, y, Q<sup>2</sup>, W, t, etc.)
- Lepton kinematics
- Hadronic system

INITIAL STATE

- Propagation/transport (default is an "effective cascade")
  - Fast and re-weightable

FINAL STATE

# Usual Pieces



## REMNANT STATE

- Decays before and after propagation
- Remnant decay
  - Just started caring about this, really...
  - Current model is very simple
    - Working on adopting other codes (Geant4, INCL++, possibly GiBUU) to handle clustering, de-excitation, evaporation
    - May be a bridge to more sophisticated transport codes
- Sometimes models can't work this way - e.g., discovering we can't separate lepton and hadron kinematics into separate modules for QE events (can't compute cross section in Q2 and then compute lepton and hadron kinematics, need to flip the procedure and then accept-reject based on Q2), etc.
  - (Actually, we should do all events this way - but the code runs much slower and so we're working on ways to make that process fast enough to be more widely used.)

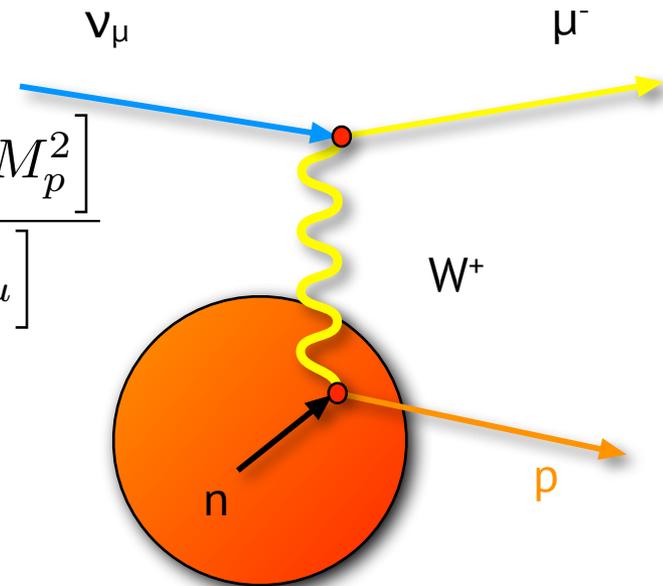
# Neutrino Energy Idea: Use Quasi-Elastics

(Flip nucleons for antineutrino scattering.)

$$E_{\nu}^{QE} = \frac{2(M_n - E_B) E_{\mu} - \left[ (M_n - E_B)^2 + m_{\mu}^2 - M_p^2 \right]}{2 \left[ (M_n - E_B) - E_{\mu} + \sqrt{E_{\mu}^2 - m_{\mu}^2} \cos \theta_{\mu} \right]}$$

$$Q_{QE}^2 = -m_{\mu}^2 + 2E_{\nu}^{QE} \left( E_{\mu} - \sqrt{E_{\mu}^2 - m_{\mu}^2} \cos \theta_{\mu} \right)$$

$E_{\mu} = T_{\mu} + m_{\mu}$	Muon Energy
$M_n, M_p, m_{\mu}$	Neutron, Proton, Muon Mass
$E_B$	Binding Energy (~30 MeV)
$\theta_{\mu}$	Muon Angle w.r.t. Neutrino Direction

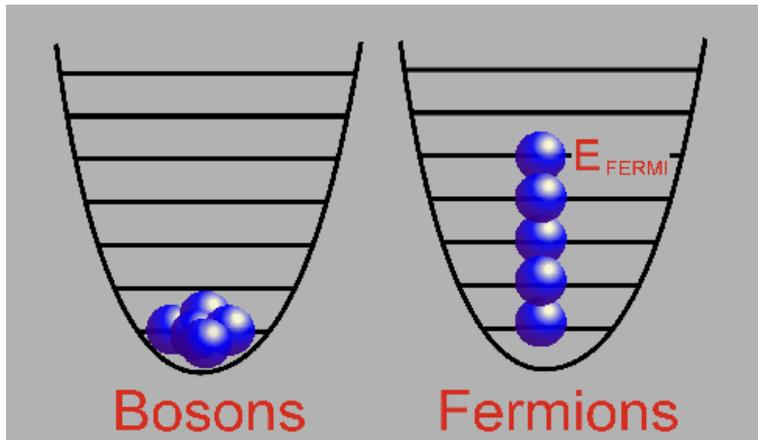


$$\nu_l + n \rightarrow l^- + p$$

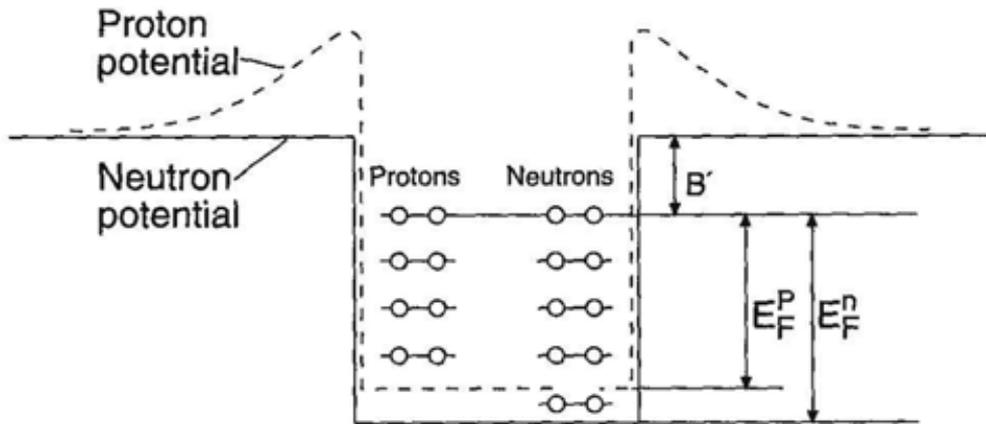
$$\bar{\nu}_l + p \rightarrow l^+ + n$$

Get everything with just the lepton!

# Nucleon not at rest: Fermi Gas Model



- **Impulse approximation:** scatter off independent single nucleons summed (incoherently) over the nucleus.
- In the FGM, all the nucleons are non-interacting and all states are filled up to  $k_F$ .
- The IA becomes problematic when the momentum transfer is *smaller* than  $\sim 300$  MeV (think about the de Broglie wavelength and remember  $1 \text{ fm} = 1/200 \text{ MeV}$ ).



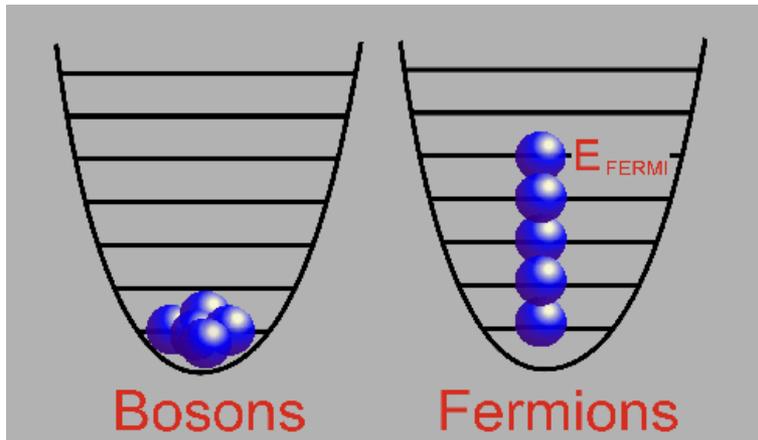
B. Povh et al, *Particles and Nuclei*, Springer 2002



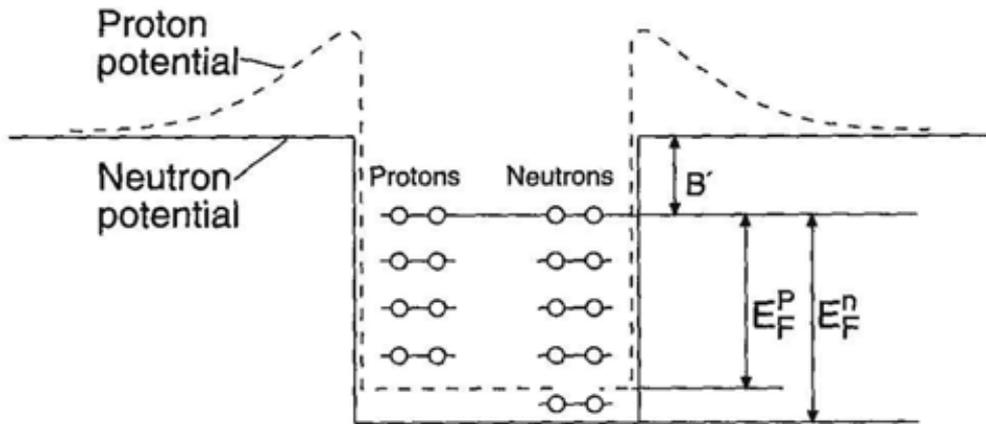
It is nice to see this problem getting high-level attention.

$^{12}\text{C}$	$E_B = 25 \text{ MeV}$	$p_F = 220 \text{ MeV}/c$
-----------------	------------------------	---------------------------

# Nucleon not at rest: Fermi Gas Model



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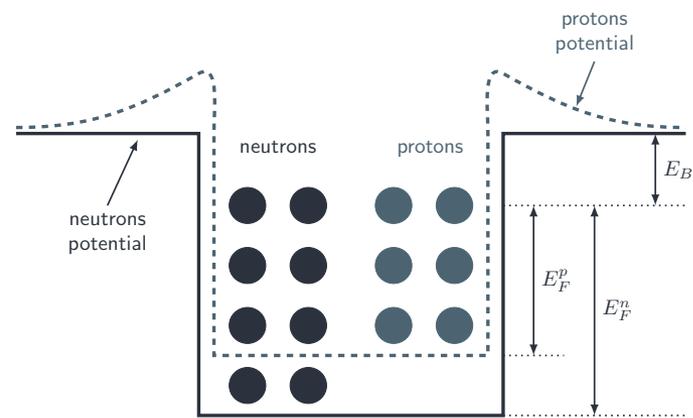
B. Povh et al, *Particles and Nuclei*, Springer 2002

$^{12}\text{C}$	$E_B = 25 \text{ MeV}$	$p_F = 220 \text{ MeV}/c$
-----------------	------------------------	---------------------------

You can't use the Fermi Gas Model anymore!



Nucleons move freely within the nuclear volume in constant binding potential.



Global Fermi Gas

Local Fermi Gas

$$p_F = \frac{\hbar}{r_0} \left( \frac{9\pi N}{4A} \right)^{1/3}$$

$$p_F(r) = \hbar \left( 3\pi^2 \rho(r) \frac{N}{A} \right)^{1/3}$$

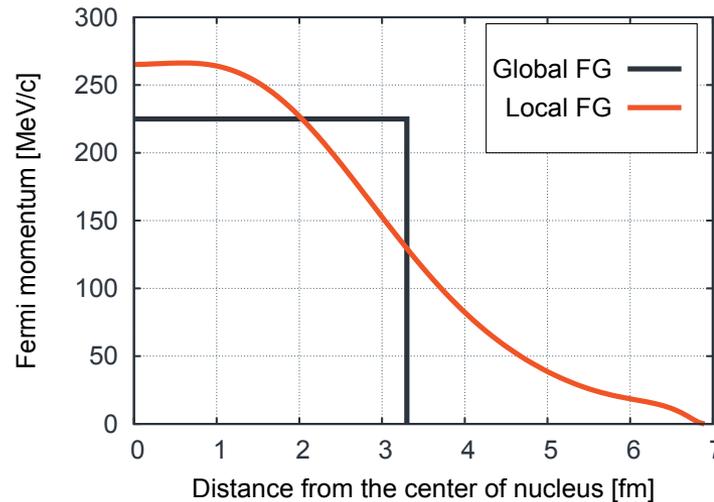
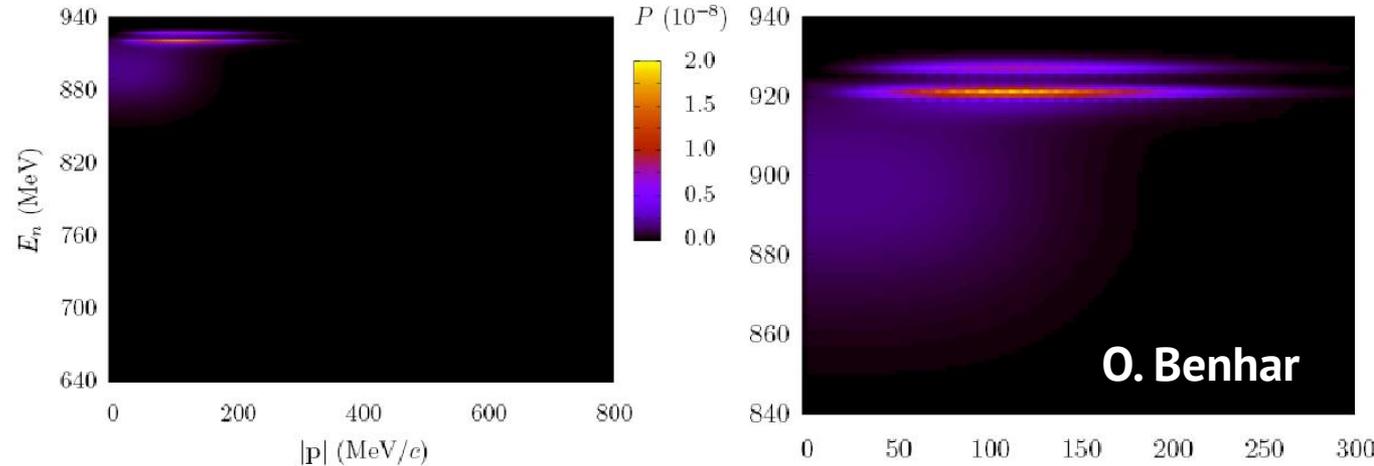


Figure by T. Golan

# Spectral Functions

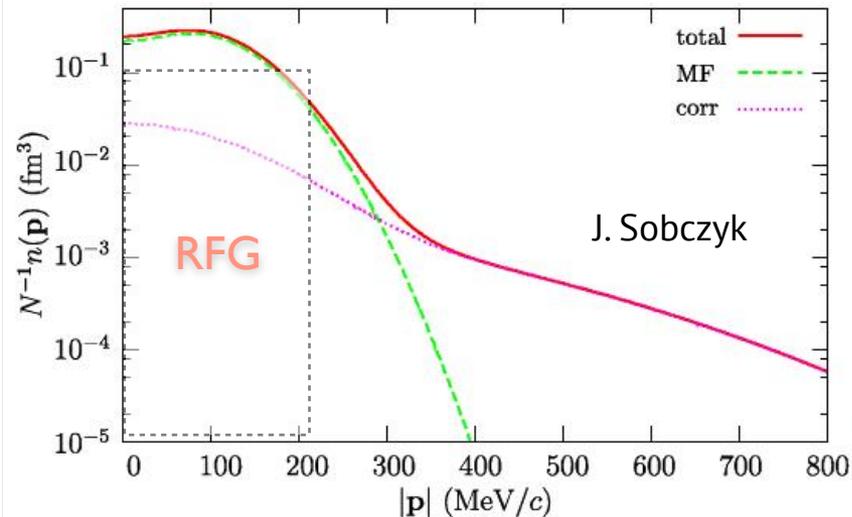
- Most event generators use the Fermi Gas model.
- But there are better options: *Spectral Functions*.
- Technically FGM is a "spectral function" also - SFs offer momentum distributions and removal energy for nuclei.

## Spectral Function for Oxygen



Shell Orbitals  
are visible:

	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$
E (MeV)	45	18.44	12.11



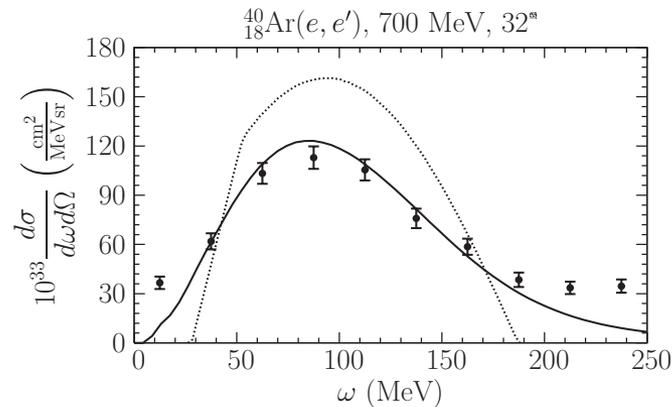
- The Mean Field (MF, "long-range") and Short-range Correlations (SRC) contributions are separated here.
- The high momentum tail (absent in the Fermi Gas Model) comes from correlated pairs of nucleons.

# Spectral Functions

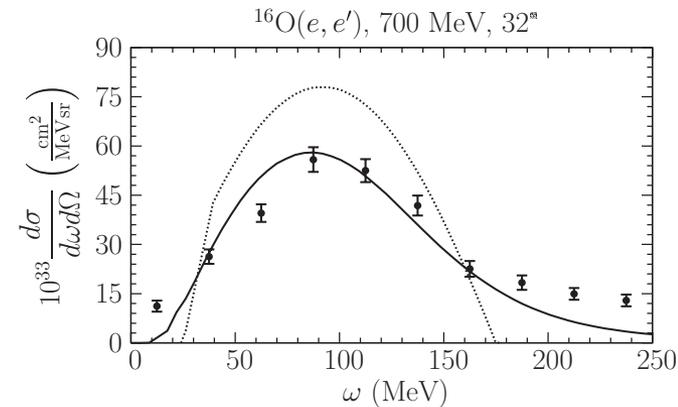
- Typically, spectral functions better reproduce the quasielastic peak.

## Electron Scattering Data

### Argon



### Oxygen



A. Ankowski and J. Sobczyk, PRC 77, 044311

- Comparison of a Gaussian Spectral Function (GSF, solid) and Fermi Gas Model (FGM, dashed) for Argon (left) and Oxygen (right) in electron scattering data.

The probability of removing of a nucleon with momentum  $\vec{p}$  and leaving residual nucleus with excitation energy  $E$ .

$$P(\vec{p}, E) = P_{MF}(\vec{p}, E) + P_{corr}(\vec{p}, E)$$

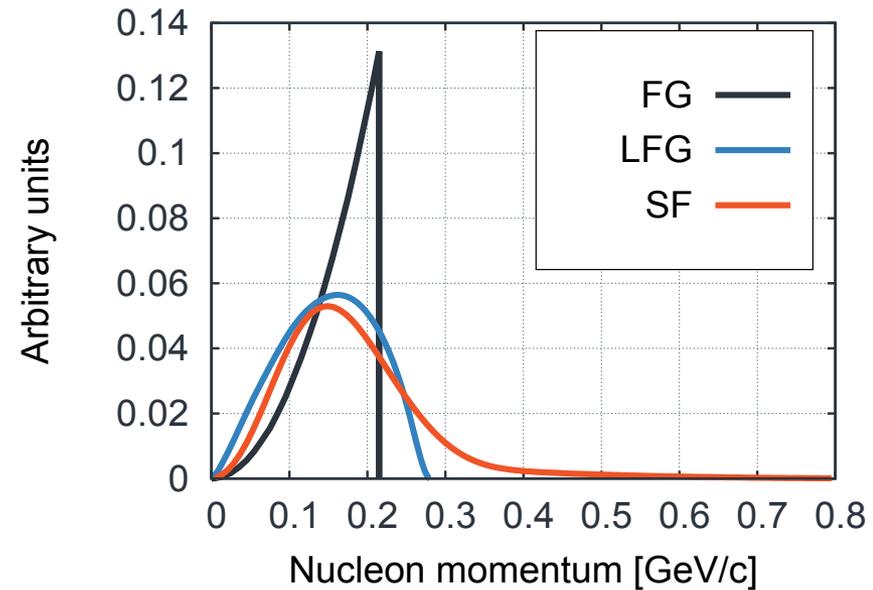
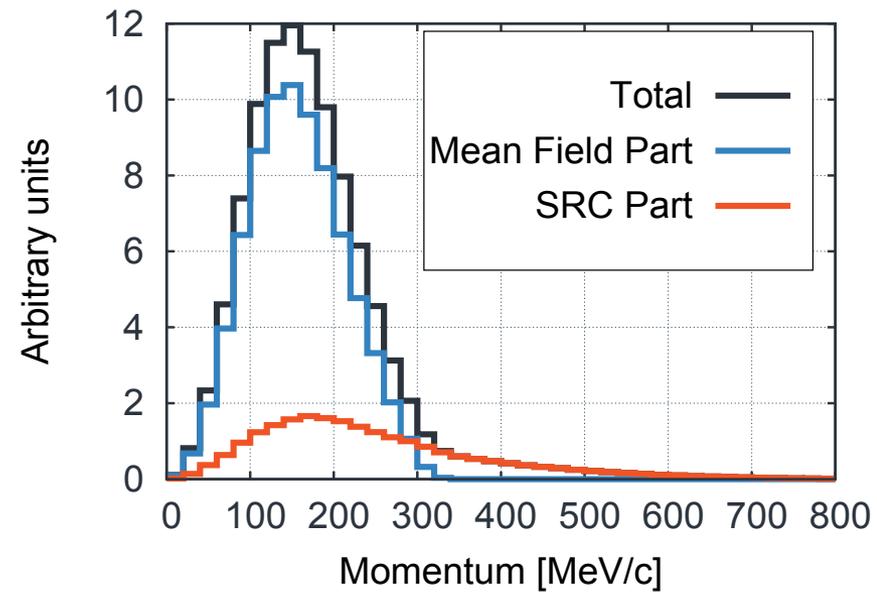
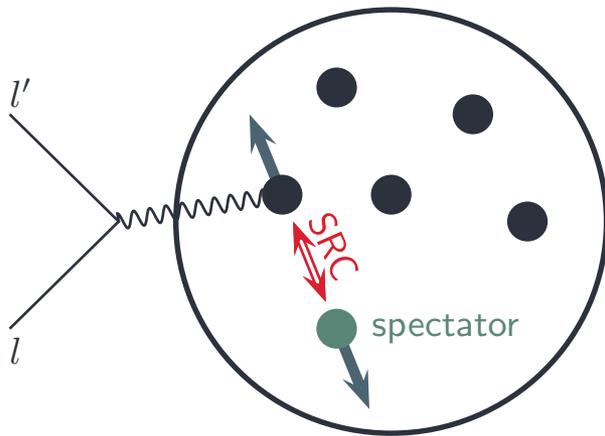
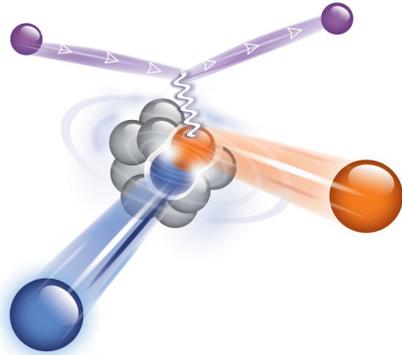


Figure by T. Golan

# Nucleon-nucleon correlations

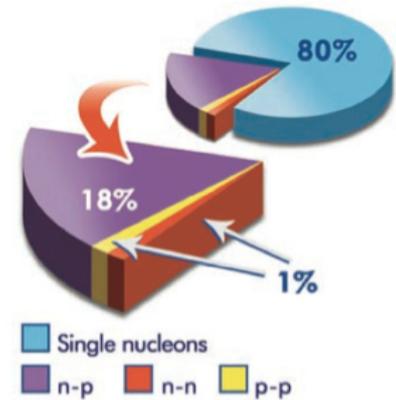
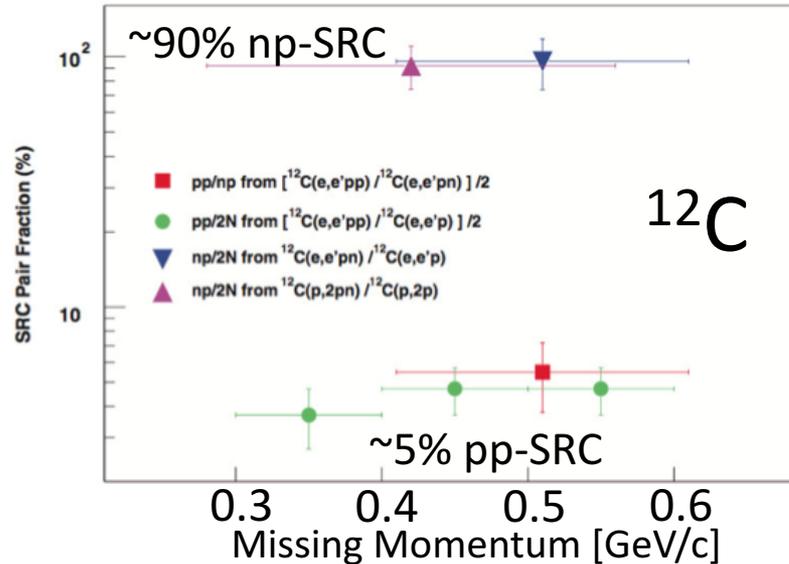


Recent(ish) Jlab analyses of  $^{12}\text{C}$  quasi-elastic scattering with **electrons** have demonstrated significant probabilities to see multiple nucleons knocked out.

See also O. Hen et al, Science 364 (2014) 614

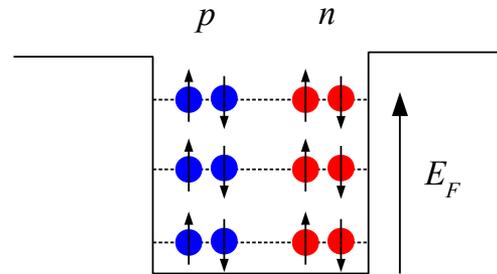
- The kinematics may be altered there is a ~20% chance of scattering from a correlated pair of nucleons rather than a single nucleon.
- This is not a new idea in quasielastic scattering, but evidence in charged lepton scattering now strengthens the case.

R. Subedi et al., Science 320 (2008) 1476

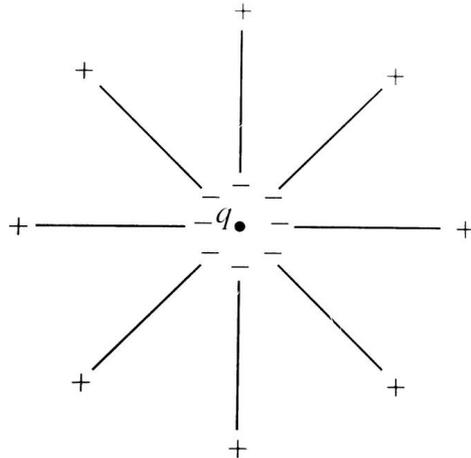


Dekker et al., PLB **266**, 249 (1991)  
 Singh, Oset, NP **A542**, 587 (1992)  
 Gil et al., NP **A627**, 543 (1997)  
 J. Marteau, NPPS **112**, 203 (2002)  
 Nieves et al., PRC **70**, 055503 (2004)  
 Martini et al., PRC **80**, 065001 (2009)

# Evidence from nuclear physics suggests two effects missing in current event generators

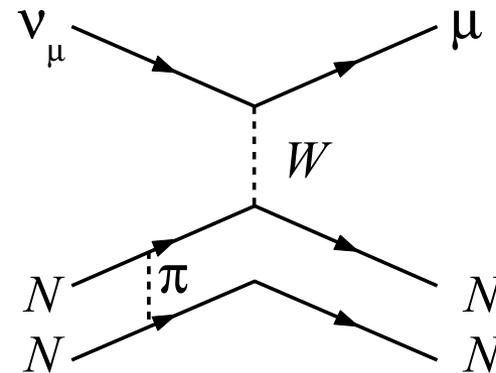


## 1. Screening from $W$ polarization

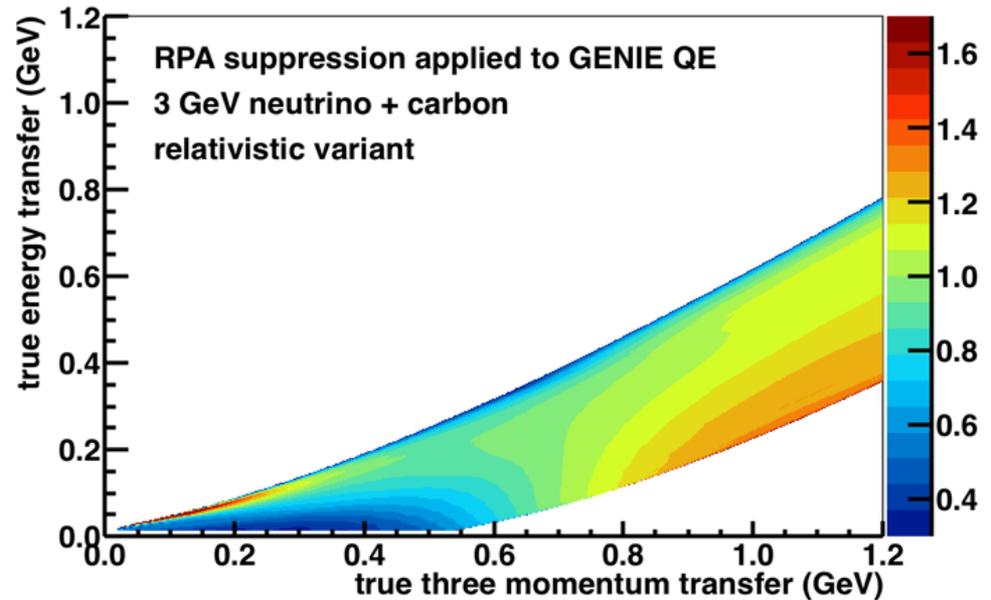
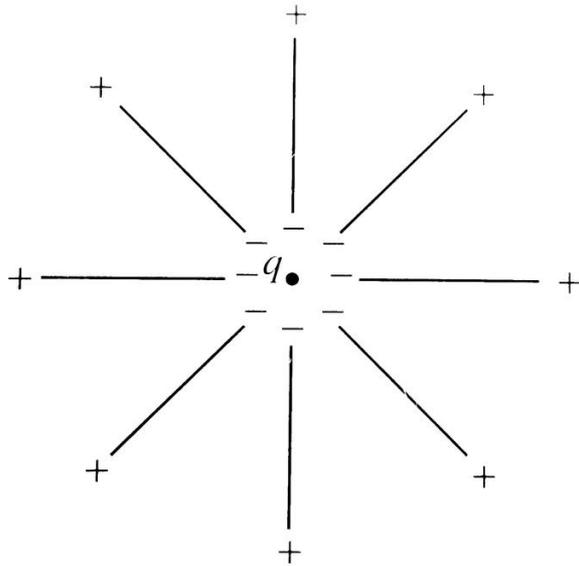


Griffiths, *Introduction to Electrodynamics*

## 2. Interactions involving multiple nucleons



# Charge screening in nuclear medium: “RPA”



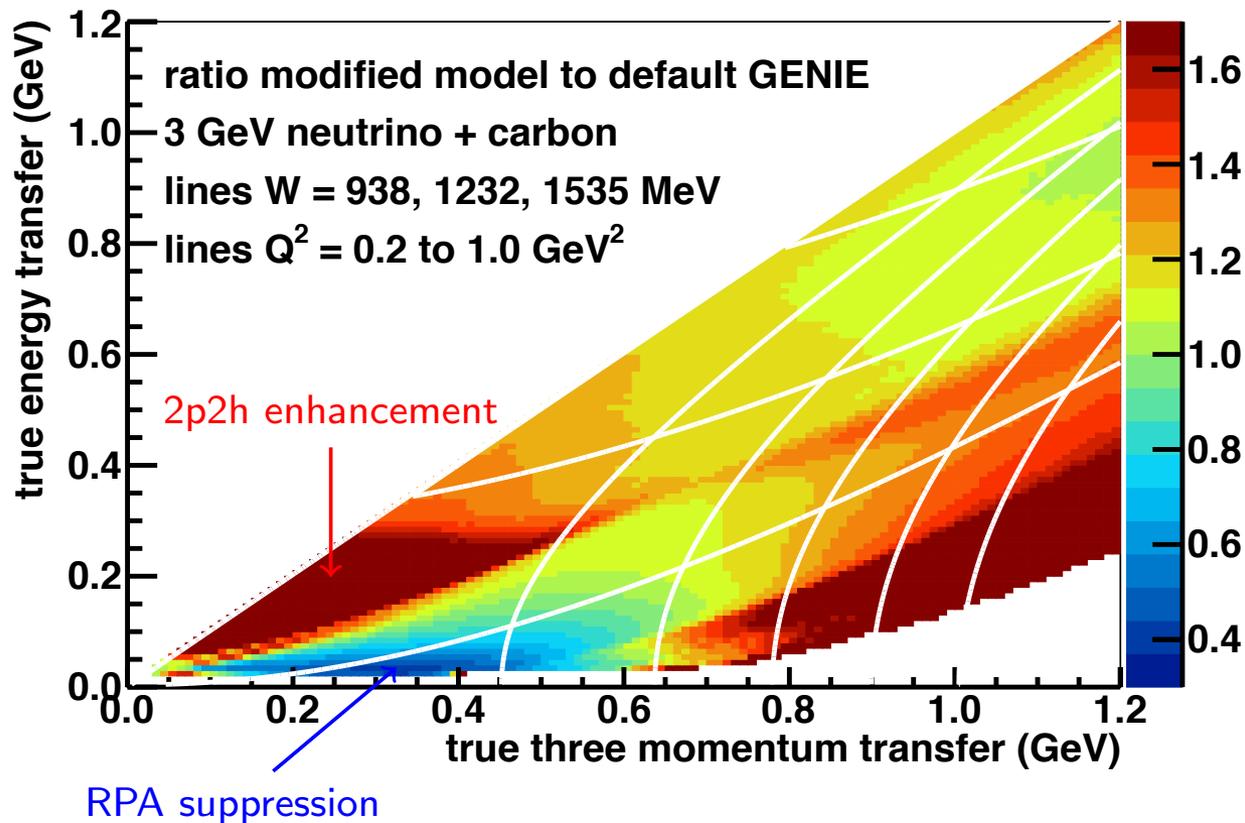
Griffiths, *Introduction to Electrodynamics*

- ▶ Analogous to screening of electric charge in a dielectric
- ▶ Calculated using Random Phase Approximation (RPA) PRC 70, 055503 (2004)
- ▶ Suppresses low energy, momentum transfer

P. Rodrigues, FNAL JETP Dec. 11, 2015

## These two effects turn up in different regions of our 2D space

- ▶ Put in both effects. take ratio to nominal:



- ▶ Use illustrative Nieves *et al.* calculations PRC 70, 055503 (2004); PRC 83, 045501 (2011)
- ▶ Calculations only for  $0\pi$  final states

P. Rodrigues, FNAL JETP Dec. 11, 2015

# Final State Interactions

- Hadrons produced at the hard-scattering vertex must propagate out of the nucleus - very complex process (everything is an off-shell, many-bodied, non-perturbative, strongly coupled mess).
- Three ways of handling it on the market: transport theory (GiBUU - <http://gibuu.hepforge.org> - , best theory), intranuclear cascade (“billiard balls”), parameterized cascade.

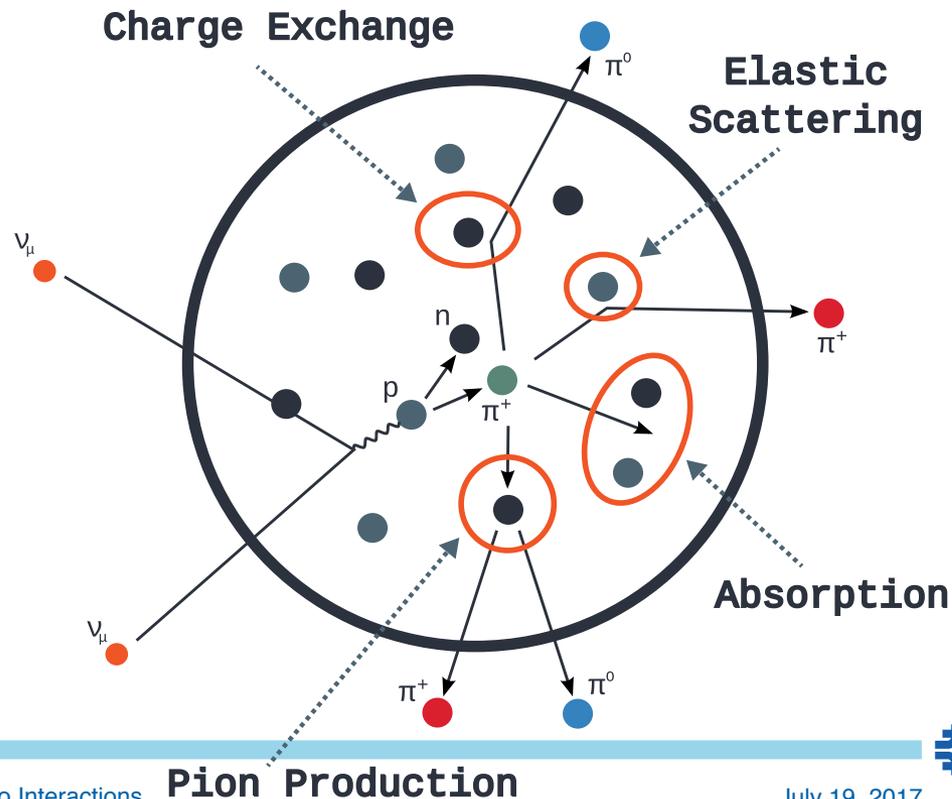
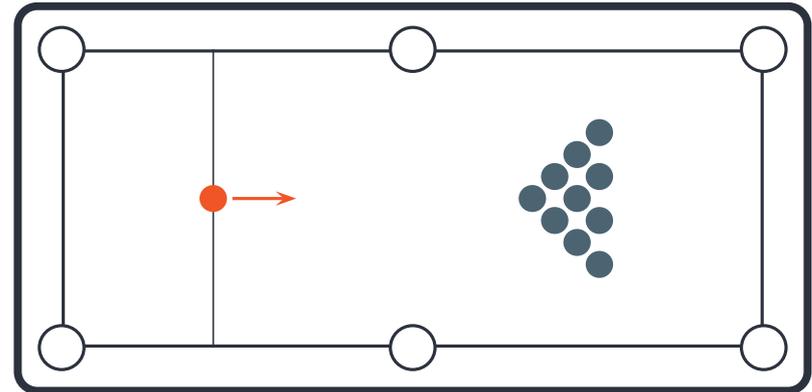


Figure by T. Golan

- In INC model particles are assumed to be classical and move along the straight line.



- The probability of passing a distance  $\lambda$  (small enough to assume constant nuclear density) without any interaction is given by:

$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

$\tilde{\lambda} = (\sigma\rho)^{-1}$  - mean free path

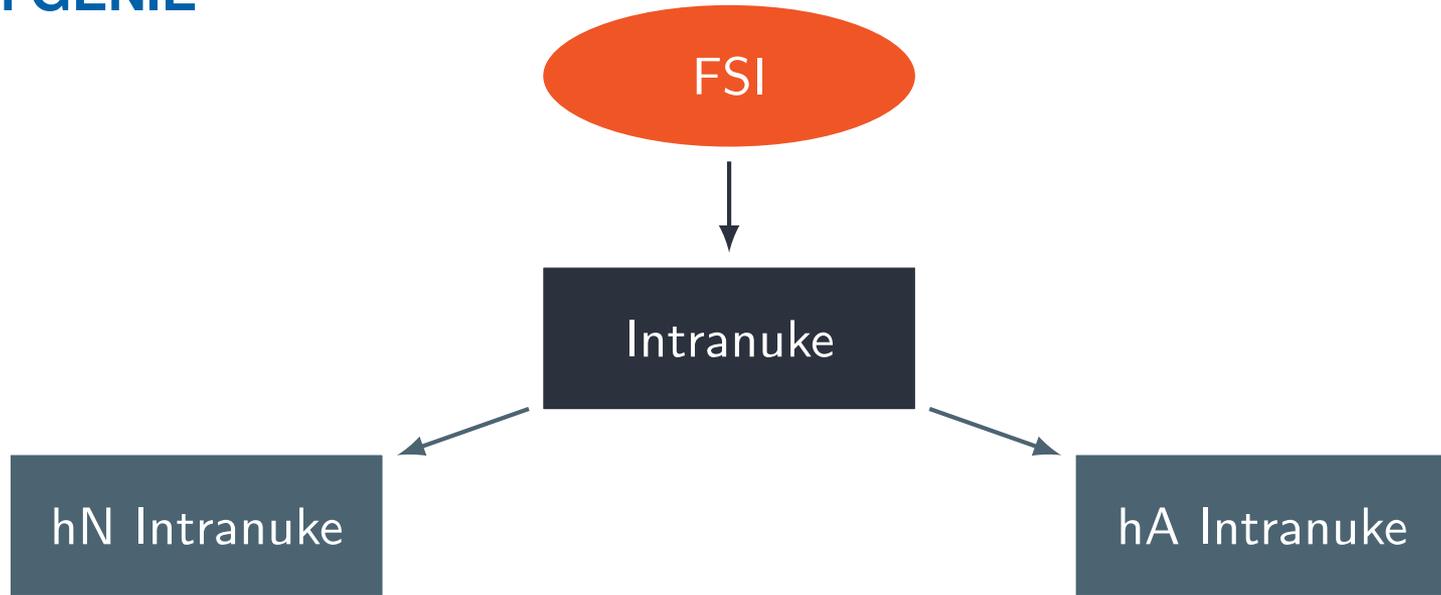
$\sigma$  - cross section

$\rho$  - nuclear density

Can be easily handled  
with MC methods.

Figure by T. Golan

# FSI in GENIE

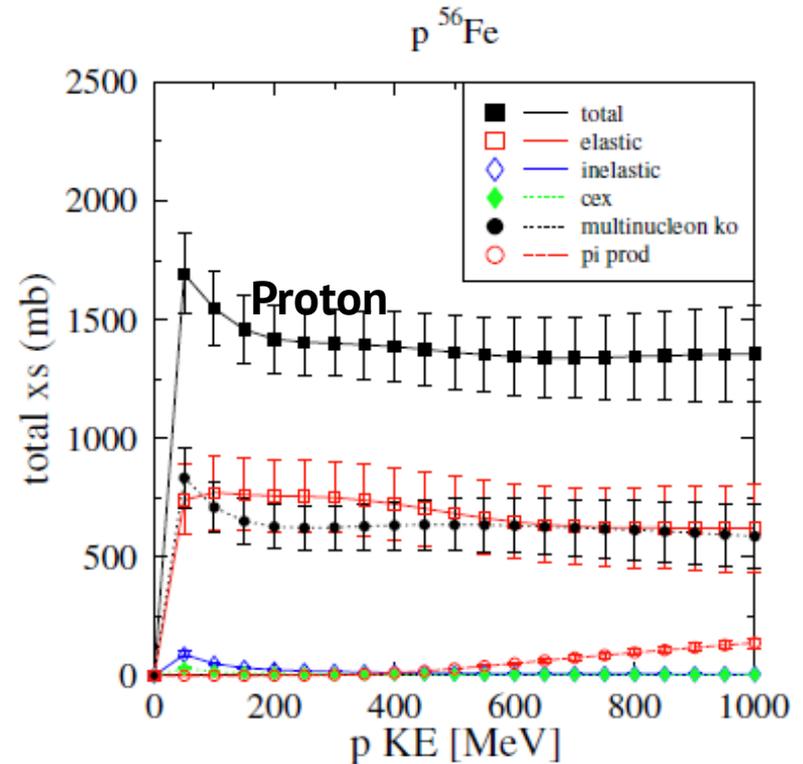
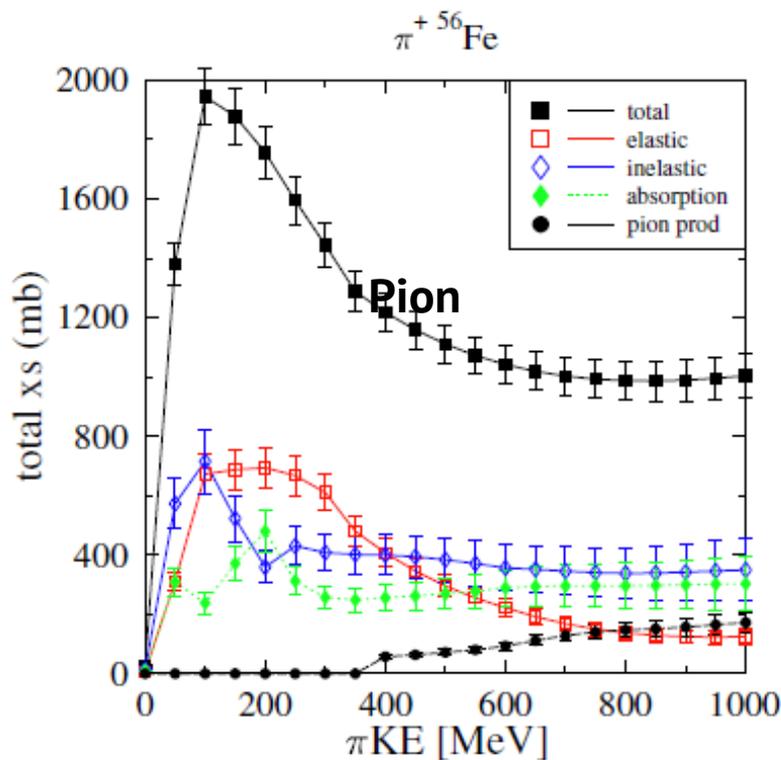


- intranuclear cascade
- data-driven cross sections
- Oset model for pions (coming soon)
- INC-like with one “effective” interaction
- tuned do hadron-nucleus data
- easy to reweight

Figure by T. Golan

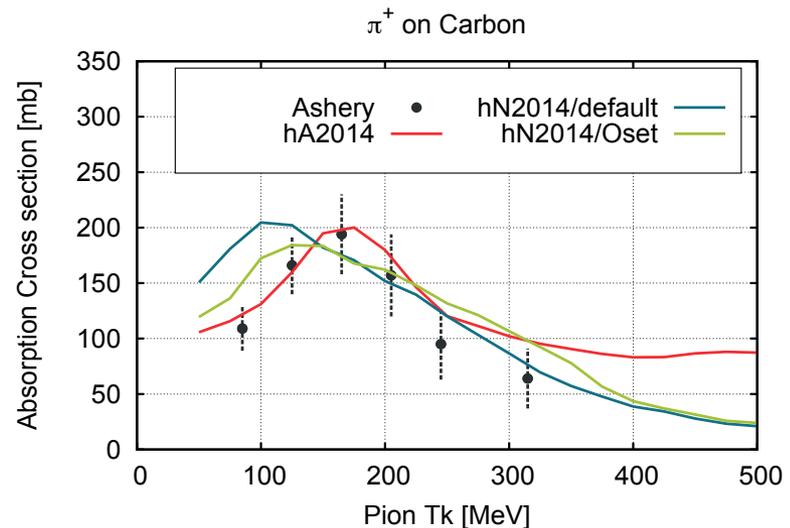
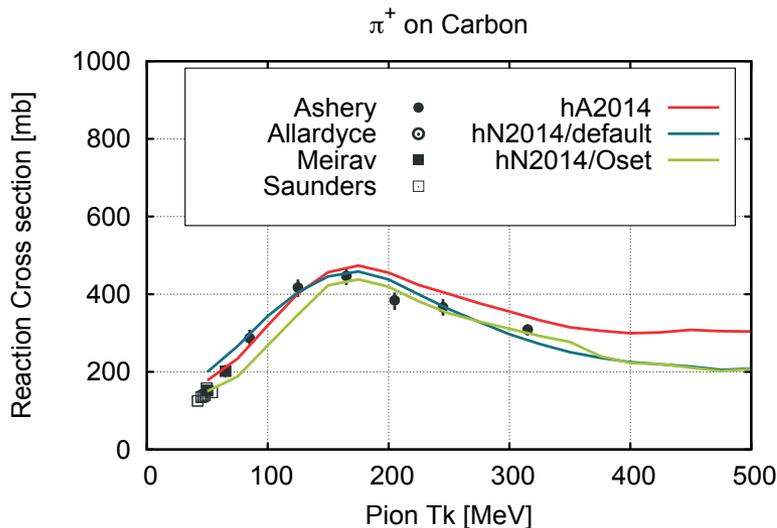
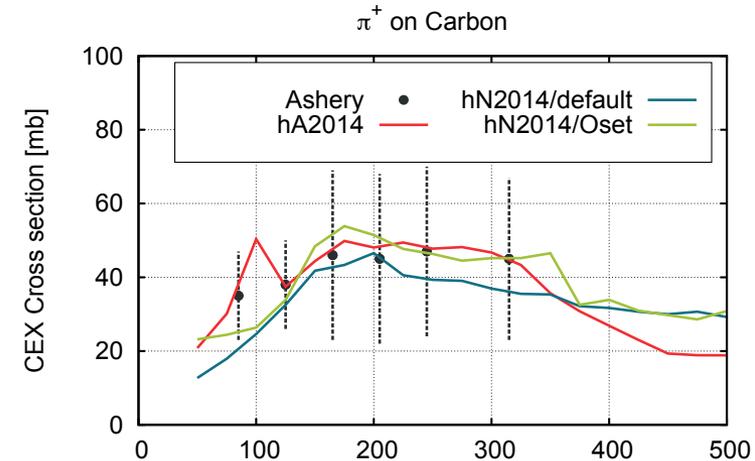
# FSI Models

- GENIE: "hA" (default) - use iron reaction cross section data, isospin symmetry, and  $A^{2/3}$  scaling to predict the FSI reaction rates.
- Individual particle energies and angles use data templates or sample from the allowed phase space.



# FSI Models

- GENIE "hN" is our cascade model.
- New to hN are: Oset et al, Nucl. Phys. A468 (1987), Oset et al, Nucl. Phys. A484 (1998)
- Model describes low energy (kinetic E around Delta peak, 85 MeV - 350 MeV) pion interactions inside nuclear matter.
  - Nuclear effects are implemented as modifications of the Delta width.
- Introduced here as a modification of the GENIE cascade model (hN). Modifications not yet filtered down into the parameterized (hA, default) model.



# GiBUU: References

## ■ Essential References:

1. Buss et al, Phys. Rept. 512 (2012) 1  
contains both the theory and the practical implementation of transport theory
2. Gallmeister et al., Phys.Rev. C94 (2016), 035502  
contains the latest changes in GiBUU2016
3. Mosel, Ann. Rev. Nucl. Part. Sci. 66 (2016) 171  
short review, contains some discussion of generators
4. Mosel et al, arXiv:1702.04932  
pion production comparison of MiniBooNE, T2K and MINERvA

Durham 04/2017



Institut für  
Theoretische Physik



Ulrich Mosel

IPPP/NuSTEC (Durham) 2017



# Conclusions?

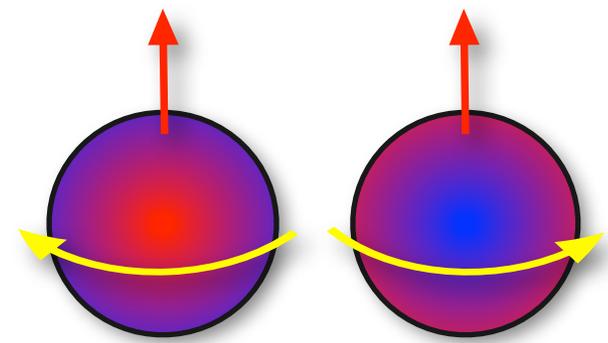
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- This is an large, important topic that we only barely scratched.
  - We need to know a huge amount of nuclear physics to do this properly!
  - Most particle physicists are not also experts in nuclear physics, and, indeed, the very structure of our funding agencies (especially in the US) conspires to push these groups apart.
- We use event generators to help us map observations in the detector back to a probability-weighted distribution of possible initial states.
  - Exactly as is the case with collider physics, no neutrino-nucleus scattering event can be interpreted unambiguously. Our description is fundamentally probabilistic.
- Understanding neutrino-nucleus interactions is fundamental to everything we do in accelerator neutrinos, so it is worth learning more about them.



# Back-up

# Helicity, Chirality, & Parody, oops, Parity!



Left-Helicity Right-Helicity

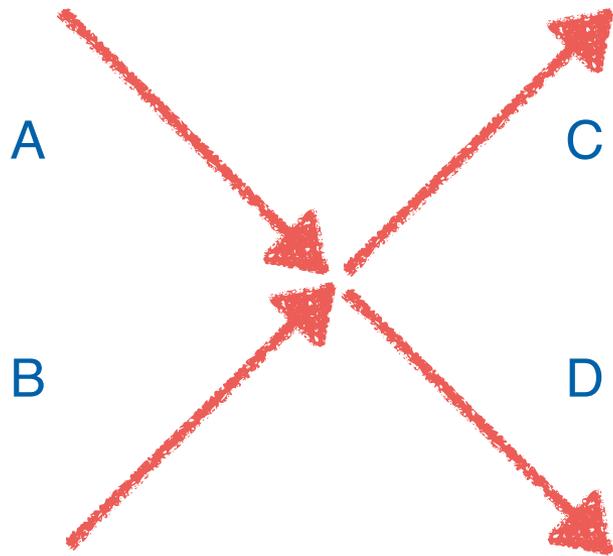
$$\Sigma_{4x4} = \begin{pmatrix} \sigma_{2x2} & 0 \\ 0 & \sigma_{2x2} \end{pmatrix}$$

- *The Weak force is left-handed.*

$$\frac{1}{2} (1 - \gamma^5) \psi \simeq (\Sigma \cdot p) \psi$$

$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L = \alpha \phi_L + \beta \phi_R$$

$$\frac{1}{2} (1 - \gamma^5) \psi = \psi_L$$



$$s = (p_A + p_B)^2$$

$$t = (p_A - p_C)^2$$

$$u = (p_A - p_D)^2$$

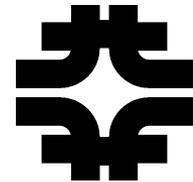
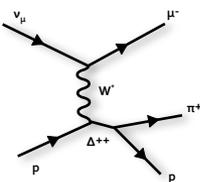
$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

If  $e^-e^+ \rightarrow e^-e^+$  is s-channel scattering:

$$s = 4(k^2 + m^2)$$

$$t = -2k^2(1 - \cos \theta)$$

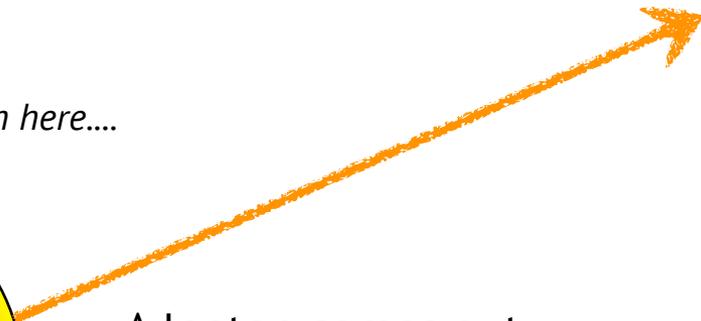
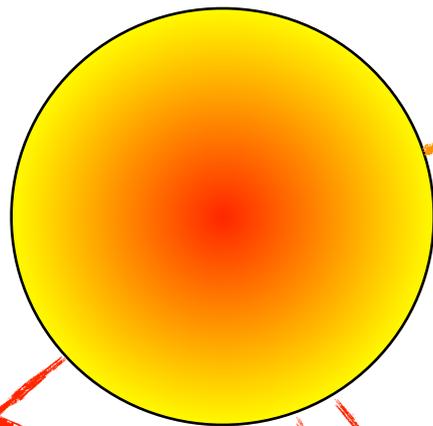
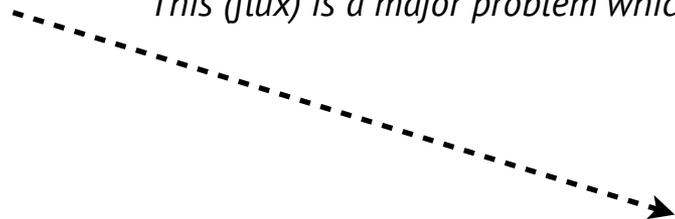
$$u = -2k^2(1 + \cos \theta)$$



# The Basic Problem

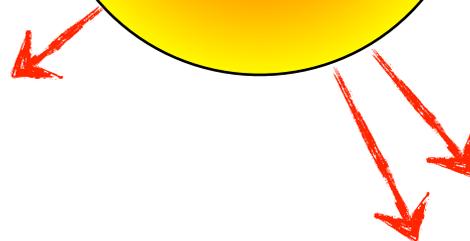
A neutrino comes in (unobserved).

*This (flux) is a major problem which we will not consider much here....*



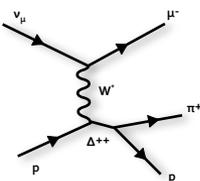
A lepton comes out...

...along with some hadrons (maybe).



*What was the neutrino's energy?*

*We really want flavor too...*



# Why do we need the energy?



$\nu_\alpha$  = Flavor Eigenstates

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$\nu_i$  = Mass Eigenstates

## PMNS matrix...

- 3 x 3 Unitary Matrix
- 3 “Euler Angles”, 1 Complex Phase\*
- 3 Masses
- 2 Independent Splittings

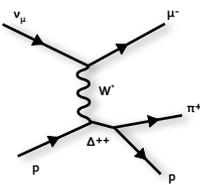
$\theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP}$

—  $m_c$

—  $m_b$

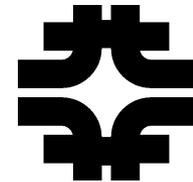
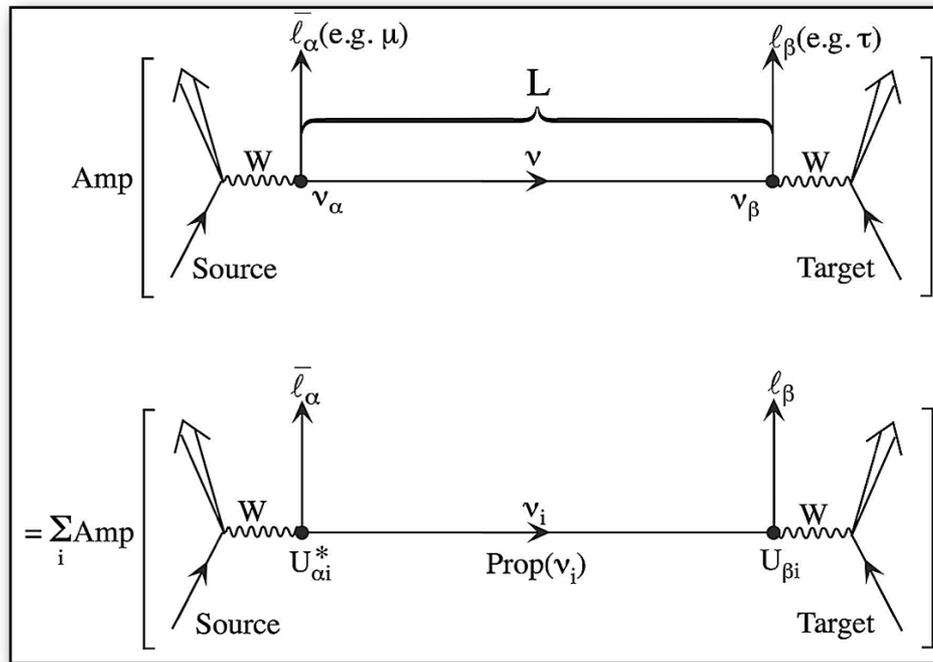
—  $m_a$

\*Plus two Majorana phases - Insanely important!



$$\text{Prop}(\nu_j) \sim e^{(-im_j \tau_j)}$$

$$m_1 \neq m_2 \neq m_3$$

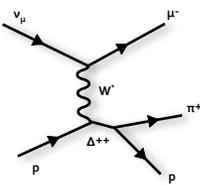


B. Kayser, arXiv  
0804.1121

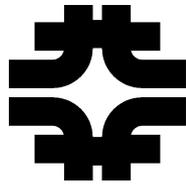
- Flavor eigenstates interact. Flavor states are superpositions of mass states.
  - Different masses  $\Rightarrow$  Different propagators.

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \left| \sum_j U_{\alpha j}^* e^{-im_j^2 \frac{L}{2E}} U_{\beta j} \right|^2$$

- $\Rightarrow$  Flavor composition evolves with time.



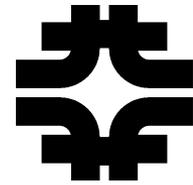
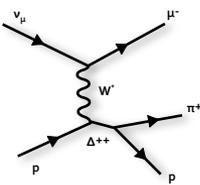
# How do we measure PMNS?



$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= \left| U_{\mu 1}^* e^{-im_1^2 L/2E} U_{e1} + U_{\mu 2}^* e^{-im_2^2 L/2E} U_{e2} + U_{\mu 3}^* e^{-im_3^2 L/2E} U_{e3} \right|^2 \\
 &= \left| 2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21} \right|^2 \\
 &\simeq \left| \sqrt{P_{atm}} e^{-i(\Delta_{32} + \delta)} + \sqrt{P_{sol}} \right|^2
 \end{aligned}$$

- We beat these probabilities against each other.
- $\delta \rightarrow -\delta$  for antineutrinos.
- Compare neutrinos to antineutrinos to measure CP violation and the mass hierarchy.

$$\Delta_{ij} = 1.27 \Delta m_{ij}^2 L/E$$



# Probabilities

**In MATTER:**

$$P_{atm} \sim \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 (\Delta_{31} - aL) \left( \frac{\Delta_{31}}{\Delta_{31} - aL} \right)^2$$

$$P_{sol} \sim \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2 (aL) \left( \frac{\Delta_{21}}{aL} \right)$$

$$a = \pm G_F N_e / \sqrt{2} \sim (4000 \text{ km})^{-1}$$

- The probabilities are a function of the matrix parameters, the mass splittings, and the *neutrino energy*!

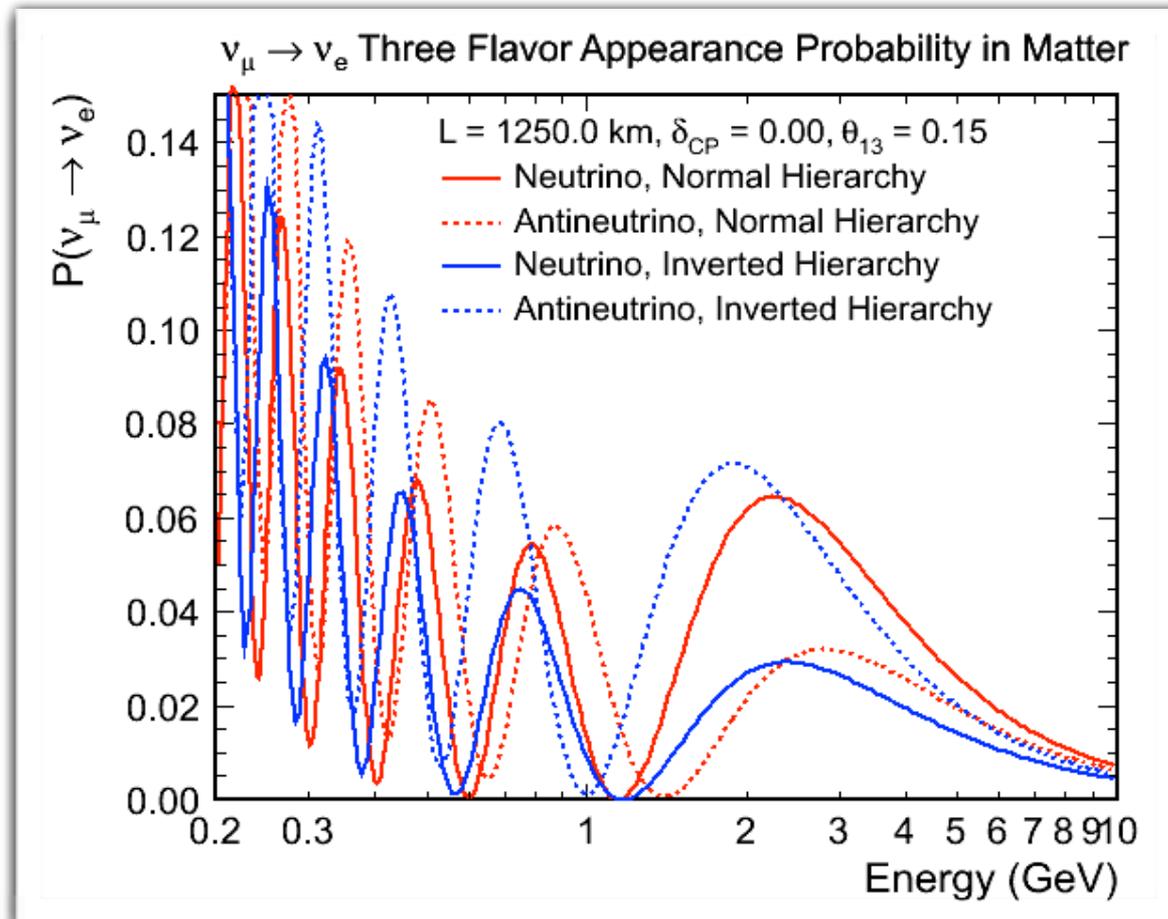
$$\Delta_{ij} = 1.27 \Delta m_{ij}^2 L / E$$

$$P \sim 2\sqrt{P'_{atm}}\sqrt{P'_{sol}} \cos \Delta_{32} \cos \delta_{CP} \mp 2\sqrt{P'_{atm}}\sqrt{P'_{sol}} \sin \Delta_{32} \sin \delta_{CP}$$

$$\delta_{CP} : 0 \rightarrow 2\pi$$

$m_2$    
 $m_1$    
 $m_3$    
 Inverted

Antineutrino:  $-\delta$

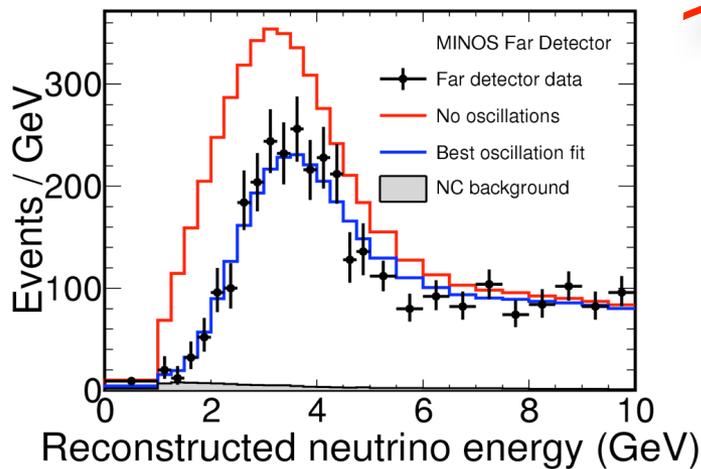


  $m_3$   
  $m_2$   
  $m_1$   
 Normal

Neutrino:  $\delta$

How do we measure these probabilities?

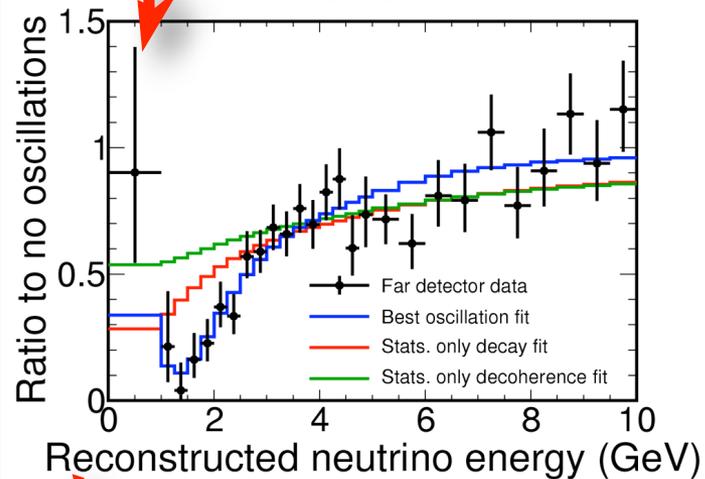
## Measure "Near"/Far



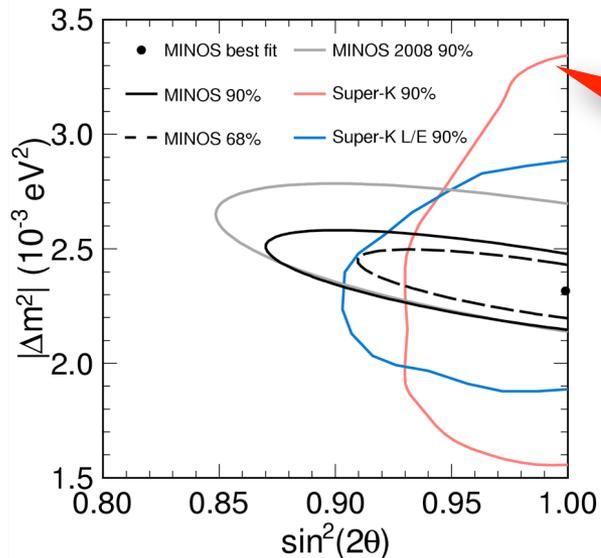
<http://www-numi.fnal.gov/PublicInfo/forScientists.html>

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{32} \sin^2 [1.27\Delta m_{32}^2 (L/E)]$$

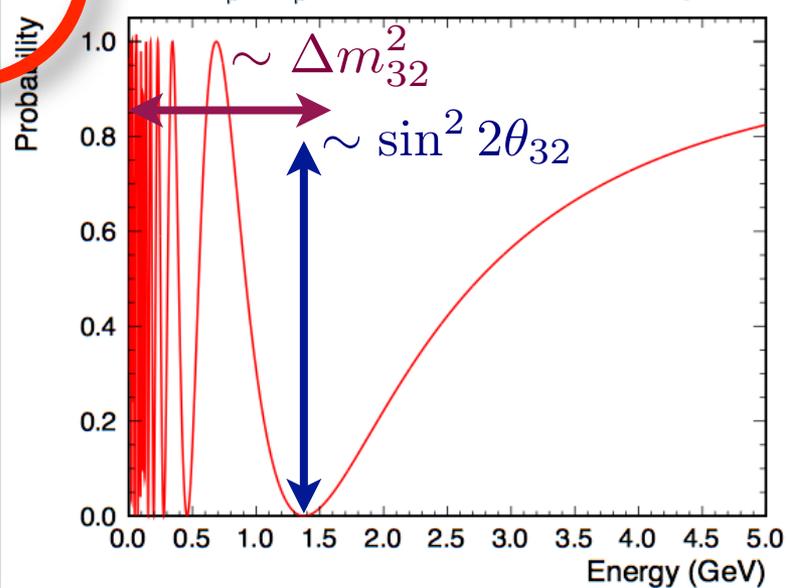
## Fit Ratio

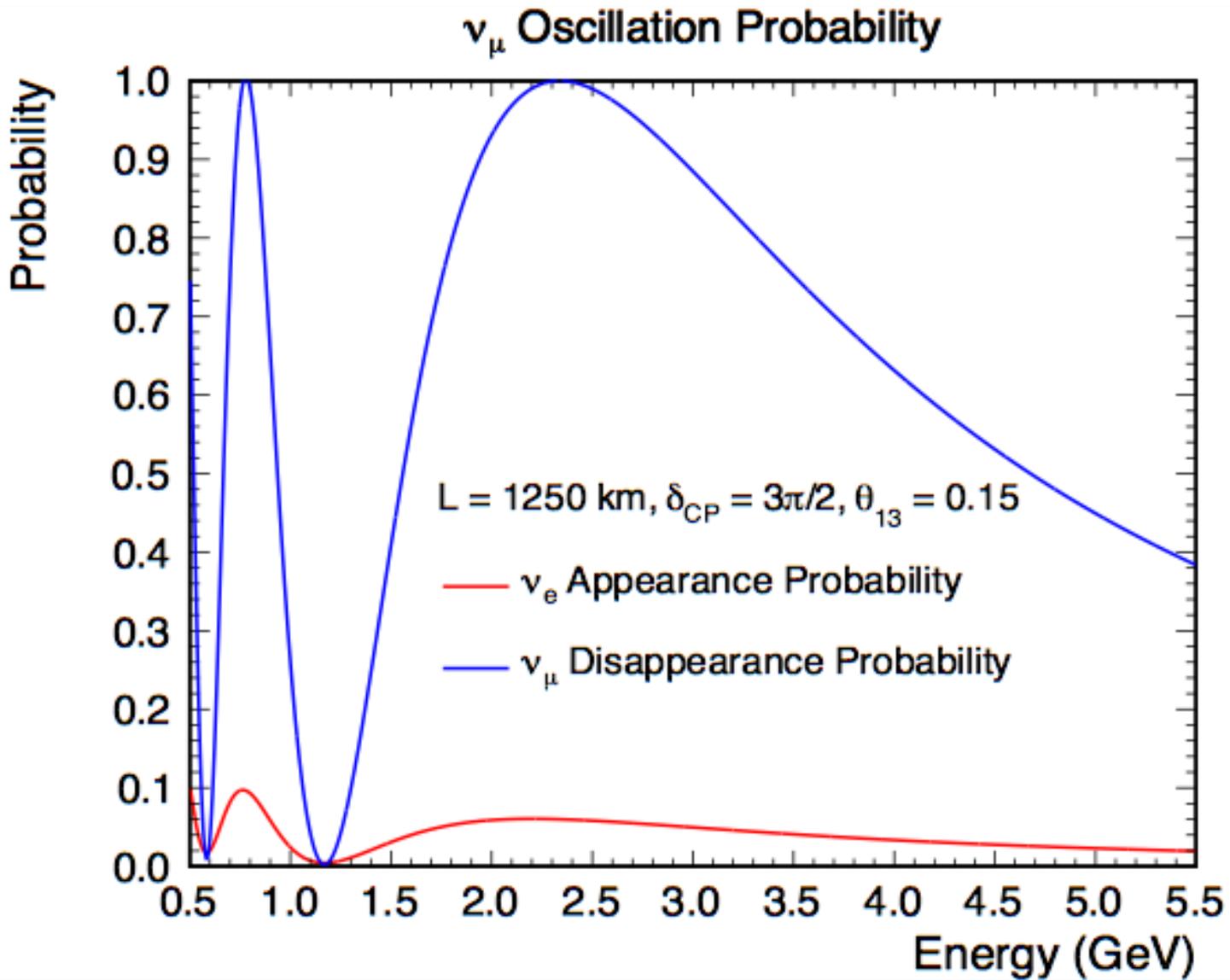


## Extract Physics!

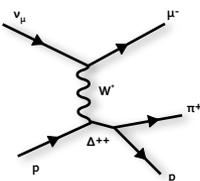


## $\nu_\mu \rightarrow \nu_\mu$ Two Flavor Survival Probability





And remember, we need to do it all over again for antineutrinos!



# Review

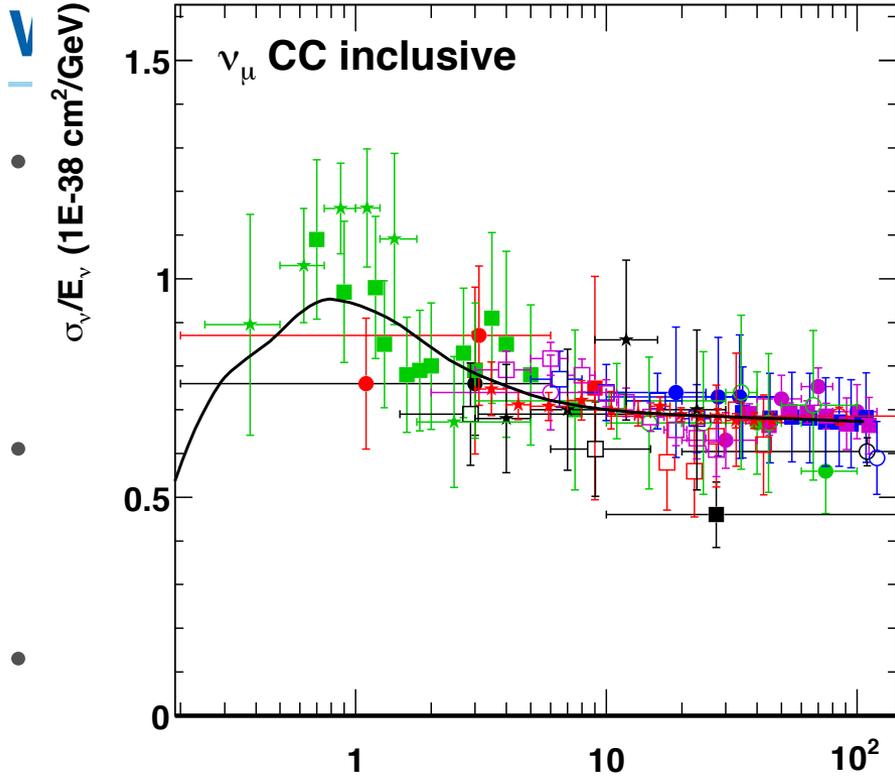


- We need neutrino energy to high precision in our far detector.
- We need neutrino energy in our near detector.
  - These may feature different detector technologies. They *definitely* see different neutrino fluxes.
- We need to understand neutrinos and antineutrinos.
- We're looking for a tiny effect, so "large" systematic uncertainties will destroy the measurement.

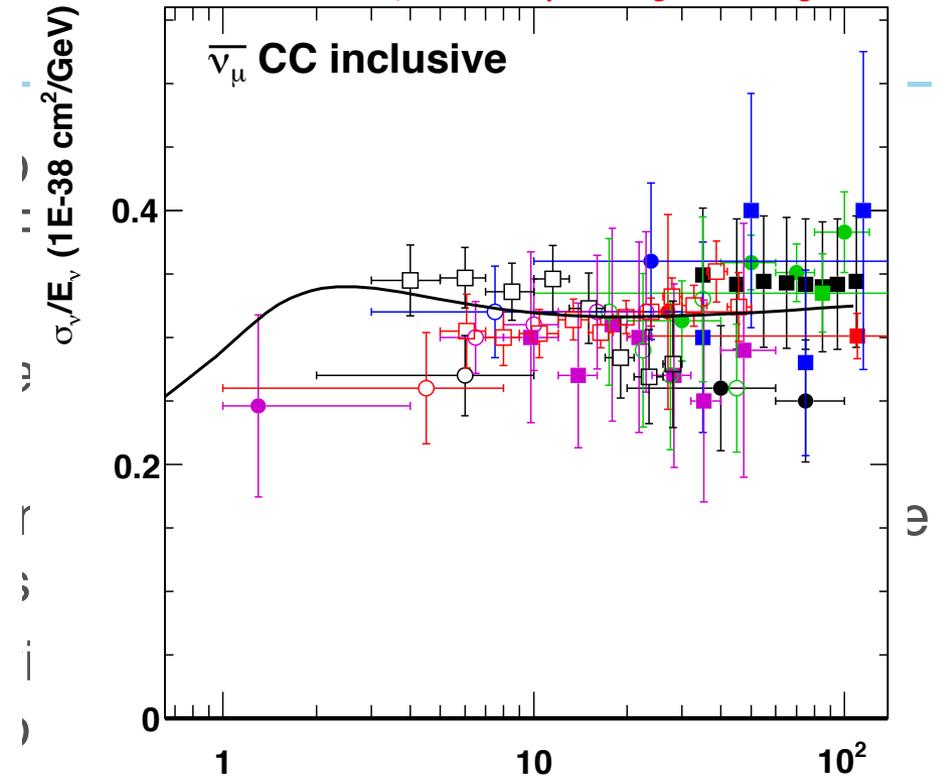
# What is GENIE?

---

- We build a global physics model from a collection of exclusive state models (e.g., Llewellyn Smith QE, Rein-Sehgal resonant pion production, Bodek-Yang DIS, etc.).
  - (Many of these are *wrong but useful*.)
- When we add a new process (e.g., Nieves group MEC), we need to retune the total cross section by controlling the strength of the exclusive processes or subtracting processes.
- We try very hard to be consistent with data for the total cross section, so inclusive cross section calculations are very valuable as an additional constraint.
- We try to agree with a many other measured distributions as possible, but there are always tensions that are difficult to understand/reconcile.



- ANL\_12FT,2 [Barish et al., Phys.Lett.B66:291 (1977)]
- ANL\_12FT,4 [Barish et al., Phys.Rev.D19:2521 (1979)]
- BEBC,0 [Bosetti et al., Phys.Lett.B70:273 (1977)]
- BEBC,2 [Colley et al., Zeit.Phys.C2:187 (1979)]
- BEBC,5 [Bosetti et al., Phys.Lett.B110:167 (1982)]
- BEBC,8 [Parker et al., Nucl.Phys.B232:1 (1984)]
- BNL\_7FT,0 [Baltay et al., Phys.Rev.Lett.44:916 (1980)]
- BNL\_7FT,4 [Baker et al., Phys.Rev.D25:617 (1982)]
- CCFR,2 [Seligman et al., Nevis Report 292 (1996)]
- CCFRR,0 [MacFarlane et al., Zeit.Phys.C26:1 (1984)]
- CHARM,0 [Jonker et al., Phys.Lett.B99:265 (1981)]
- CHARM,4 [Allaby et al., Zeit.Phys.C38:403 (1988)]
- FNAL\_15FT,1 [Kitagaki et al., Phys.Rev.Lett.49:98 (1982)]
- FNAL\_15FT,2 [Baker et al., Phys.Rev.Lett.51:735 (1983)]
- Gargamelle,0 [Eichten et al., Phys.Lett.B46:274 (1973)]
- Gargamelle,10 [Ciampolillo et al., Phys.Lett.B84:281 (1978)]
- Gargamelle,12 [Morfin et al., Phys.Lett.B104:235 (1981)]
- IHEP\_IJEP,0 [Asratyan et al., Phys.Lett.B76:239 (1978)]
- IHEP\_IJEP,2 [Vovenko et al., Sov.J.Nucl.Phys.30:528 (1978)]
- IHEP\_JINR,0 [Anikeev et al., Zeit.Phys.C70:39 (1996)]
- ★ SKAT,0 [Baranov et al., Phys.Rev.B81:255 (1979)]
- ★ MINOS,0 [Adamson et al., Phys.Rev.D81:072002 (2010)]
- ★ SciBooNE,0 [Nakajima et al., Phys.Rev.D83:012005 (2011)]
- G00\_00a



- BEBC,1 [Bosetti et al., Phys.Lett.B70:273 (1977)]
- BEBC,3 [Colley et al., Zeit.Phys.C2:187 (1979)]
- BEBC,6 [Bosetti et al., Phys.Lett.B110:167 (1982)]
- BEBC,7 [Parker et al., Nucl.Phys.B232:1 (1984)]
- BNL\_7FT,1 [Fanourakis et al., Phys.Rev.D21:562 (1980)]
- CCFR,3 [Seligman et al., Nevis Report 292 (1996)]
- CHARM,1 [Jonker et al., Phys.Lett.B99:265 (1981)]
- CHARM,5 [Allaby et al., Zeit.Phys.C38:403 (1988)]
- FNAL\_15FT,4 [Taylor et al., Phys.Rev.Lett.51:739 (1983)]
- FNAL\_15FT,5 [Asratyan et al., Phys.Lett.B137:122 (1984)]
- Gargamelle,1 [Eichten et al., Phys.Lett.B46:274 (1973)]
- Gargamelle,11 [Erriquez et al., Phys.Lett.B80:309 (1979)]
- Gargamelle,13 [Morfin et al., Phys.Lett.B104:235 (1981)]
- IHEP\_IJEP,1 [Asratyan et al., Phys.Lett.B76:239 (1978)]
- IHEP\_IJEP,3 [Vovenko et al., Sov.J.Nucl.Phys.30:528 (1978)]
- IHEP\_JINR,1 [Anikeev et al., Zeit.Phys.C70:39 (1996)]
- MINOS,1 [Adamson et al., Phys.Rev.D81:072002 (2010)]
- G00\_00a

## How does GENIE work?

---

- With the total cross sections in hand, event generation proceeds by projecting rays through the detector geometry and computing the total path length of all the materials along a trajectory.
- At the start of a run, we find the longest path length through the detector and normalize the interaction probability to 1 on that path, scaling the interaction probabilities appropriately, and incorporating this information into the flux driver.
  - Necessary to keep running times reasonable.
- Then for any given path, events are chosen randomly by channel according to their contribution to the total cross section in an accept-reject loop.

# The GENIE default model:

```
<param_set name="Default">
  <param type="int" name="NGenerators"> 13 </param>
  <param type="alg" name="Generator-0"> genie::EventGenerator/QEL-CC </param>
  <param type="alg" name="Generator-1"> genie::EventGenerator/QEL-NC </param>
  <param type="alg" name="Generator-2"> genie::EventGenerator/RES-CC </param>
  <param type="alg" name="Generator-3"> genie::EventGenerator/RES-NC </param>
  <param type="alg" name="Generator-4"> genie::EventGenerator/DIS-CC </param>
  <param type="alg" name="Generator-5"> genie::EventGenerator/DIS-NC </param>
  <param type="alg" name="Generator-6"> genie::EventGenerator/COH-CC </param>
  <param type="alg" name="Generator-7"> genie::EventGenerator/COH-NC </param>
  <param type="alg" name="Generator-8"> genie::EventGenerator/DIS-CC-CHARM </param>
  <param type="alg" name="Generator-9"> genie::EventGenerator/QEL-CC-CHARM </param>
  <param type="alg" name="Generator-10"> genie::EventGenerator/NUE-EL </param>
  <param type="alg" name="Generator-11"> genie::EventGenerator/IMD </param>
  <param type="alg" name="Generator-12"> genie::EventGenerator/IMD-ANH </param>
</param_set>
```

## Interesting additions / alternatives:



```
<param_set name="DFR">
  <param type="int" name="NGenerators"> 2 </param>
  <param type="alg" name="Generator-0"> genie::EventGenerator/DFR-CC </param>
  <param type="alg" name="Generator-1"> genie::EventGenerator/DFR-NC </param>
</param_set>
```

# GENIE Physics Models



- GENIE 2.0 (~2007) used identical physics models as NEUGEN, a Fortran generator that was developed over a number of years by a succession of physicists, and used by MINOS. GENIE has evolved with each subsequent release.
- There are currently dozens of different physics models.
- The default nuclear model is the relativistic Fermi gas with Bodek and Ritchie high-momentum tails. GENIE also implements the Effective Spectral Function, and the Local Fermi Gas. Other spectral function implementations exist in development branches and need a bit more effort to become public.
- The quasielastic process defaults to Llewellyn-Smith, but we also have the Nieves et al model. The axial form factor model is the dipole but we offer (and are preparing to default to) the z-expansion model as well.
- Excitation of nucleon resonances (decaying by meson emission) and coherent pion production are both described by models by Rein and Sehgal, but we offer a number of alternatives (Berger and Sehgal, different form factor models, etc.).
  - We also offer a diffractive pion production model (Rein).
- Models for neutrino-electron scattering and inverse muon decay are included and mostly complete (additional radiative corrections required for neutrino-electron scattering).

# GENIE Physics Models

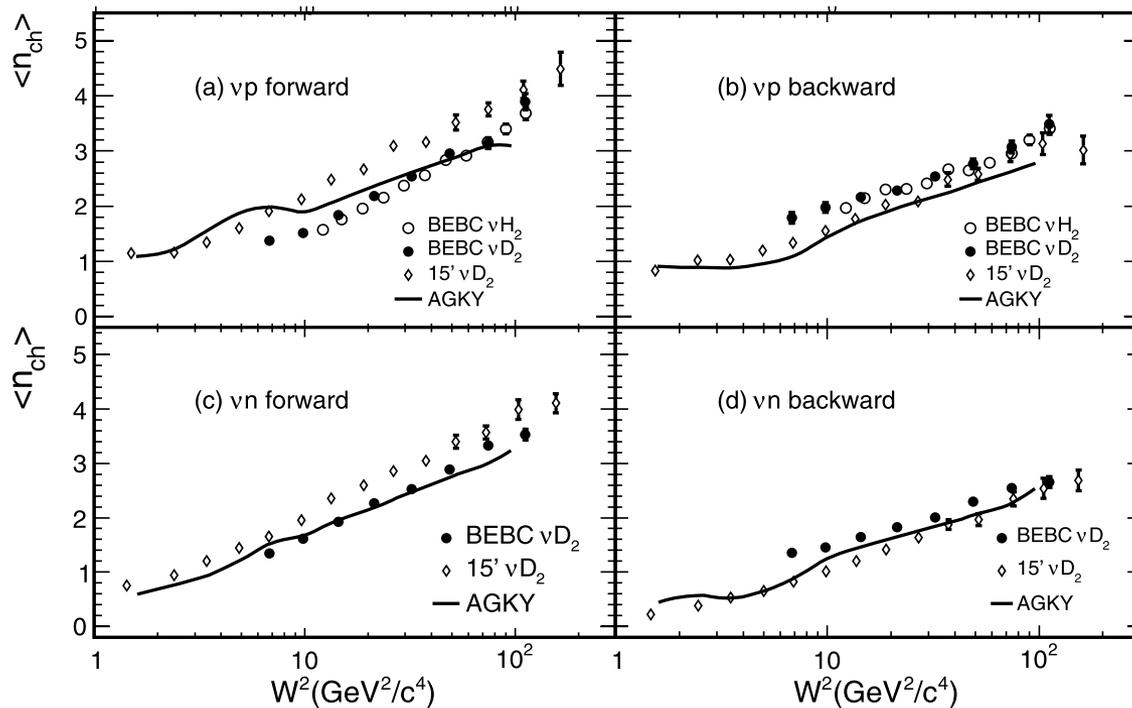
---



- We offer (non-default) a custom built and the Valencia 2p2h models.
- Bodek and Yang (2003) is used for nonresonant inelastic scattering.
- Other interesting exclusive states (QEL hyperon production, single Kaon production, etc.) are optional (making them default would lead to double counting in the hadronization model).
- The custom "AGKY" hadronization model, developed internally, covers the transition between PYTHIA at high ( $W > 3\text{GeV}/c^2$ ) invariant masses and an empirical model based on KNO-scaling at lower invariant masses.
- GENIE has two\* internally developed models for final-state interactions; one is a cascade model and the other (the default) parameterizes the cascade a single effective interaction for easy re-weighting.
  - Actually many more than two - we are snap-shotting major changes with dated timestamps as we make improvements. Users can choose from our long-standing default and the bleeding edge, with a variety of options in between.
- GENIE uses the SKAT parametrization of formation zones (the effective distance over which a quark hadronizes).
- More detail in the back-ups...

# Modeling Nuclear Effects

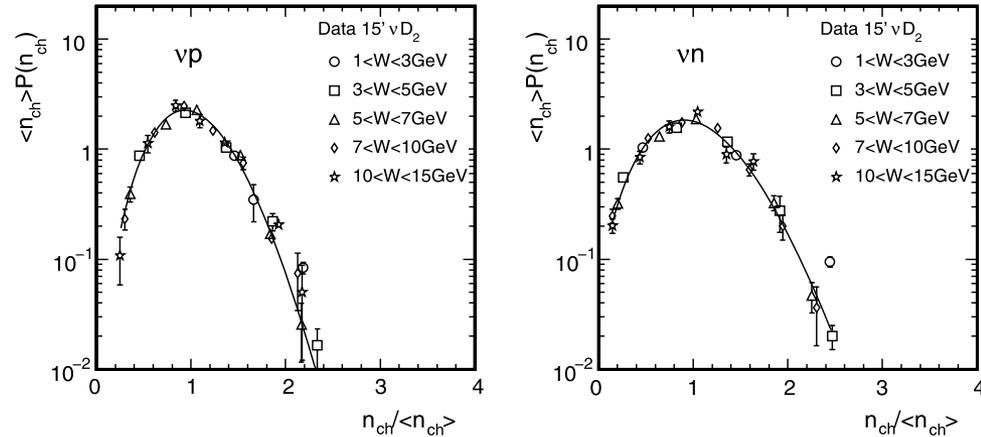
- What about hadronization in the nuclear medium?
- We use Pythia (currently version 6, migration to 8 is on-going).
- GENIE does reasonably well, but the validation uses deuterium or hydrogen - little influence from nuclear effects.



T. Yang et al, Eur. Phys. J C (2009) 63:1-10

# AGKY Hadronization

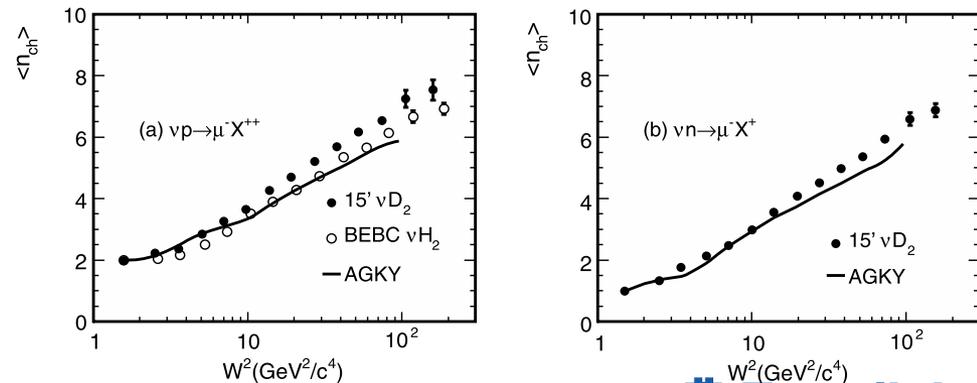
**Fig. 1** KNO scaling distributions for  $\nu p$  (left) and  $\nu n$  interactions. The curve represents a fit to the Levy function. Data points are taken from [7]



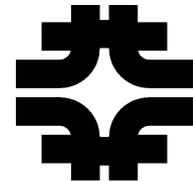
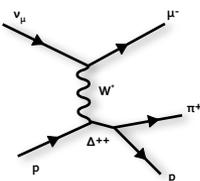
The AGKY model, which is now the default hadronization model in the neutrino Monte Carlo generators NEU-GEN [9] and GENIE-2.0.0 [10], includes a phenomenological description of the low invariant mass region based on Koba–Nielsen–Olesen (KNO) scaling [11], while at higher masses it gradually switches over to the PYTHIA/JETSET model. The transition from the KNO-based model to the PYTHIA/JETSET model takes place gradually, at an intermediate invariant mass region, ensuring the continuity of all simulated observables as a function of the invariant mass. This is accomplished by using a transition window  $[W_{\min}^{\text{tr}}, W_{\max}^{\text{tr}}]$  over which we linearly increase the fraction of neutrino events for which the hadronization is performed by the PYTHIA/JETSET model from 0% at  $W_{\min}^{\text{tr}}$  to 100% at  $W_{\max}^{\text{tr}}$ . The default values used in the AGKY model are

$$W_{\min}^{\text{tr}} = 2.3 \text{ GeV}/c^2, \quad W_{\max}^{\text{tr}} = 3.0 \text{ GeV}/c^2.$$

**Fig. 3** Average charged-hadron multiplicity  $\langle n_{ch} \rangle$  as a function of  $W^2$ . (a)  $\nu p$  events. (b)  $\nu n$  events. Data points are taken from [7, 20]



T. Yang et al, Eur. Phys. J C (2009) 63:1-10



# QE Cross Section

ν Cross Section: 
$$\frac{d\sigma}{dQ^2} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[ A(Q^2) \pm B(Q^2) \frac{s-u}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right]$$

- Early formalism by Llewellyn Smith.
- Vector and Axial-Vector Components.
  - Vector piece can be lifted from (“easier”) electron scattering data.
  - We have to measure the Axial piece.
- $Q^2$  is the 4-momentum transfer ( $-q^2$ ).
- $s$  and  $u$  are Mandelstam variables.
- The lepton vertex is known; the nucleon structure is parameterized with 2 vector ( $F_1, F_2$ ) and 1 axial-vector ( $F_A$ ) form factors.
  - Form factors are  $f(Q^2)$  and encoded in  $A, B,$  and  $C$ .

C. H. Llewellyn Smith, Phys. Rept. 3 261 (1972).

R. Johnson, [http://www.physics.uc.edu/~johnson/Boone/cross\\_sections/free\\_nucleon/quasielastic.pdf](http://www.physics.uc.edu/~johnson/Boone/cross_sections/free_nucleon/quasielastic.pdf)

# Form Factors

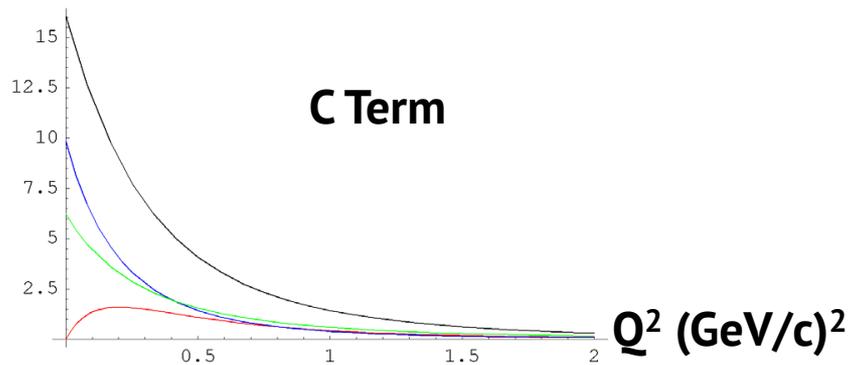
$$A \simeq \frac{t}{M^2} \left( |f_{1V}|^2 - |f_A|^2 \right) + \frac{t^2}{4M^2} \left( |f_{1V}|^2 + \xi^2 |f_{2V}|^2 + |f_A|^2 + 4\xi \operatorname{Re}(f_{1V} f_{2V}^*) \right) + \frac{t^3 \xi^2}{16M^6} |f_{2V}|^2$$
$$B \simeq \frac{1}{M^2} \left( \operatorname{Re}(f_{1V} f_A^*) + \xi \operatorname{Re}(f_{2V} f_A^*) \right) t$$
$$C = \frac{1}{4} \left( |f_{1V}|^2 + |f_A|^2 - \frac{\xi^2 |f_{2V}|^2}{4M^2} t \right)$$

$$f_A(q^2) = \frac{f_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

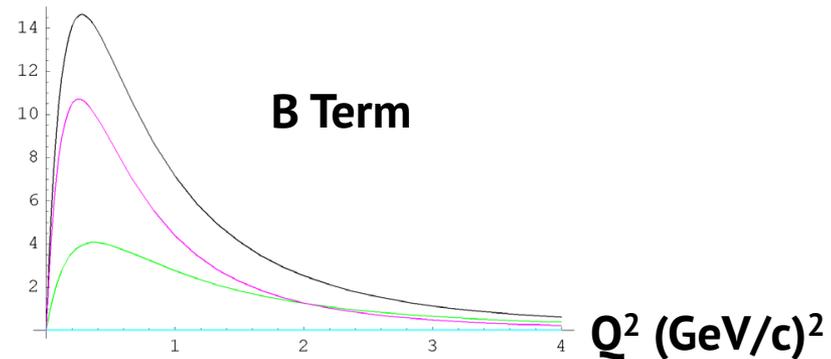
$f_A$  is the axial-vector form factor. We must measure this in  $\nu$ -scattering. Typically, we assume a dipole form (not required!)\*.

The **form factors** ( $f$ ) contain parameterized information about the target (general shape of the form factors comes from symmetry arguments).

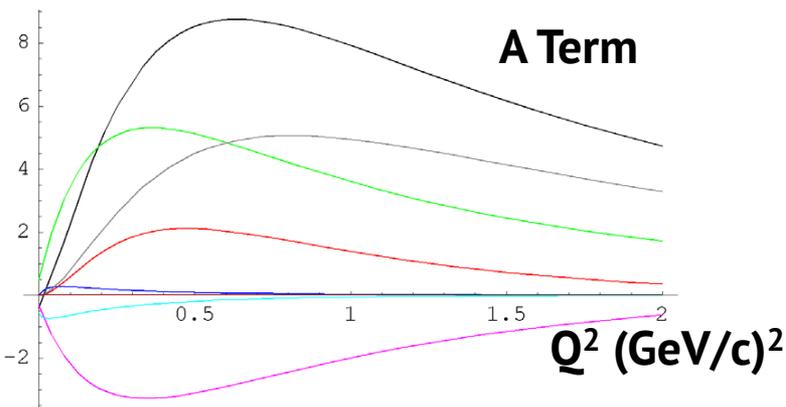
Not calculable from first principles, instead we measure them experimentally.



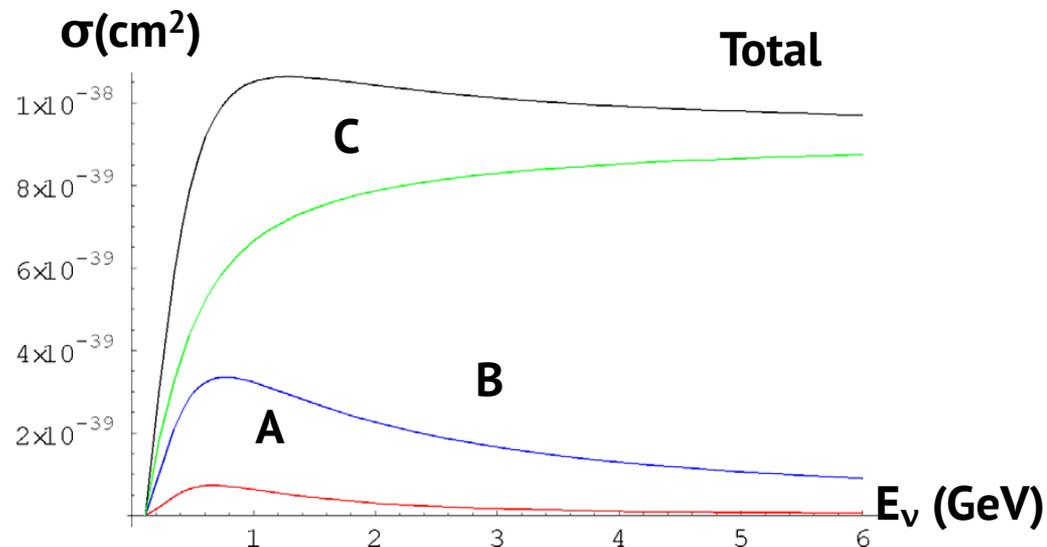
**Figure 1.** Sum of all terms in C is black. The contribution from the  $|f_{1V}|^2$  is in red, the  $|f_{2V}|^2$  in blue and the  $|f_A|^2$  term is in green.



**Figure 2.** “B” as a function of  $Q^2$ . Sum of all terms is black. The  $\text{Re}(f_{1V}f_A^*)$  term is magenta and the  $\text{Re}(f_{2V}f_A^*)$  term is green. All other terms are small and plotted along the x axis.



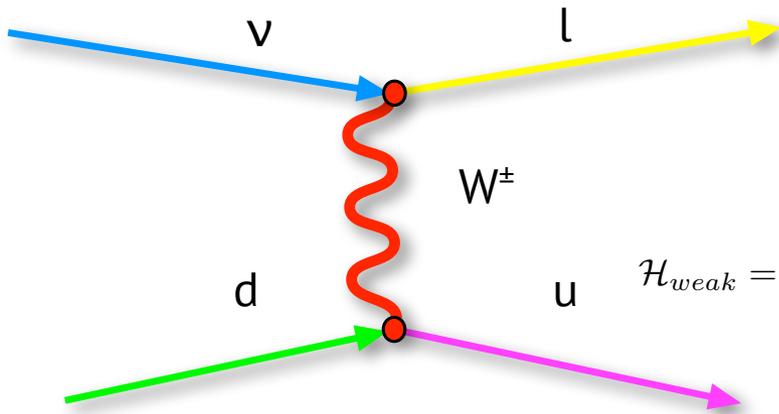
**Figure 3.** The “A” term. Sum of all terms is black. The term proportional to  $|f_{1V}|^2$  is blue, the term with  $|f_{2V}|^2$  is red, the term with  $|f_A|^2$  is green, the term with  $|f_P|^2$  is magenta, the term with  $\text{Re}(f_{1V}f_{2V}^*)$  is light blue, the term with  $\text{Re}(f_{1V}f_A^*)$  is yellow (almost on the x axis), the term with  $\text{Re}(f_{2V}f_A^*)$  is gray, and the term with  $\text{Re}(f_A f_P^*)$  is brown (again, almost on the x axis).



**Figure 4.** Total neutrino neutron quasielastic cross section (black) and the contributions to the cross section from the “C” term (green), the “B” term (blue) and the “A” term (red).

# Form Factors

## “Intuition” for the axial form factor & $M_A$ ...



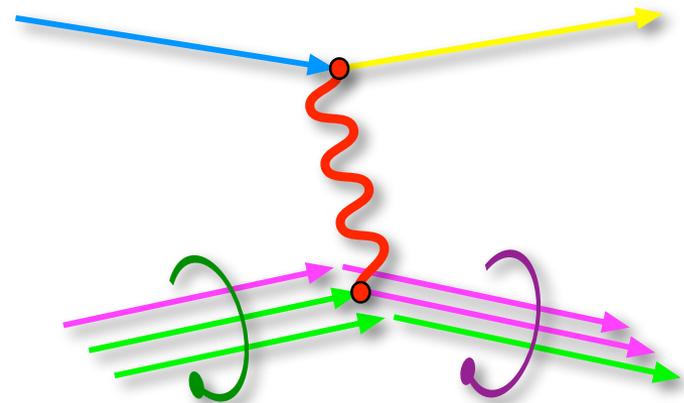
We know how to handle scattering for Dirac particles:

$$\mathcal{H}_{weak} = \frac{4 G_F}{\sqrt{2}} \left[ \bar{l}/\bar{\nu} \gamma_\mu \frac{1-\gamma_5}{2} \nu \right] \left[ \bar{f}' \gamma_\mu \left( g_L \frac{1-\gamma_5}{2} + g_R \frac{1+\gamma_5}{2} \right) f \right] + h.c.$$

Real protons are more complicated!

Form Factor : Fourier Transform of the Charge Distribution

$$\rho(r) = \rho_0 e^{-mr}$$



# Form Factor :

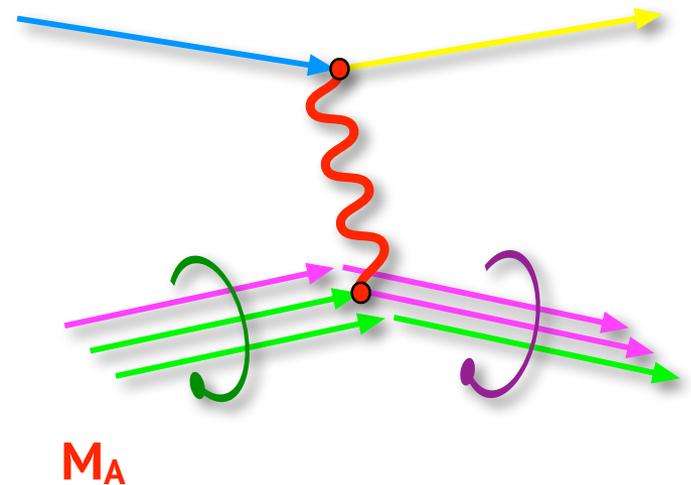
## Fourier Transform of the Charge Distribution

$$\begin{aligned}
 F(|q|^2) &= N \int e^{-mr} e^{i\vec{q}\cdot\vec{x}} d^3x & \rho(r) &= \rho_0 e^{-mr} \\
 &= 2\pi N \int r^2 e^{-mr} e^{i|q|r \cos\theta} dr d(\cos\theta) \\
 &= \frac{2\pi N}{i|q|} \int_0^\infty r \left[ e^{-(m-i|q|)r} - e^{-(m+i|q|)r} \right] dr \\
 &= \frac{8\pi N}{m^3 \left(1 + \frac{|q|^2}{m^2}\right)^2}
 \end{aligned}$$

Normalization:

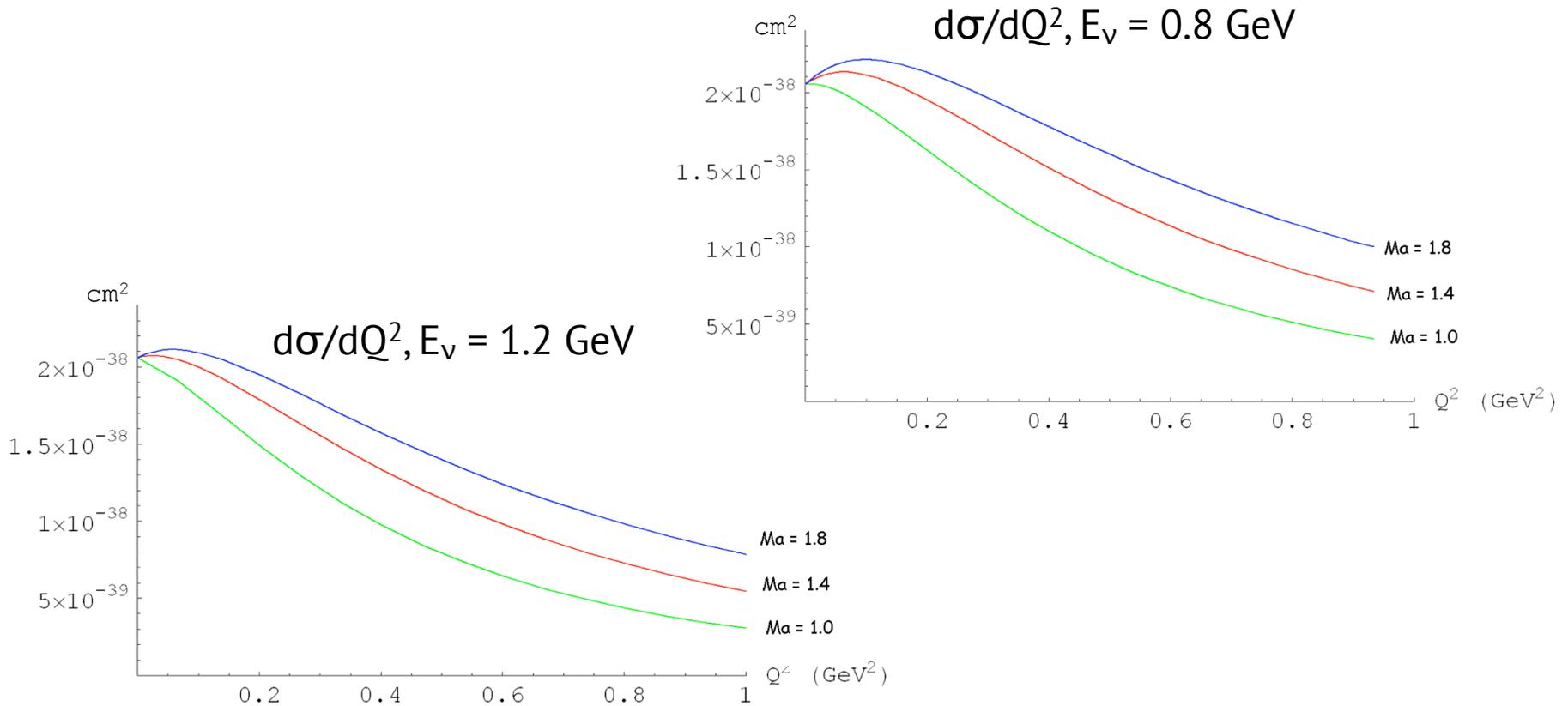
$$N \int e^{-mr} d^3x = 1 \Rightarrow N = m^3/8\pi$$

$$\Rightarrow F(q^2) = \frac{1}{\left(1 - \frac{q^2}{m^2}\right)^2}$$



$Q^2$  dependence  $\iff$  Finite nucleon size.

# The Effect of $M_A$

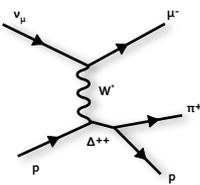


[http://www.physics.uc.edu/~johnson/Boone/cross\\_sections/free\\_nucleon/Varying\\_MA\\_plots.html](http://www.physics.uc.edu/~johnson/Boone/cross_sections/free_nucleon/Varying_MA_plots.html)

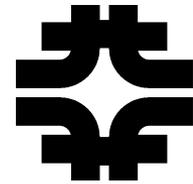
# The Effect of $M_A$



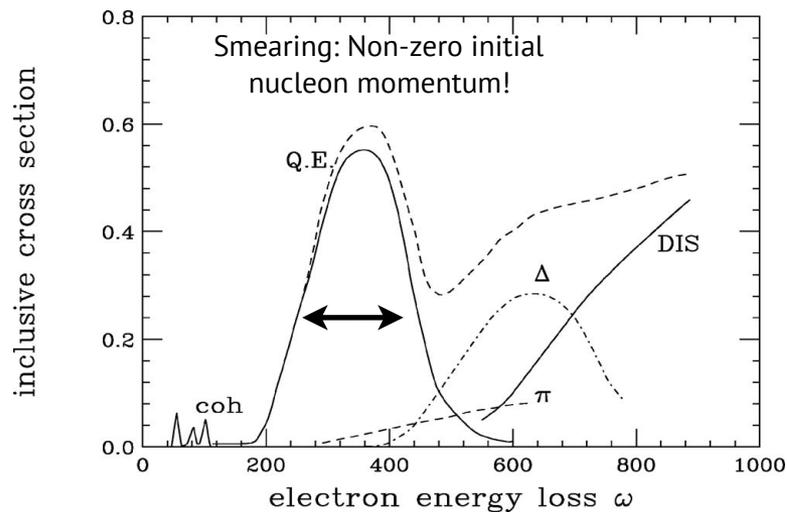
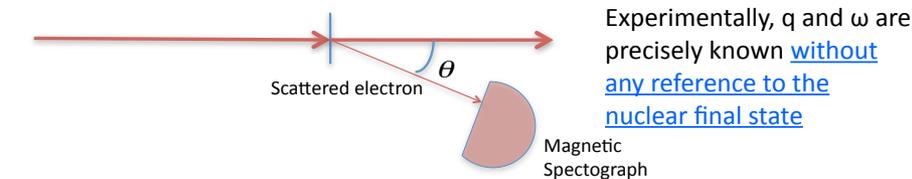
[http://www.physics.uc.edu/~johnson/Boone/cross\\_sections/free\\_nucleon/Varying\\_MA\\_plots.html](http://www.physics.uc.edu/~johnson/Boone/cross_sections/free_nucleon/Varying_MA_plots.html)



# Vector Form Factors

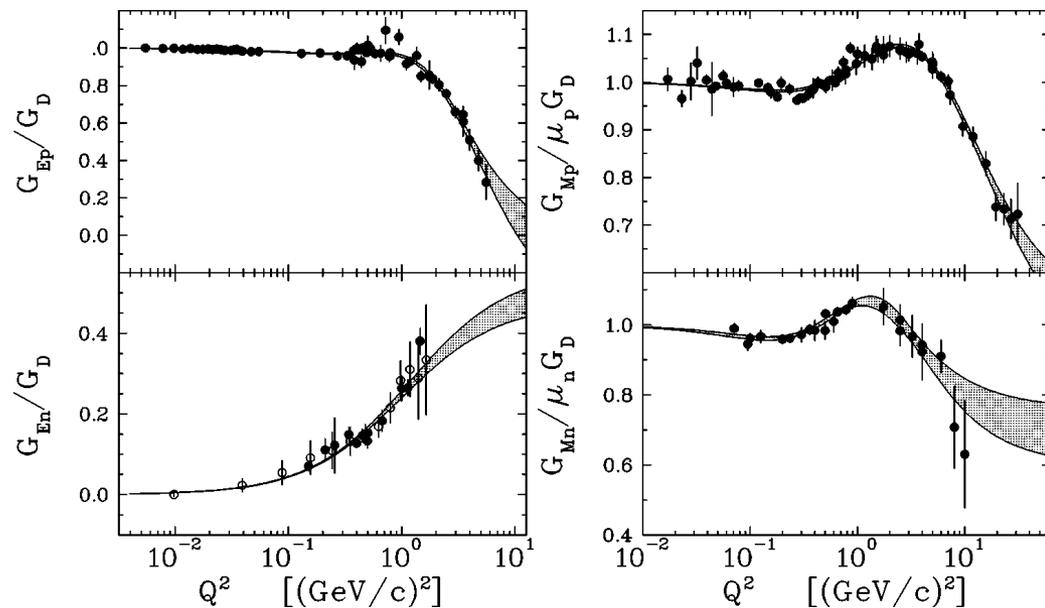


- $f_{1V}$  &  $f_{2V}$  come from high precision electron scattering experiments.
- Notice the small error bars...



J. Carlson, FNAL Short-Baseline Neutrino Workshop, 2011

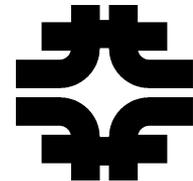
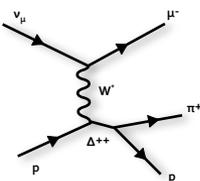
## Nucleon Electromagnetic Form Factors (presented as a ratio to a dipole form factor)



J.J. Kelly, PRC 70, 068202 (2004)

$$G_D = (1 + Q^2/\Lambda^2)^{-2}$$

$$\Lambda^2 = 0.71 \text{ (GeV/c)}^2$$



# Llewellyn Smith & CCQE Cross Sections

- Standard Application:

- Assume a Fermi Gas Model with parameters from electron scattering (or a favorite nuclear model).

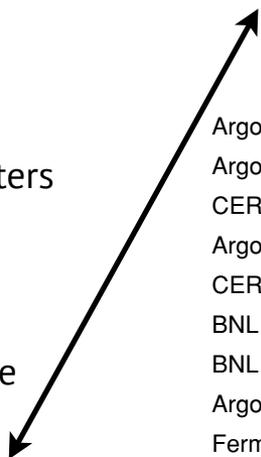
- Typically (FGM) assume the Impulse Approximation.

- Vector form factors from electron scattering.

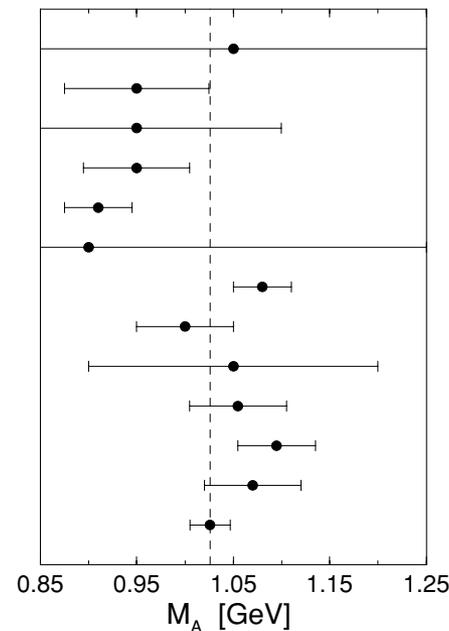
- Assume dipole form for Axial-vector form factor. Everything now follows from  $M_A$ . Measure the x-section, get  $M_A$ .

- $F_A(0)$  is measured in beta-decay.

$$F_A(Q^2) = \frac{-g_A}{(1 + Q^2/M_A^2)^2}$$



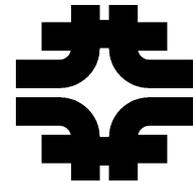
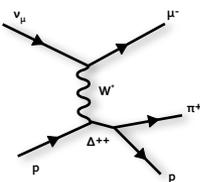
- Argonne (1969)
- Argonne (1973)
- CERN (1977)
- Argonne (1977)
- CERN (1979)
- BNL (1980)
- BNL (1981)
- Argonne (1982)
- Fermilab (1983)
- BNL (1986)
- BNL (1987)
- BNL (1990)
- Average



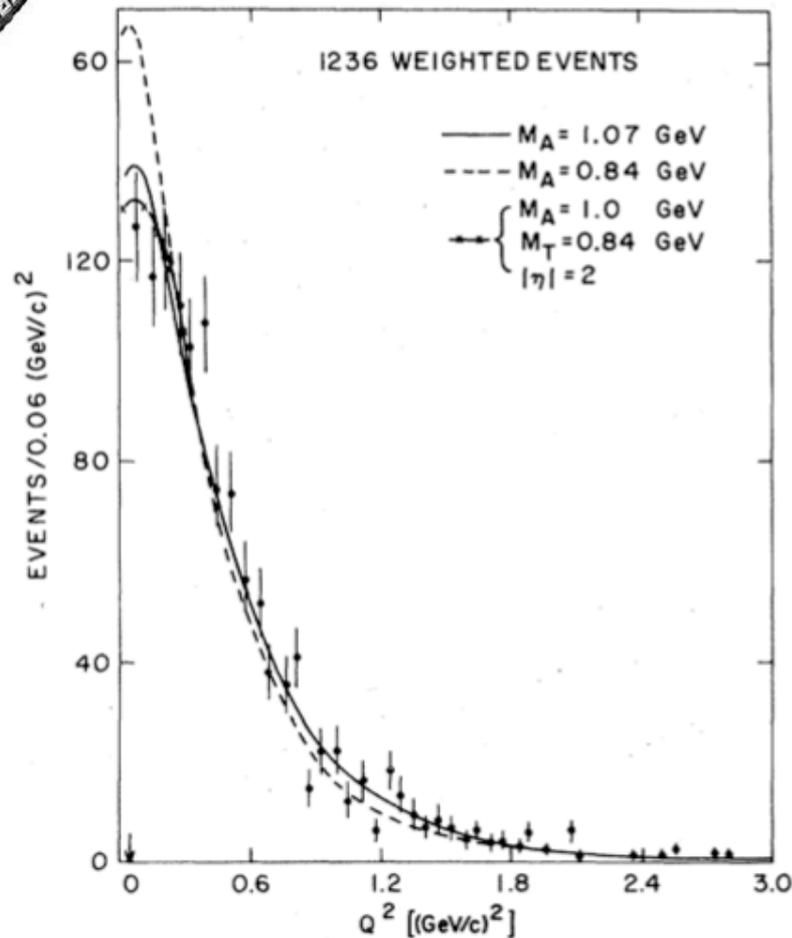
Bernard et al 2002 J. Phys. G: Nucl. Part. Phys. 28 R1

Relativistic Fermi Gas: Smith, Moniz, NPB 43, 605 (1972)

Llewellyn Smith, C.H., 1972, Phys. Rep. C3, 261.



- Aside...
- In the bad old days:
  - Fit CCQE  $d\sigma/dQ^2$  for best Axial Mass parameter.
    - You only need the shape, not the level, to get  $M_A$ .
  - Use Llewellyn-Smith to calculate the cross section.
  - Use the cross-section to calculate the flux.
  - Use the flux to measure the cross-section!



## z-Expansion of the axial form factor

---

- Model independent determination of axial mass parameter, PRD 84 (2011) 073006
  - No need to assume a dipole shape.
- Change of variable from  $q^2$  to  $z$  for actual expansion parameter.
- Current (configurable via xml) parameters derived from fits to deuterium bubble chamber data, in Meyer, Betancourt, Gran, Hill arXiv 1603.03048
- Also includes new re-weighting routines to re-weight from the dipole model to the  $z$ -expansion of the axial form factor.

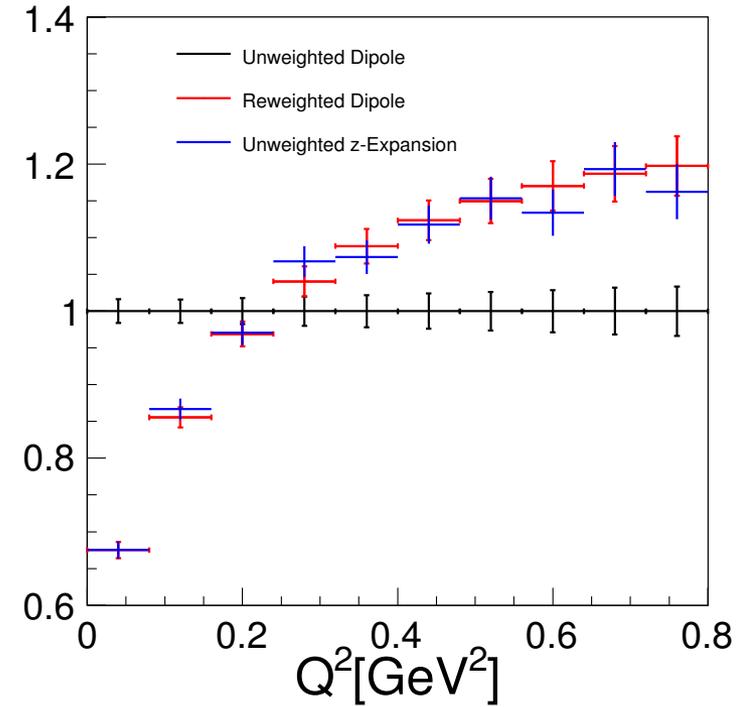
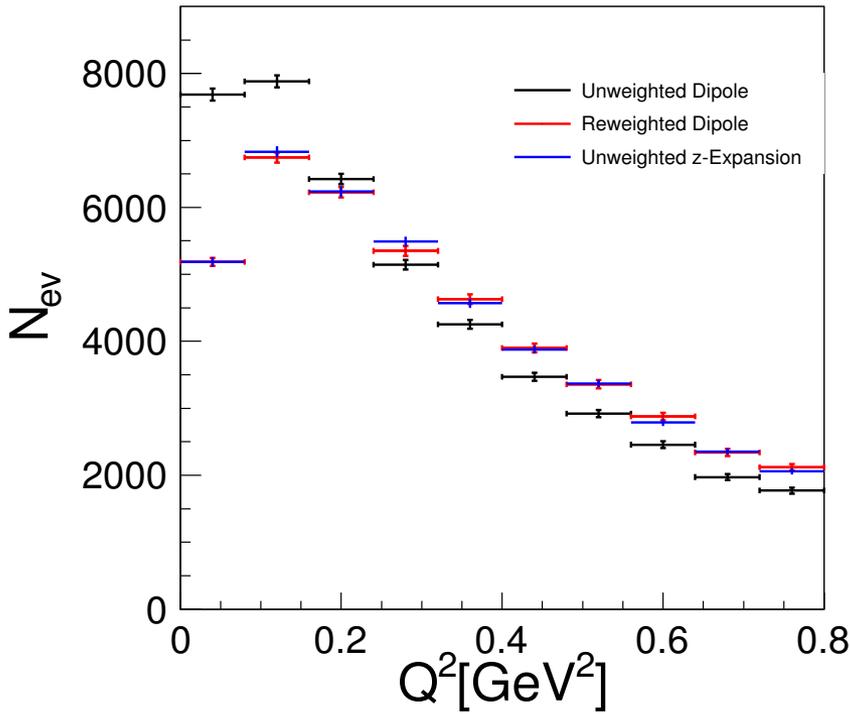


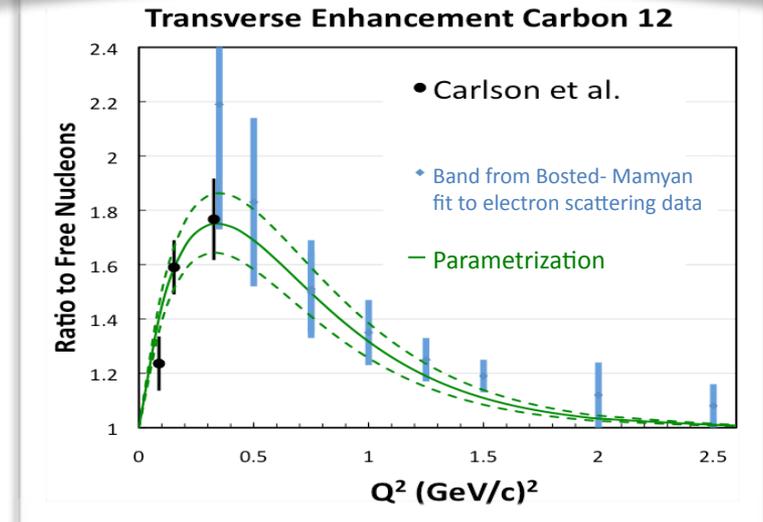
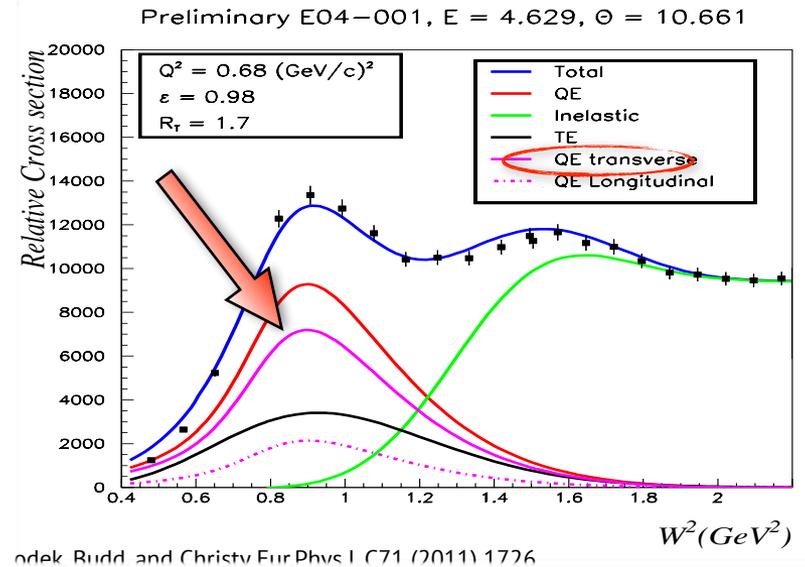
Figure 2: A nominal dipole event sample which has been reweighted to a z-expansion sample. The dipole Monte Carlo sample is represented in black, with statistical error bars. The reweighted dipole sample is shown in red, and the independent sample with  $z$  expansion values is shown in blue. The left plot shows the raw number of events in each bin for a 50k event sample of pure CCQE, and the right plot shows the events normalized by the nominal sample. The agreement between red and blue is a validation of the reweighting procedure. The study was done using a carbon target at 1 GeV.

# Transverse Enhancement

Bodek, Budd, and Christy Eur.Phys.J. C71 (2011) 1726

- The sort of model experimenters love - it may or may not be right, but it matches data (MiniBooNE - NOMAD).
- Separate the cross section into "longitudinal" and "transverse" components (polarization of the virtual photon) in electron scattering.
- Modify only vector magnetic form factors with  $e^-$  scattering data - everything else is single free nucleon.
- $e^-$  scattering data suggests only the longitudinal portion of the QE x-section is  $\sim$ universal free nucleon response function - the transverse component shows an enhancement relative to this approach.

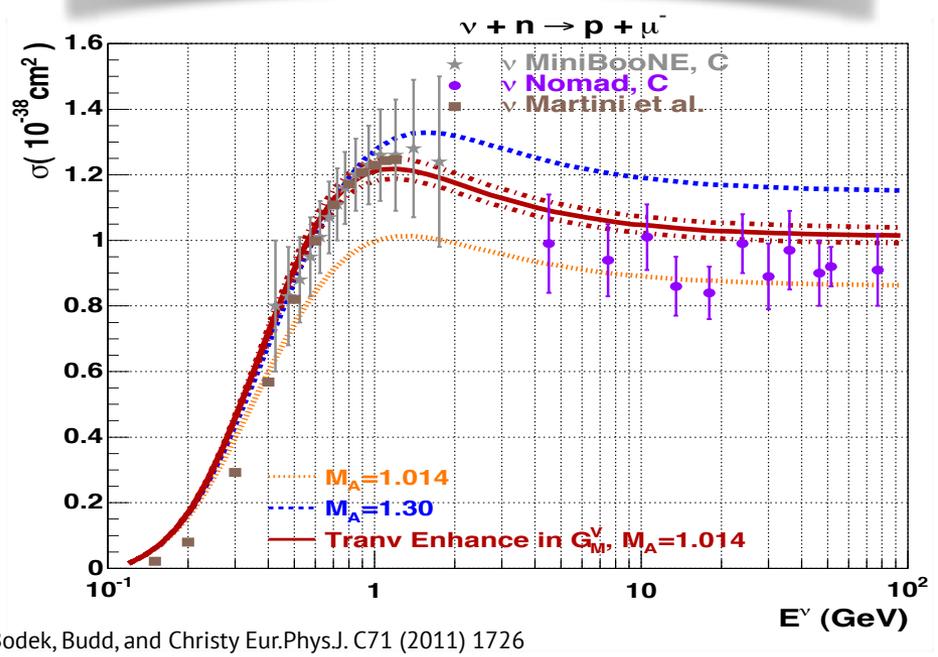
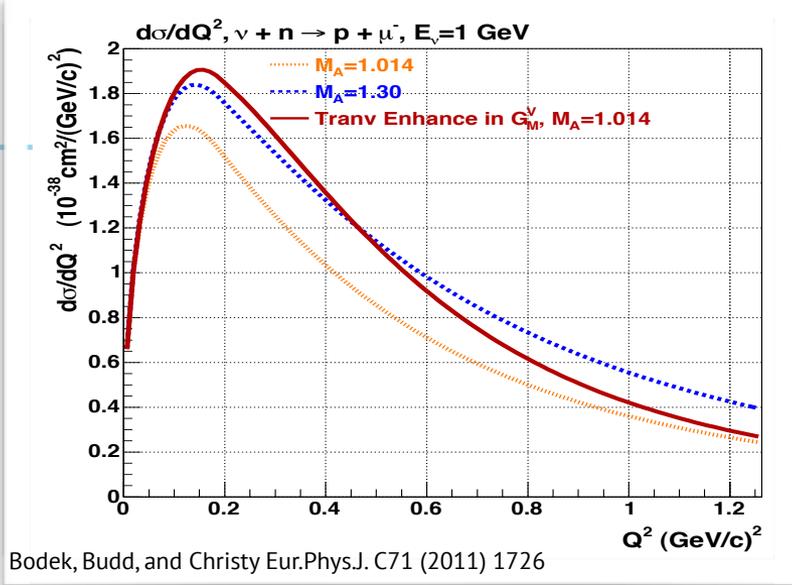
$$\frac{d^2\sigma}{d\Omega d\omega} = \Gamma [R_T(q, \omega) + \epsilon \cdot R_L(q, \omega)]$$



Fit to electron scattering data from JUPITER (JLab E04-001) to extract enhancement as a function of  $Q^2$ .

# Transverse Enhancement

- $d\sigma/dQ^2$  w/  $M_A = 1.014$  GeV & TEM is very similar to the result for  $M_A = 1.3$  GeV for  $Q^2 < 0.6$  (GeV/c)<sup>2</sup>.
- For high  $Q^2$ , the TEM contribution is small.
- Experiments at high energy often remove low  $Q^2$  values from their  $M_A$  fits - predict an even lower  $M_A$  due to steep slope for  $d\sigma/dQ^2$  at  $M_A = 1.014$  GeV.



# Nuclear Effects in *Electron Scattering*

## EMC Effect and Quark Distributions in Nuclei

Measurements of  $F_2^A / F_2^D$  (EMC, SLAC, BCDMS,...) have shown definitively that quark distributions are modified in nuclei.

*Nucleus is not simply an incoherent sum of protons and neutrons*

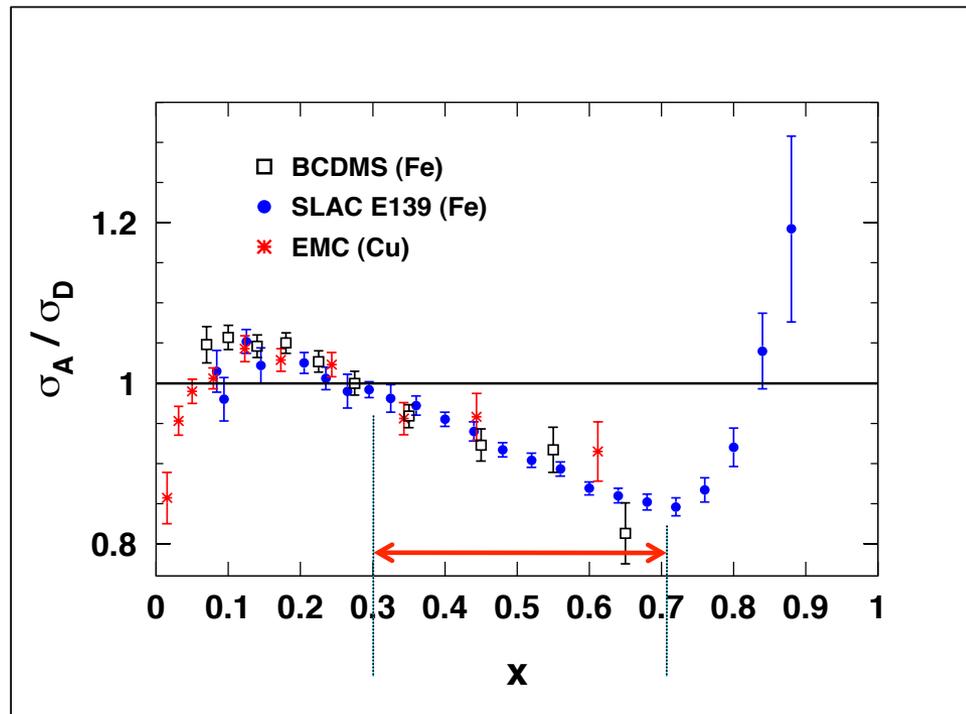
Observed properties:

1.  $x$ -dependence same for all  $A$

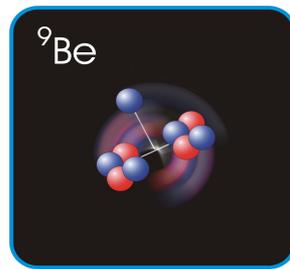
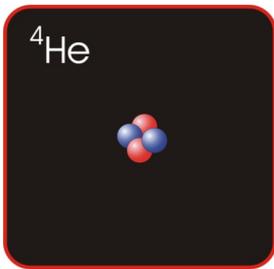
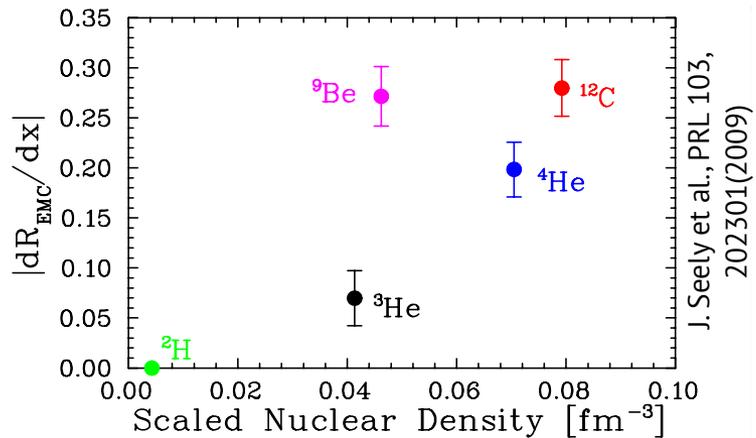
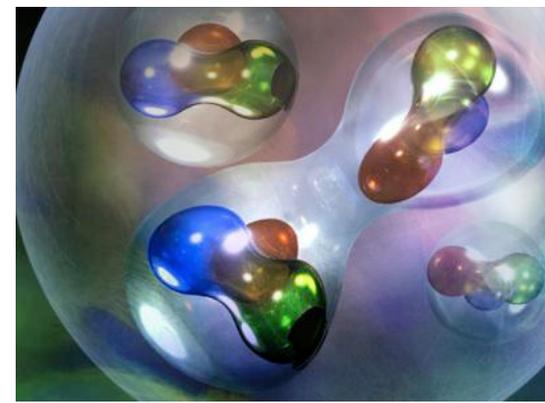
Shadowing:  $x < 0.1$   
Anti-shadowing:  $0.1 < x < 0.3$   
EMC effect:  $x > 0.3$

2. Size of EMC effect depends on  $A$  (i.e. minimum at  $x=0.7$ )

D. Gaskell, ECT 2012, Trento  
Hadrons in the Nuclear Medium

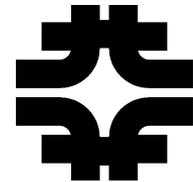
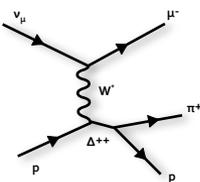


# Short-Range Correlations and the EMC Effect



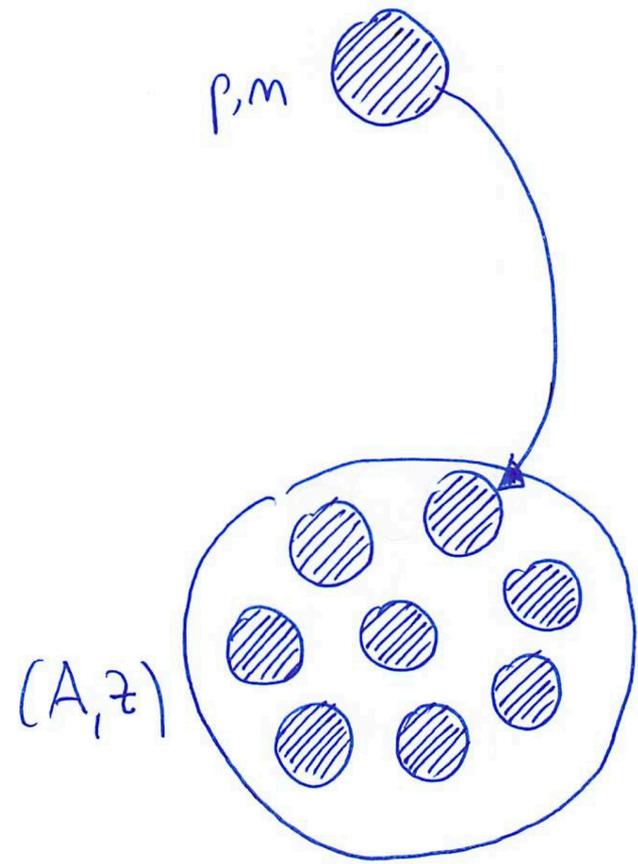
D. Gaskell, ECT 2012, Trento  
Hadrons in the Nuclear Medium

- <sup>9</sup>Be has a low average density - structure  $\sim 2\alpha + n$ .
- Most nucleons are tightly-grouped ( $\alpha$ -like).
- EMC effect modulated by local instead of average density?
- Is there a relation to MEC in neutrino scattering?



**DESCRIPTION  
OF NUCLEON**

**DESCRIPTION  
OF NUCLEUS**



**STRUCTURE FUNCTIONS**

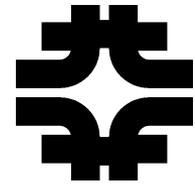
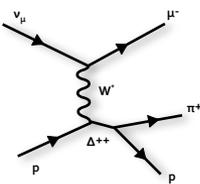
$$F_1(x, Q^2), F_2(x, Q^2), xF_3(x, Q^2), \dots$$

$$\delta f(x)$$

**SPECTRAL FUNCTION**

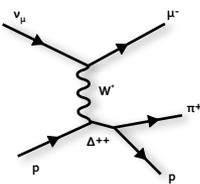
$$\mathcal{P}(\varepsilon, \mathbf{p})$$

R. Petti, ECT Trento, 2012

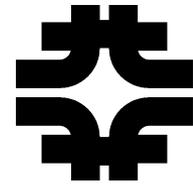


# Heavy-Target Scattering

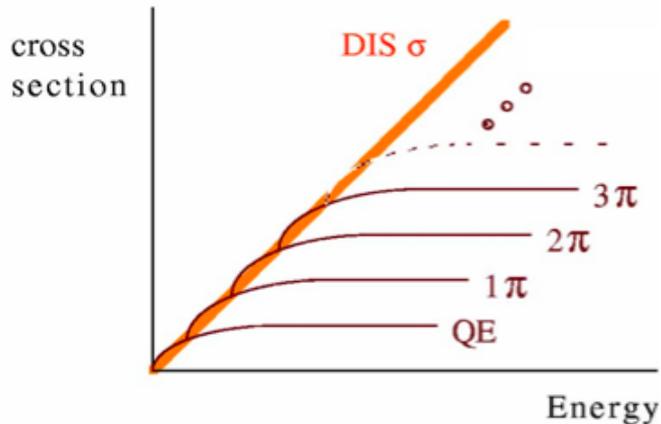
- Inelastic Scattering
  - Produce new particles, probe inner structure of the nucleon.
- (Quasi-)Elastic Scattering
  - Resolve nuclear structure, scatter off of (independent?) nucleons.



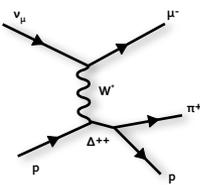
# Features



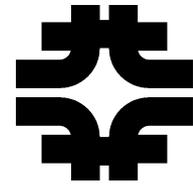
- Cross-sections scale ~linearly with the number of targets.
- Experiments often report cross-sections per:
  - Isoscalar nucleon (sum of protons and neutrons)
  - Atom (e.g. per  $^{12}\text{C}$ , etc.)
  - Per proton / neutron (typically for anti-nu / nu)



The total cross-section increases linearly with energy!



# Inelastic Reactions



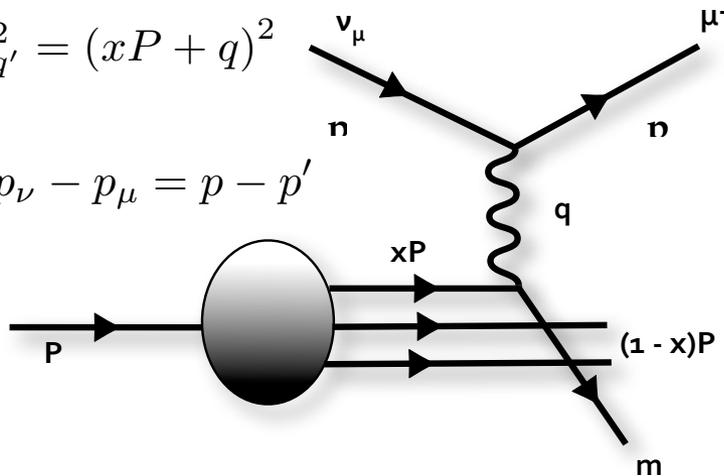
- “Real” scattering involves very complicated targets. Electroweak theory does not provide couplings for composite particles (e.g. nucleons).

In DIS, the neutrino scatters against an individual parton, carrying momentum fraction  $x$ , inside the nucleon.

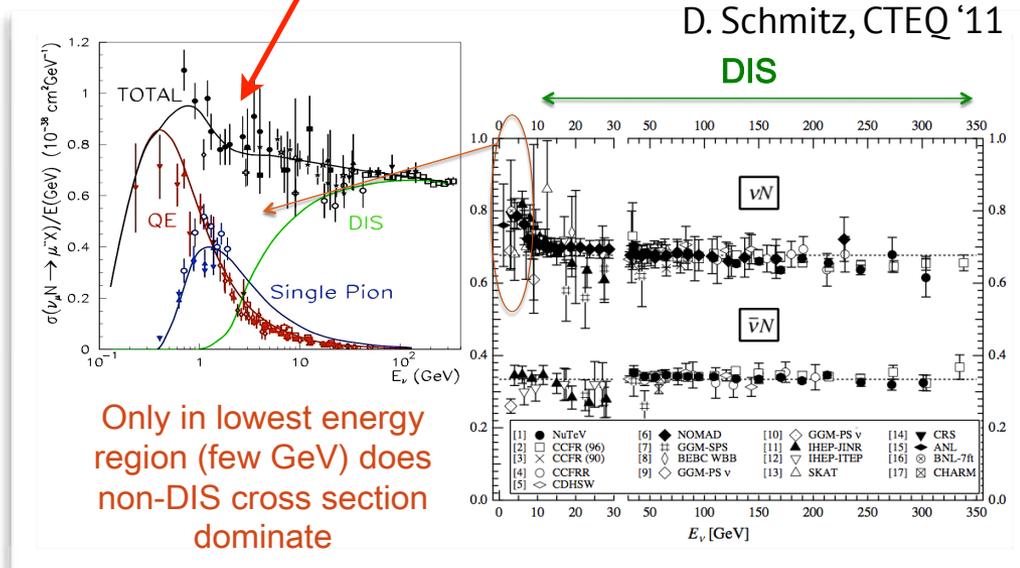
$$m_q^2 = x^2 P^2 = x^2 M_T^2$$

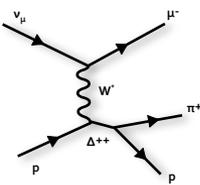
$$m_{q'}^2 = (xP + q)^2$$

$$q = p_\nu - p_\mu = p - p'$$

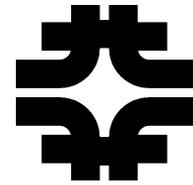


**Transition Region** - Messy Final States, but not scattering cleanly off partons.



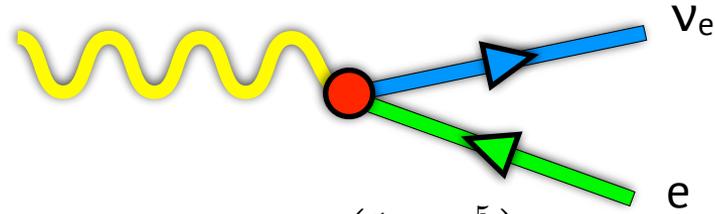
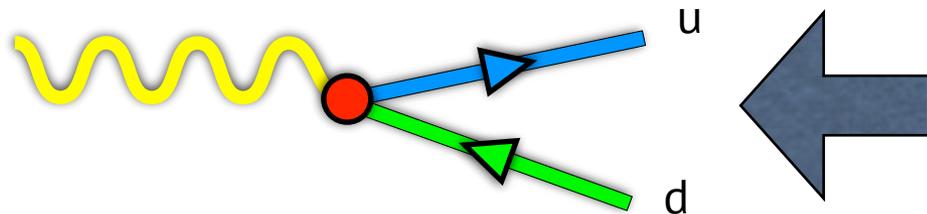


# Neutrino-Quark Scattering



“Charge-raising” quark current

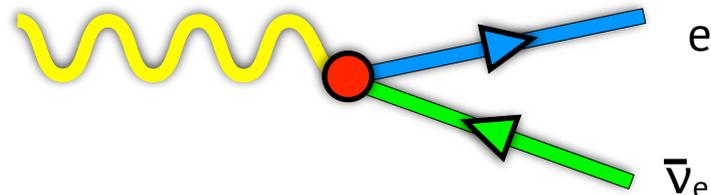
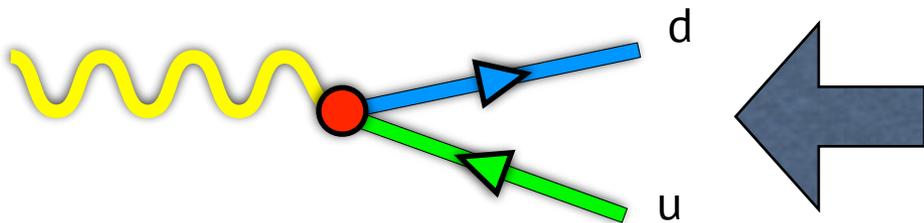
Electron weak current



$$J_q^\mu = \bar{u}_u \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) u_d$$

$$J_e^\mu = \bar{u}_\nu \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) u_e$$

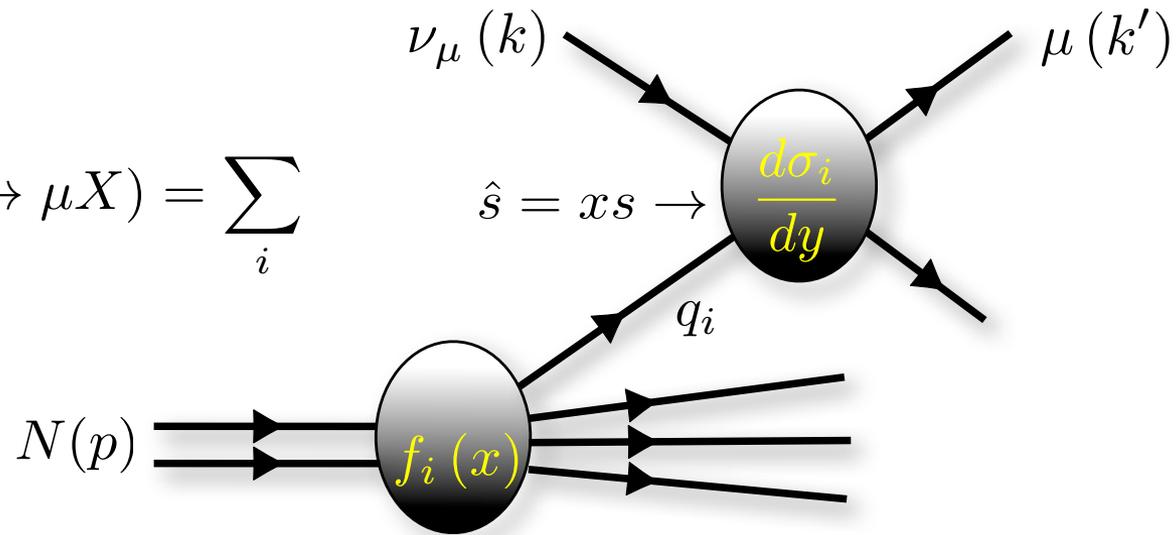
Hermitian Conjugates give the charge-lowering weak currents...



$$\frac{d\sigma}{d\Omega} (\nu_\mu d \rightarrow \mu^- u) = \frac{G_F^2 s}{4\pi^2}$$

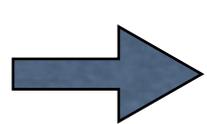
$$\frac{d\sigma}{d\Omega} (\bar{\nu}_\mu u \rightarrow \mu^+ d) = \frac{G_F^2 s}{16\pi^2} (1 + \cos \theta)^2$$

$$\frac{d^2\sigma}{dx dy} (\nu N \rightarrow \mu X) = \sum_i$$



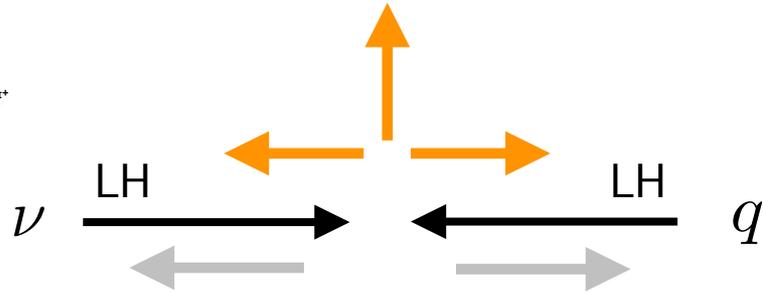
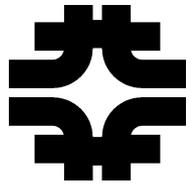
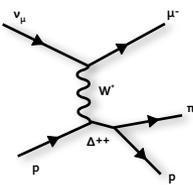
$$= \sum_i f_i(x) \left( \frac{d\sigma_i}{dy} \right)_{\hat{s}=xs}$$

$$1 - y \equiv \frac{p \cdot k'}{p \cdot k} = \frac{1}{2} (1 + \cos \theta) \quad \& \text{ Center-of-Mass Energy} = xs$$



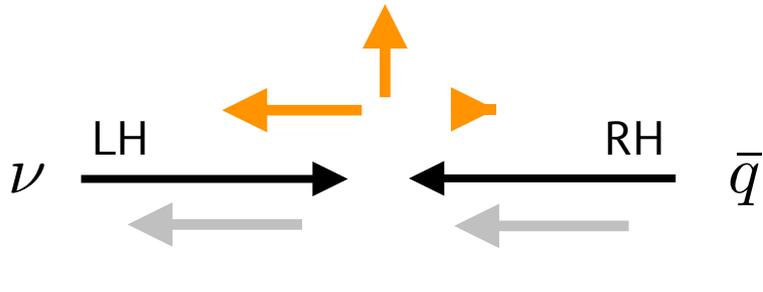
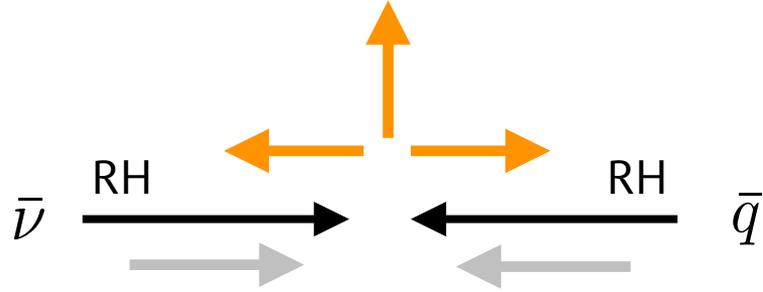
$$\frac{d\sigma}{dy} (\nu_\mu d \rightarrow \mu^- u) = \frac{G_F^2 xs}{\pi}$$

$$\frac{d\sigma}{dy} (\bar{\nu}_\mu u \rightarrow \mu^+ d) = \frac{G_F^2 xs}{\pi} (1 - y)^2$$



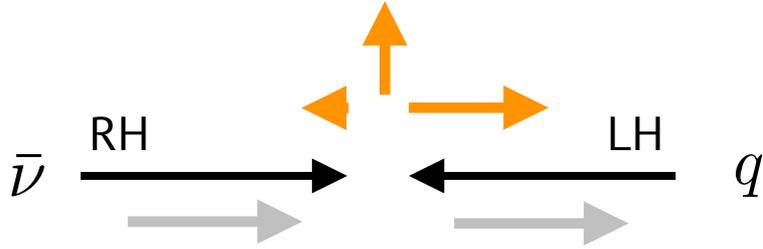
neutrino + quark  
anti-neutrino + anti-quark

$$\frac{d\sigma}{dy}(\nu q) = \frac{d\sigma}{dy}(\bar{\nu} \bar{q}) = \frac{G_F^2}{\pi} s x$$

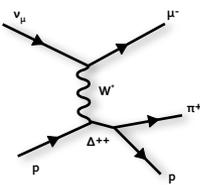


neutrino + anti-quark  
anti-neutrino + quark

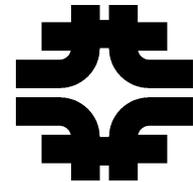
$$\frac{d\sigma}{dy}(\bar{\nu} q) = \frac{d\sigma}{dy}(\nu \bar{q}) = \frac{G_F^2}{\pi} s x (1 - y)^2$$



$$1 - y \simeq \frac{1}{2} (1 + \cos \theta)$$



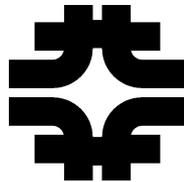
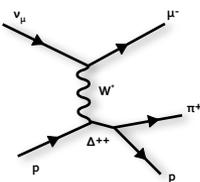
# Parton Distribution Functions $q(x)$ : Charge and Helicity



- Neutrinos and anti-neutrinos “taste” different quark flavors!
  - Neutrinos: d, s, u-bar, c-bar *ONLY*
  - Anti-neutrinos: u, c, d-bar, s-bar *ONLY*
- Scattering is *not* from free quarks though! We must use *parton distribution functions!*
  - We cannot calculate these with QCD, but we do know they are universal:

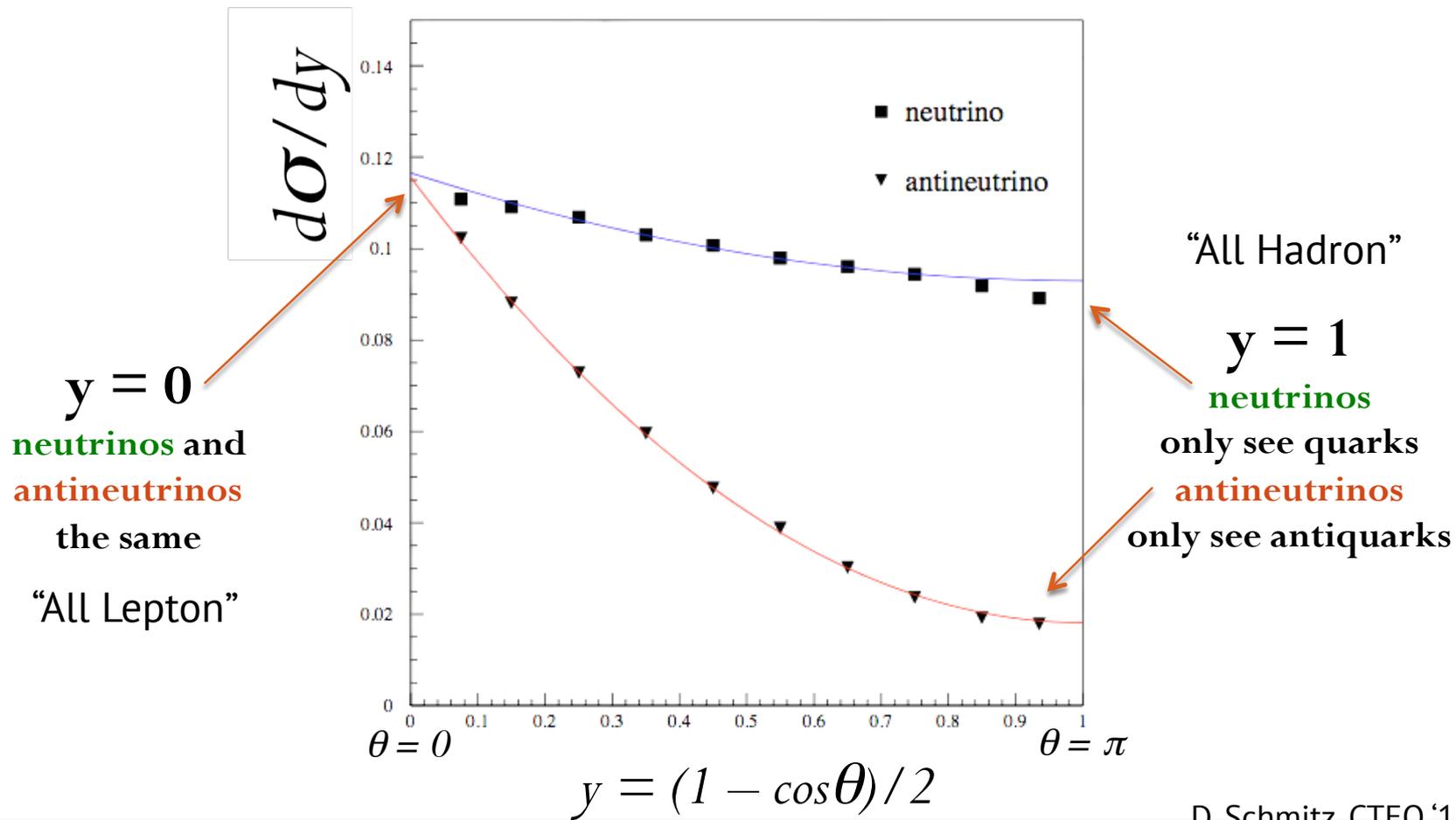
$$\frac{d^2\sigma}{dx dy} (\nu + \text{proton}) = \frac{G_F^2 s}{\pi} x \left[ d(x) + s(x) + [\bar{u}(x) + \bar{c}(x)] (1 - y)^2 \right]$$

$$\frac{d^2\sigma}{dx dy} (\bar{\nu} + \text{proton}) = \frac{G_F^2 s}{\pi} x \left[ \bar{d}(x) + \bar{s}(x) + [u(x) + c(x)] (1 - y)^2 \right]$$

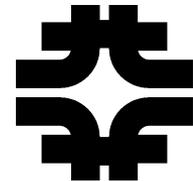
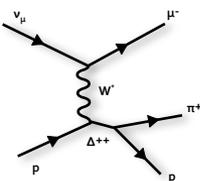


$$y = 1 - \frac{E_l}{E_\nu} \quad \text{Inelasticity}$$

## Neutrino CC DIS cross section vs. $y$



D. Schmitz, CTEQ '11



# Nucleon Structure Functions

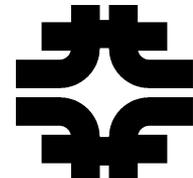
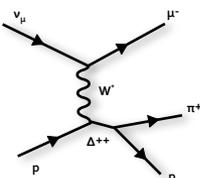
- We may write the ν-N cross-sections in a model-independent way using three nucleon structure functions:  $F_1, F_2, xF_3$ :

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dx dy} = \frac{G_F^2 M_T E}{\pi} \left[ xy^2 F_1(x, Q^2) + \left(1 - y - \frac{xyM_T}{2E}\right) F_2(x, Q^2) \pm y \left(1 - \frac{1}{2}y\right) xF_3(x, Q^2) \right]$$

- We may invoke Callan-Gross ( $2xF_1 = F_2$ ) to simplify. *Deviations:*

$$R \equiv \left(1 + \frac{4M_T^2 x^2}{Q^2}\right) \frac{F_2}{2xF_1} - 1$$

- The functions  $F_2(x, Q^2)$ ,  $xF_3(x, Q^2)$ , and  $R(x, Q^2)$  may now be experimentally charted from the measured DIS cross-section,  $d\sigma/dy$ , in bins of  $x$  and  $Q^2$ .



# Nucleon Structure Functions

neutrino... (top)

$$\frac{d^2 \sigma^{\nu A}}{dx dy} \propto [F_2^{\nu A}(x, Q^2) + xF_3^{\nu A}(x, Q^2)] + (1-y)^2 [F_2^{\nu A}(x, Q^2) - xF_3^{\nu A}(x, Q^2)] + f(R)$$

$$\frac{d^2 \sigma^{\bar{\nu} A}}{dx dy} \propto [F_2^{\bar{\nu} A}(x, Q^2) - xF_3^{\bar{\nu} A}(x, Q^2)] + (1-y)^2 [F_2^{\bar{\nu} A}(x, Q^2) + xF_3^{\bar{\nu} A}(x, Q^2)] + f(R)$$

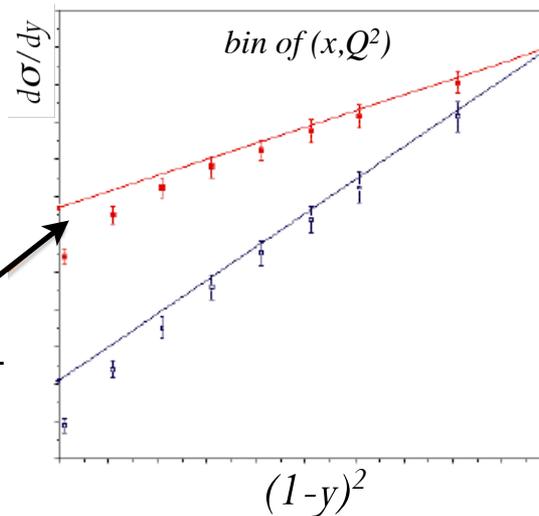
anti-neutrino... (bottom)

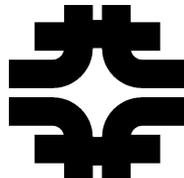
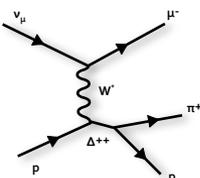
Equations of lines!

$$y \propto m \times x + b$$

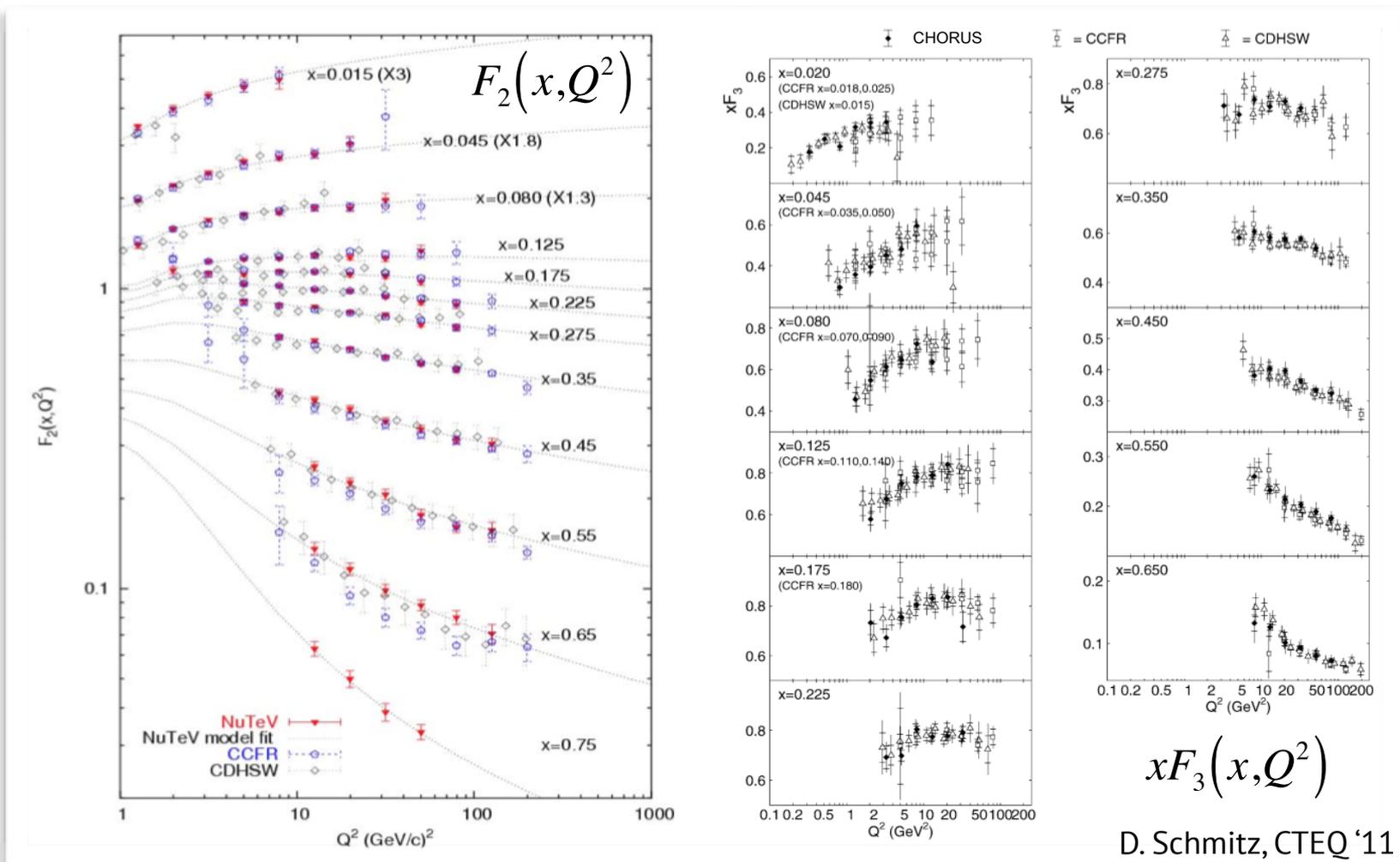
Fit for  $F_2, xF_3$  in bins of  $(x, Q^2)$ .

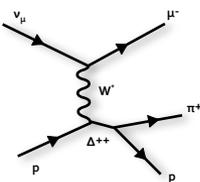
C.G. R is related to excursions from a straight-line slope.



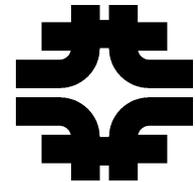


# Nucleon Structure Functions





# Structure Functions & PDFs (Charged Current)



Leading order expressions to relate SFs to PDFs:

$$F_2^{\nu N}(x, Q^2) = x [u + \bar{u} + d + \bar{d} + 2s + 2\bar{c}]$$

$$F_2^{\bar{\nu} N}(x, Q^2) = x [u + \bar{u} + d + \bar{d} + 2\bar{s} + 2c]$$

$$xF_3^{\nu N}(x, Q^2) = x [u - \bar{u} + d - \bar{d} + 2s - 2\bar{c}]$$

$$xF_3^{\bar{\nu} N}(x, Q^2) = x [u - \bar{u} + d - \bar{d} - 2\bar{s} + 2c]$$

Assuming  $c = \bar{c}$  &  $s = \bar{s}$ :

$$F_2^{\nu} - xF_3^{\nu} = 2(\bar{u} + \bar{d} + 2\bar{c}) = 2U + 4\bar{c}$$

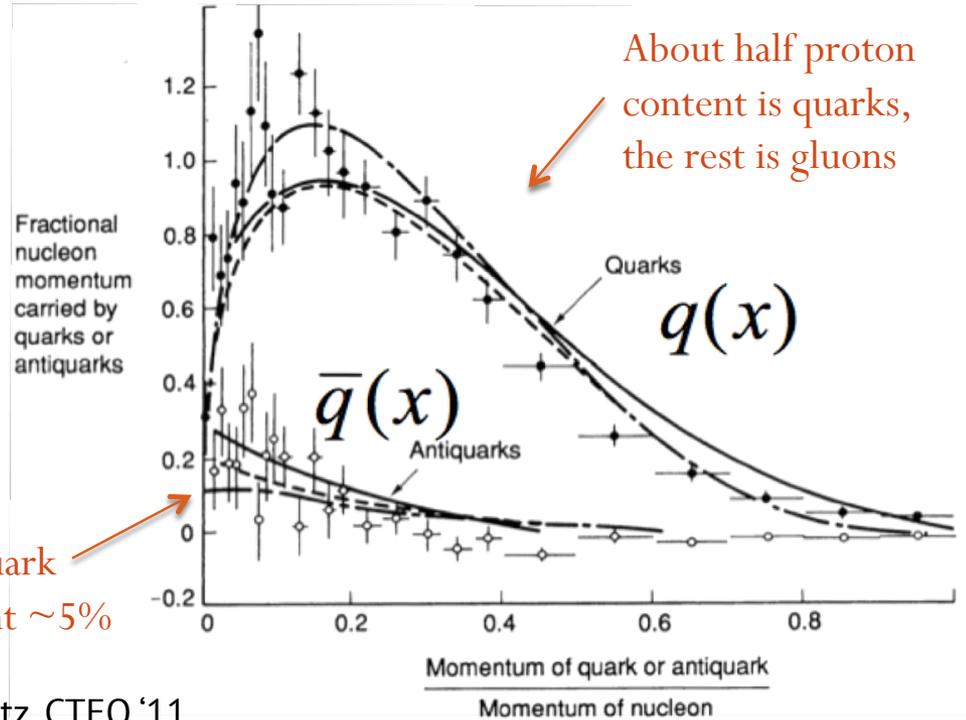
$$F_2^{\bar{\nu}} - xF_3^{\bar{\nu}} = 2(\bar{u} + \bar{d} + 2\bar{s}) = 2U + 4\bar{s}$$

$$xF_3^{\nu} - xF_3^{\bar{\nu}} = 2[(s + \bar{s}) - (c + \bar{c})] = 4\bar{s} - 4\bar{c}$$

# Parton Distribution Functions

If there were no valence quarks  
( $\bar{Q} = 0$ ):

$$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{\int_0^1 dy (1-y)^2}{\int_0^1 dy} = \frac{1}{3}$$



D. Schmitz, CTEQ '11

$$\frac{d^2\sigma}{dx dy} (\nu + \text{proton}) = \frac{G_F^2 s x}{2\pi} \left[ Q(x) + (1-y)^2 \bar{Q}(x) \right]$$

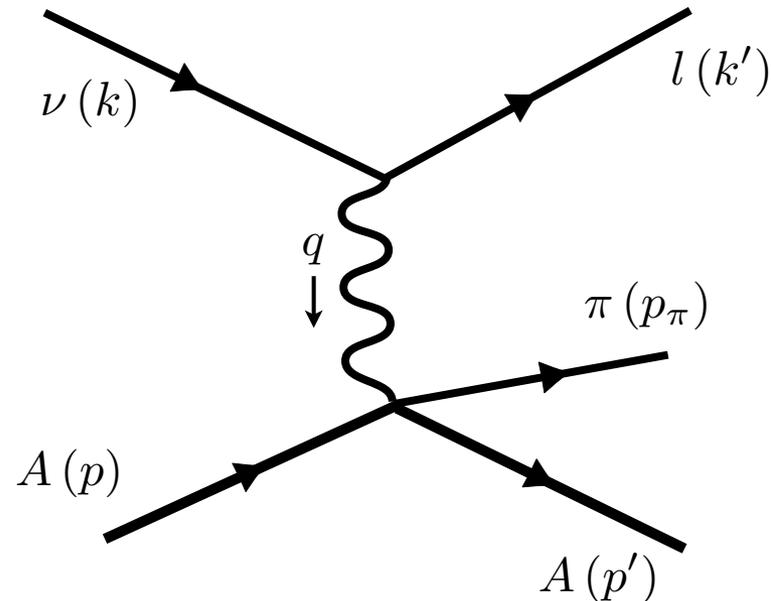
$$\frac{d^2\sigma}{dx dy} (\bar{\nu} + \text{proton}) = \frac{G_F^2 s x}{2\pi} \left[ \bar{Q}(x) + (1-y)^2 Q(x) \right]$$

# PCAC & Coherent Pion Production

- In the limit of  $Q^2 \rightarrow 0$ , the lepton emerges with momentum parallel to the neutrino.
- In this case we may use Adler's theorem, along with the PCAC (partially conserved axial current) hypothesis to write the matrix element as:

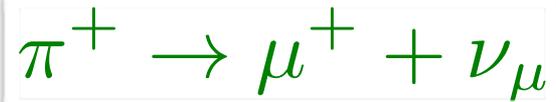
$$|\mathcal{M}|^2 = \frac{8G_F^2 E E'}{\nu^2} f_\pi^2 \sigma(\pi + A \rightarrow \pi + A)$$

- Weak neutrino scattering related to elastic pion scattering!
- The approximations are valid for neutral current and "high energy" charged current ( $Q^2 \ll m^2$ ).
- For  $Q^2 \neq 0$ , we may have a vector component and neutrino/anti-neutrino cross sections can differ.



# PCAC

- The non-conservation of the axial current leads to the decay of the pion



- With matrix elements given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \langle 0 | J_\mu | \pi(q) \rangle \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l$$

- Lorentz invariance of  $\langle 0 | J_\mu | \pi(q) \rangle$  requires that the amplitude to be either vector or axial. Since the pion has not spin the only vector available is the four-momentum  $q$

$$\langle 0 | J_\mu^A(x) | \pi(q) \rangle = i f_\pi q_\mu e^{-iqx}$$

# PCAC

- Taking the divergence of the previous equation we have

$$\langle 0 | \partial^\mu J_\mu^A(x) | \pi(q) \rangle = f_\pi q^2 e^{-iqx} = f_\pi m_\pi^2 e^{-iqx}$$

- We conclude from this relation that the axial current is not conserved, because neither  $f_\pi$  nor  $m_\pi$  is zero. However, the above expression also shows that the divergence of the axial current is small because the pion mass is small in comparison with the mass of all other hadrons
- This lead to the idea that the axial current is “partially” conserved

# NuWro

- ▶ NuWro is not an official MC in any experiment and serves as a laboratory for new developments.
- ▶ New (or relatively new) ingredients:
  - ▶ Berger-Sehgal coherent pion production
  - ▶  $\pi$  momentum distribution from  $\Delta$  decay
  - ▶ effective density and momentum dependent potential for CCQE (C. Juszczak, J. Nowak, J. Sobczyk)
- ▶ eWro - electron scattering module (a work in progress) C. Juszczak, K. Graczyk, JTS, J. Zmuda

Jarek Nowak, Lancaster University

IPPP/NuSTEC topical meeting on  
Neutrino-Nucleus scattering

- <http://school.genie-mc.org> (lecture by T. Golan)
- <https://github.com/NuWro/nuwro>
- <https://nuwro.github.io/user-guide/>





## EWro (work in progress)

J. Nowak

- ▶ All major interaction channels are implemented, for charged and neutral current, covering neutrino energy region from a few hundreds MeV (Impulse Approximation limit) to several TeV:
- ▶ QEL (quasi-)elastic scattering
- ▶ RES pion production through a  $\Delta$  resonance excitation
- ▶ DIS more inelastic processes
- ▶ COH coherent pion production
- ▶ np-nh two body current contribution
- ▶ **Transition region treatment:** smooth transition from full RES( $\Delta$ ) to full DIS starting from  $W=1.3-1.6 \text{ GeV}/c^2$

The main idea: to test NuWro nuclear model using electron scattering data

- ▶ Fermi gas and local Fermi gas
- ▶ QE and  $\Delta$  regions only
- ▶ for  $\Delta$  non-resonant background after E. Hernandez, J. Nieves, M. Valverde, Phys. Rev. D76 033005 (2007)
- ▶ EM form factors from J. Zmuda, K.M. Graczyk, arXiv:1501.03086v4
- ▶  $\Delta$  self-energy following E. Oset, L.L. Salcedo, Nucl. Phys. A468 631 (1987)

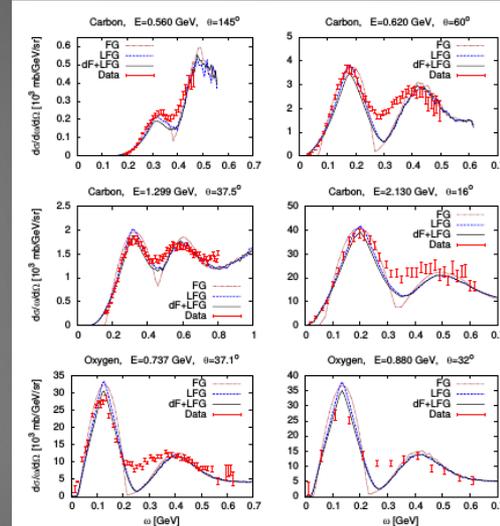
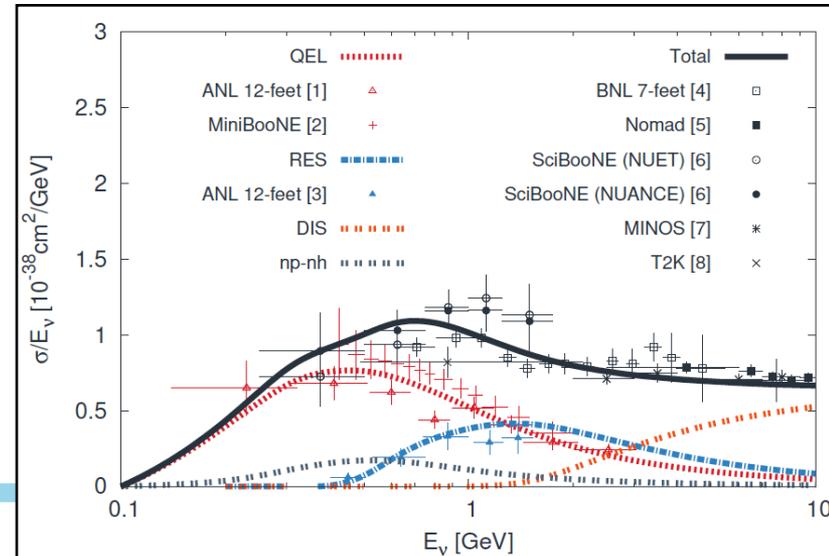
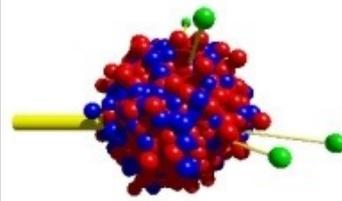


Fig. 1. Differential cross sections for electron scattering off carbon and oxygen obtained within eWro (for various beam energies,  $E$ , and scattering angles,  $\theta$ ).

K. Graczyk, C. Juszczak, JTS, J. Zmuda, arXiv:1510.03268

- Re-weighting utilities are new.





- <http://gibuu.hepforge.org>
- Strives to use the “best possible theory” in all cases.

### ■ *Initial interactions:*

- Mean field potential with local Fermigas momentum distribution, nucleons are bound (not so in generators!)
- Initial interactions calculated by summing over interactions with all bound, Fermi-moving nucleons
- 2p2h from electron phenomenology

### ■ *Final state interaction:*

- propagates outgoing particles through the nucleus using *quantum-kinetic transport theory*, fully relativistic (off-shell transport possible).  
Initial and final interactions come from the same Hamiltonian.  
CONSISTENCY of inclusive and semi-inclusive X-sections

Ulrich Mosel

New in 2016:

IPPP/NuSTEC (Durham) 2017

- Stable groundstate implemented -> improved hole spectral functions
- 2p2h structure function for all kinematics, fitted to e-scattering, is used for neutrinos as well

# Quantum-kinetic Transport Theory for FSI

On-shell drift term

Off-shell transport term

Collision term

$$\mathcal{D}F(x, p) - \text{tr} \left\{ \Gamma f, \text{Re} S^{\text{ret}}(x, p) \right\}_{\text{PB}} = C(x, p) .$$

$$\mathcal{D}F(x, p) = \{p_0 - H, F\}_{\text{PB}} = \frac{\partial(p_0 - H)}{\partial x} \frac{\partial F}{\partial p} - \frac{\partial(p_0 - H)}{\partial p} \frac{\partial F}{\partial x}$$

$H$  contains  
mean-field  
potentials

Describes time-evolution of  $F(x, p)$

$$F(x, p) = 2\pi g f(x, p) \mathcal{P}(x, p)$$

Spectral function

Phase space distribution

Kadanoff-Baym equations with BM offshell term

Durham 04/2017



Institut für  
Theoretische Physik

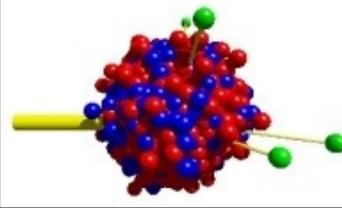


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# “Nature”

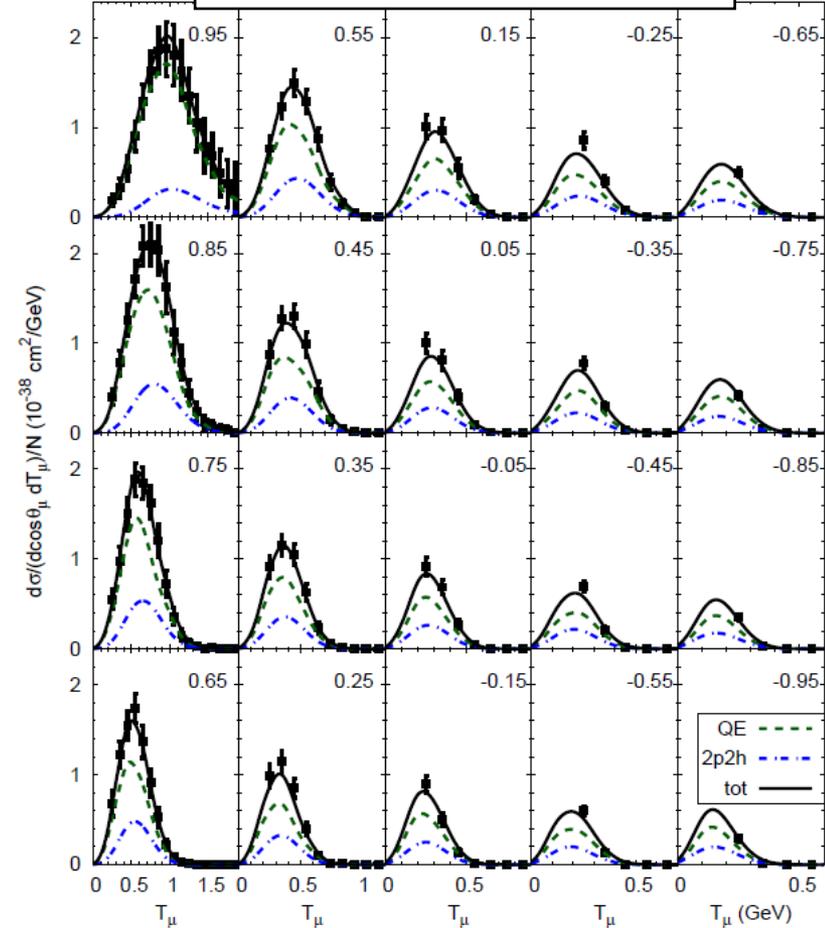
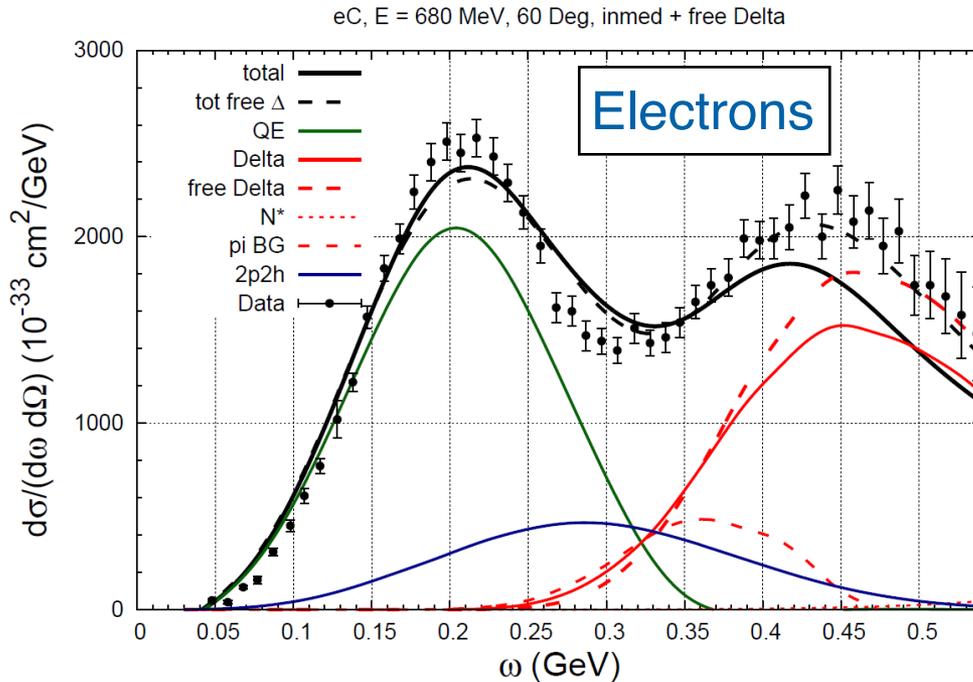


GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- Compares well to many electron and neutrino data sets.
- Typically not re-weightable, no geometry/flux

## MiniBooNE Neutrinos



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