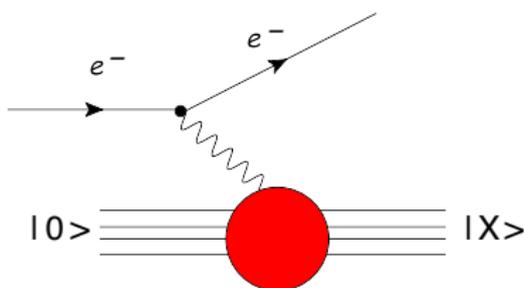


Unified treatment of the inclusive electron-nucleus scattering within the Spectral Function formalism

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- ▶ In collaboration with: **Omar Benhar, Alessandro Lovato,**
- ▶ Based on: Phys. Rev. C **92**, 024602 (2015); *arXiv:1512.07426*.

- Motivations
 - ▶ Accurate determination of neutrino oscillation parameter
 - ▶ Differences between electron- and neutrino- nucleus scattering
- Non relativistic & relativistic regimes: the factorization scheme.
- Meson-exchange currents in the extended factorization ansatz
- Comparison to experimental data
- Summary & Outlook

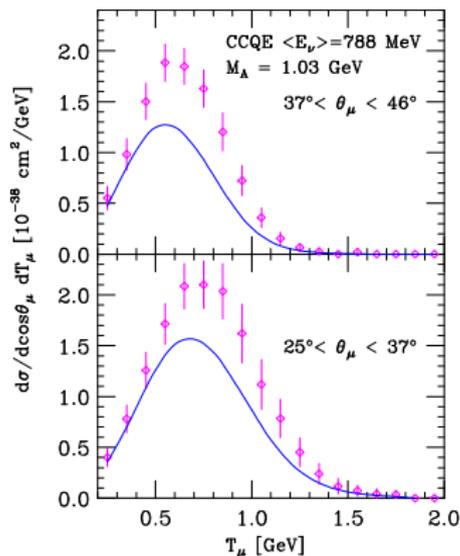
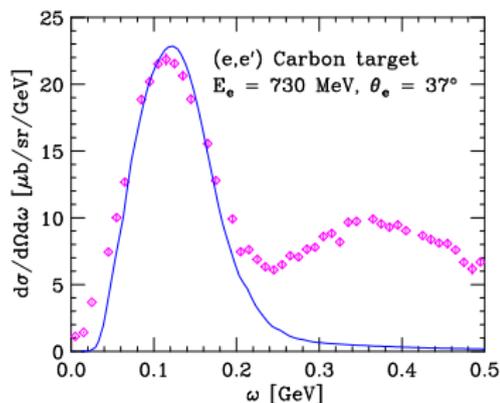
- Quantitative understanding of the nuclear response to neutrino interactions needed for the interpretation of neutrino oscillation signals
- The description of the neutrino-nucleus cross section involves non trivial additional difficulties, mostly owing to the broad distribution of the incoming neutrino energies
- Accurate theoretical models of electron- nucleus scattering provide a satisfactory description of the experimental data.

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- Data: MiniBooNE Collaboration

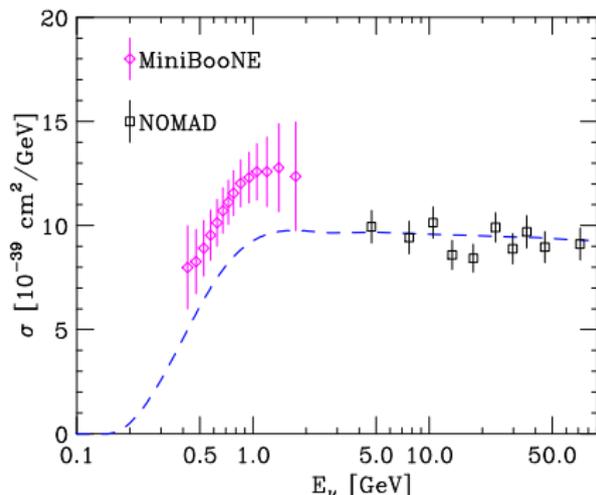
- Data: J.S. O'Connell *et al*



- The calculations performed using the spectral function and the measured nuclear vector form factors accurately reproduce the QE peak measured in electron scattering
- The same scheme largely fails to explain the MiniBooNE data.

The axial mass puzzle

- Unfolded total CCQE cross section



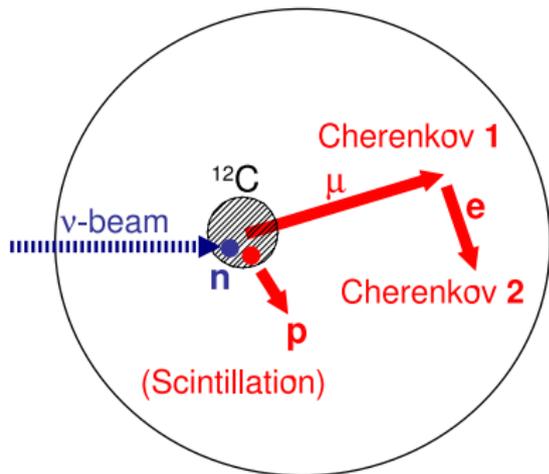
- ▶ The axial form factor is generally parametrized in the dipole form

$$F_A(Q^2) = \frac{g_A}{[1 + (Q^2/M_A^2)]^2},$$

- ▶ Deuteron data $\Rightarrow M_A \approx 1.03 \text{ GeV}$
- ▶ MinibooNE $\Rightarrow M_A \approx 1.35 \text{ GeV}$
- ▶ K2K $\Rightarrow M_A \approx 1.2 \text{ GeV}$
- ▶ NOMAD $\Rightarrow M_A \approx 1.05 \text{ GeV}$

- Interpret the value of M_A reported by MiniBooNE as an *effective* axial mass, modified by nuclear effects not included in the RFGM.

Role of Multi Nucleon knockout

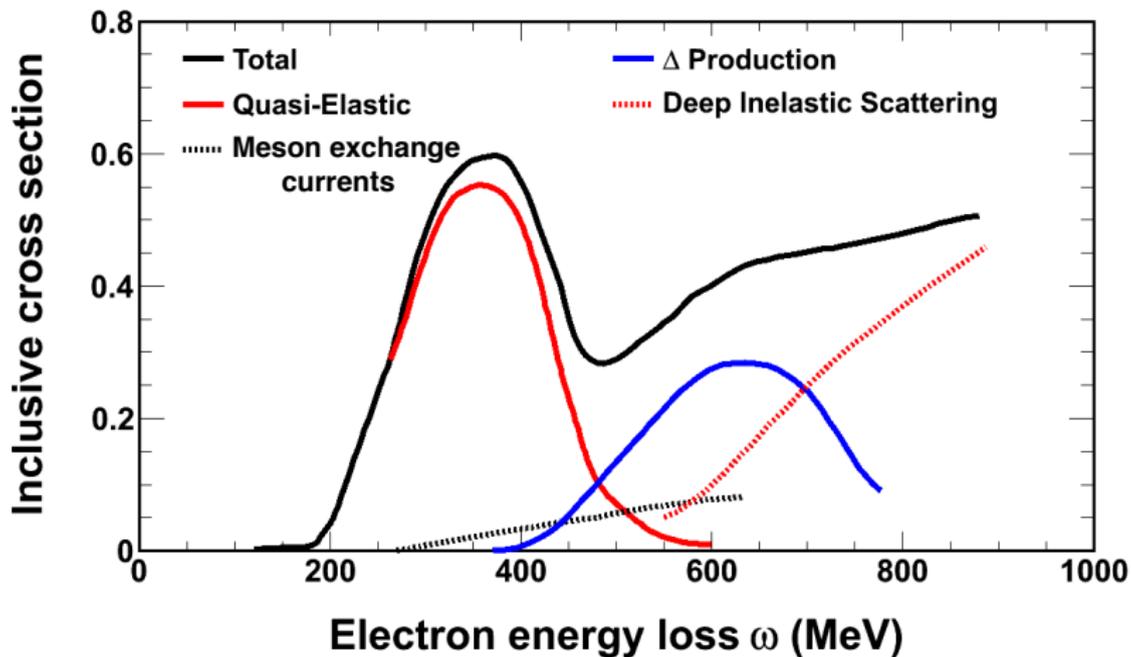


- ▶ In MiniBooNE data analysis an event is labeled as CCQE if no final state pions are detected in addition to the outgoing muon.
- ▶ The simplest reaction mechanism compatible with this definition is single nucleon knockout

The observed excess of CCQE cross-section may be traced back to the occurrence of events with two particle-two hole final states, which are often referred to CCQE-like.

Inclusive lepton-nucleus cross section at fixed beam energy

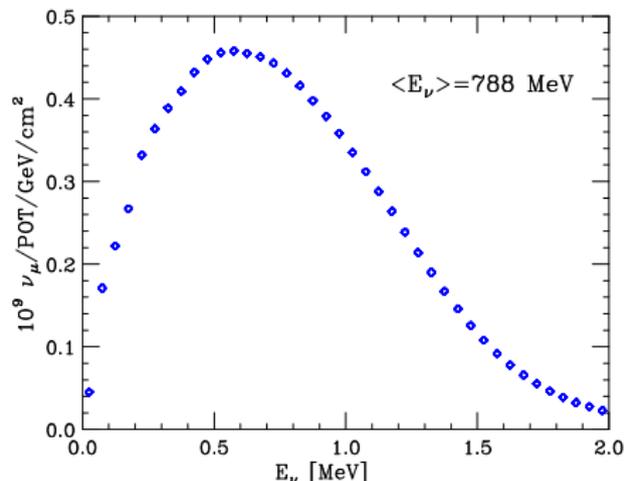
Inclusive electron-nucleus cross section at $E_e \sim 1$ GeV, as a function of ω .



The different reaction mechanisms, contributing to the cross section at different values of ω , can be easily identified.

QE neutrino-nucleus scattering

- ▶ The measured double differential CCQE cross section is averaged over the neutrino flux



- ▶ Energy distribution of MiniBooNE neutrino flux
- ▶ Different reaction mechanisms contribute to the cross section at fixed θ_μ and T_μ .

A description of neutrino-nucleus interactions, has to be validated through extensive comparison to the large body of electron-nucleus scattering data.

The electron-nucleus x-section

- The double differential x-section of the process $e^- + A \rightarrow e^- + X$, can be written

$$\frac{d^2\sigma}{d\Omega_{\mathbf{k}'} dk'_0} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} L_{\mu\nu} W_A^{\mu\nu} .$$

- $L_{\mu\nu}$ is completely determined by the lepton kinematics
- The hadronic tensor describes the **response** of the target nucleus.

$$W_A^{\mu\nu} = \sum_X \langle 0 | J_A^{\mu\dagger} | X \rangle \langle X | J_A^\nu | 0 \rangle \delta^{(4)}(p_0 + q - p_X) ,$$

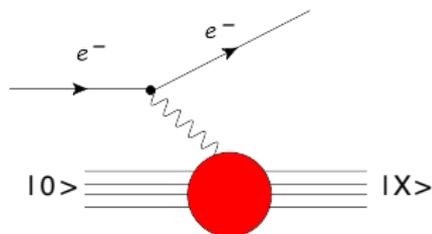
initial state

$$|0\rangle ; p_0$$

final state

$$|X\rangle = |1p; 1h\rangle, |2p; 2h\rangle \dots ; p_X$$

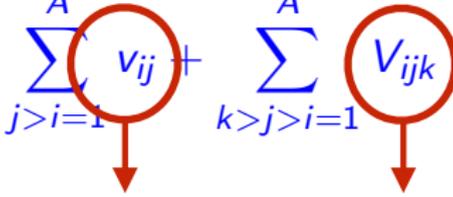
Non relativistic nuclear many-body theory (NMBT) provides a fully consistent theoretical approach allowing for an accurate description of $|0\rangle$, independent on momentum transfer.



CCQE interactions at moderate ($|q| \lesssim 500$ MeV)

- Within NMBT the nucleus is described as a collection of A pointlike nucleons, the dynamics of which are described by the nonrelativistic Hamiltonian

$$H = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{j>i=1}^A v_{ij} + \sum_{k>j>i=1}^A V_{ijk}$$



Argonne v_{18} Fujita-Miyazawa

- Initial state definition:

$$H|0\rangle = E_0|0\rangle$$

- Final state definition

$$H|X\rangle = E_X|X\rangle$$

The above Schrödinger equation can only be exactly solved for the ground- and low-lying excited states of nuclei with $A \leq 12$.

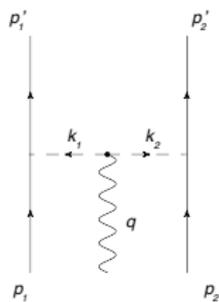
The nuclear current operator

- The nuclear Hamiltonian does not commute with the charge density operator: $[H, J^0] \neq 0$
- In order for the continuity equation to be satisfied two body currents are needed:

$$\frac{\partial}{\partial t} J^0 + \vec{\nabla} \cdot \vec{J} = 0$$

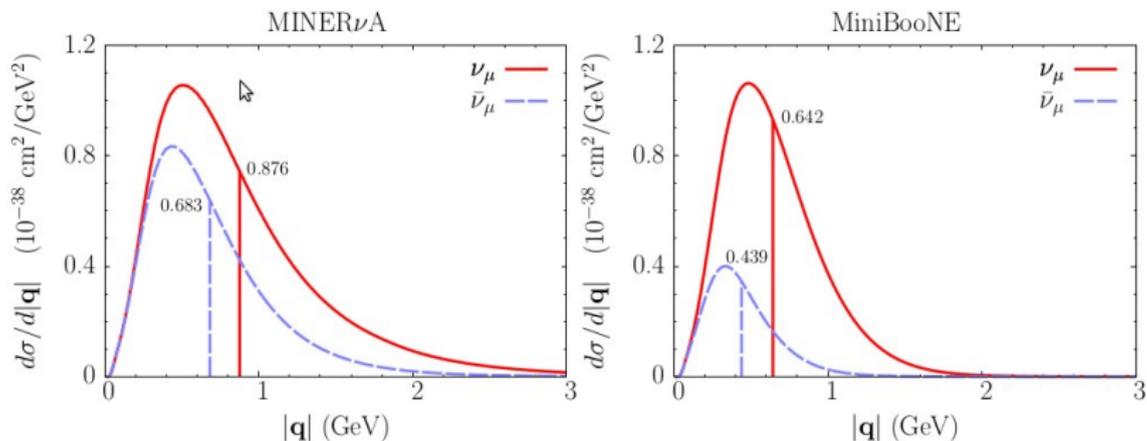
- The nuclear current includes one- and two-nucleon contributions

$$J_A^\mu(q) = \sum_{i=1}^A j_i^\mu(q) + \sum_{j>i=1}^A j_{ij}^\mu(q_1, q_2) \delta(q - q_1 - q_2)$$



- non relativistic reduction of the current (q/m expansions) .

Kinematical range of accelerator-based neutrino experiments



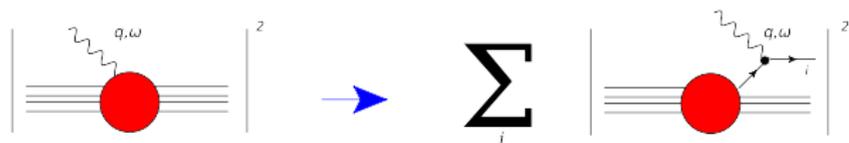
- $|q|$ -dependence of CCQE cross section averaged with the Minervνa and MiniBooNE fluxes

WARNING!

unlike the ground state, **the nuclear current operator and the nuclear final state depend on momentum transfer**. At large q non relativistic approximations become inadequate.

The factorization “paradigm”

- Simplest implementation: **Impulse Approximation (IA)**



- At $|\mathbf{q}|^{-1} \ll d$:

$$J_A^\mu \longrightarrow \sum_i j_i^\mu, \quad |X\rangle \longrightarrow |x, \mathbf{p}_x\rangle \otimes |R, \mathbf{p}_R\rangle,$$

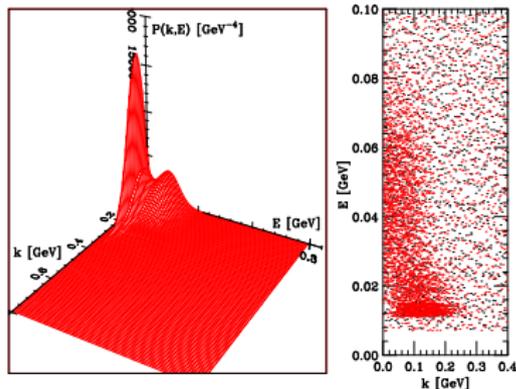
- The nuclear cross section can be traced back to the one describing the interaction with individual bound nucleons

$$d\sigma_A = \int dE d^3k d\sigma_N P(k, E)$$

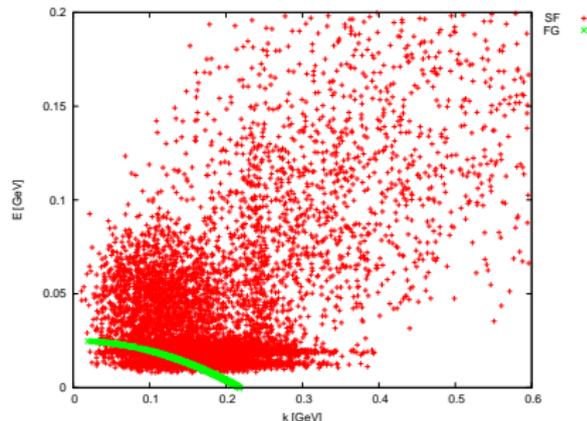
- ▶ Provided the four momentum transfer q is replaced by $\tilde{q} = (\tilde{\omega}, \mathbf{q})$
- ▶ An integration on the nucleon momentum and removal energy is carried out, with a weight given by the spectral function

Spectral function and energy-momentum distribution

- ▶ Oxygen spectral function, obtained within LDA.

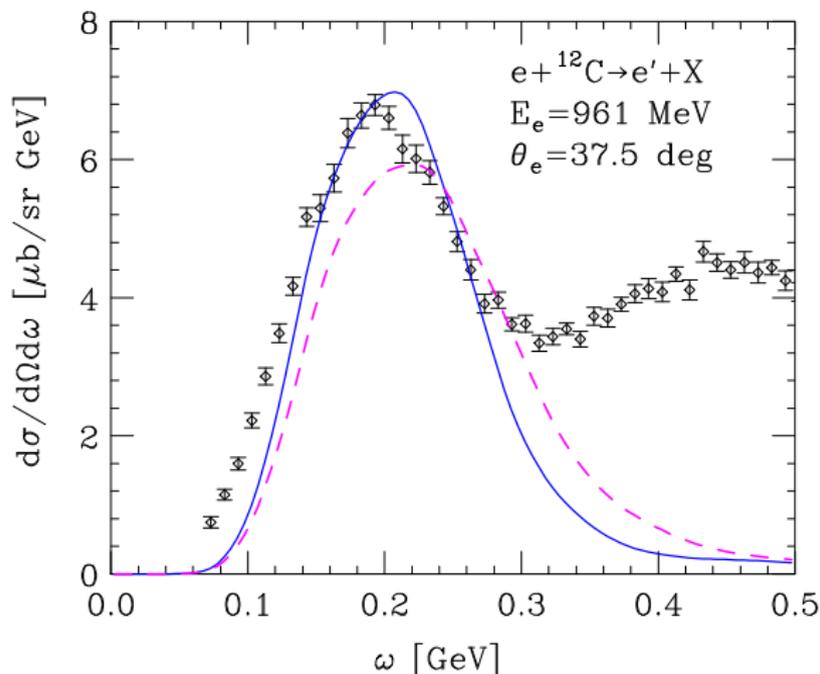


- ▶ Momentum and removal energy sampled from LDA (red) and RFGM (green) oxygen spectral functions



- FG model: $P_{RFGM}(p, E) \propto \theta(p_F - p) \delta(E_p - \epsilon + E)$,
- Scattering off high momentum **and** high removal energy nucleons, providing $\sim 20\%$ of the total strength, gives rise to 2p2h final states.

The impact of relativistic effects

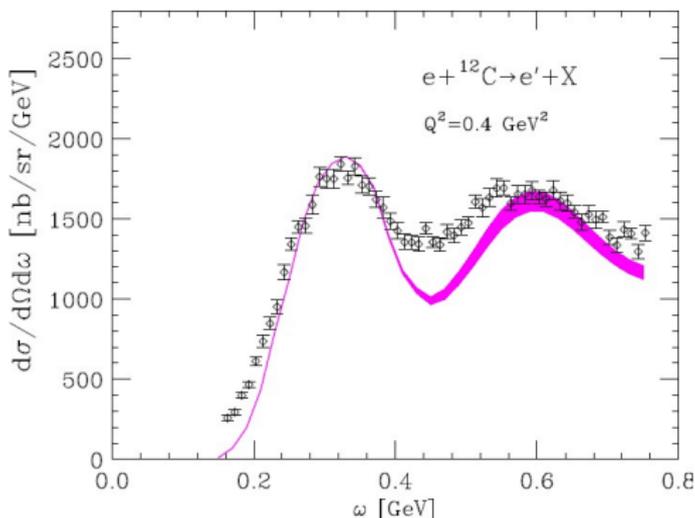


Electron-carbon cross section obtained within the IA approach using relativistic (solid line) and non relativistic (dashed line) kinematics.

- ▶ In a kinematical setup corresponding to $|q| \sim 585 \text{ MeV}$ at $\omega = \omega_{QE}$ relativistic kinematics sizeably affects both position and width of the quasi elastic peak.

Range of applicability of the IA

- Electron-Carbon cross section for $E_e = 1.3$ GeV, $\theta_e = 37.5$.



- The IA provides a unified framework, suitable to describe the measured cross section in different kinematical regimes, except in the *dip region*, where two-body currents are expected to contribute.

Nuclear and nucleon structure functions

The most general expression of the target tensor fulfilling the requirements of Lorentz covariance, conservation of parity and gauge invariance:

$$W_A^{\mu\nu} = W_1^A \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2^A}{m^2} \left(P_0^\mu - \frac{(P_0 q)}{q^2} q^\mu \right) \left(P_0^\nu - \frac{(P_0 q)}{q^2} q^\nu \right) .$$

Substituting this expression and carrying out the contraction with the lepton tensor $L^{\mu\nu}$ one finds

$$\left(\frac{d^2\sigma}{dE_{e'} d\Omega_{e'}} \right)_A = \left(\frac{d\sigma}{d\Omega_{e'}} \right)_M \left[W_2^A(|\mathbf{q}|, \omega) + 2W_1^A(|\mathbf{q}|, \omega) \tan^2 \frac{\theta}{2} \right] ,$$

where $(d\sigma/d\Omega_{e'})_M = \alpha^2 \cos^2(\theta/2)/4E_e \sin^4(\theta/2)$ is the Mott cross section.

Nuclear and nucleon structure functions

The explicit expressions of the nuclear structure functions W_1^A and W_2^A can be written in terms of the nucleon structure functions according to

$$W_1^A(|\mathbf{q}|, \omega) = \int d^3k dE \left\{ ZP_p(\mathbf{k}, E) \left(\frac{m}{E_k} \right) \times \left[w_1^p(|\mathbf{q}|, \tilde{\omega}) + \frac{1}{2} \frac{w_2^p(|\mathbf{q}|, \tilde{\omega})}{m^2} \frac{|\mathbf{k} \times \mathbf{q}|^2}{|\mathbf{q}|^2} \right] + \dots \right\},$$

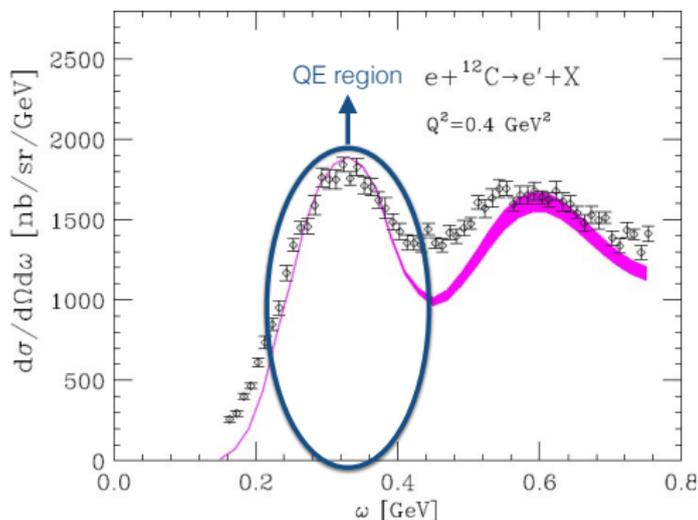
and

$$W_2^A(|\mathbf{q}|, \omega) = \int d^3k dE \left\{ ZP_p(\mathbf{k}, E) \left(\frac{m}{E_k} \right) \left[w_1^p(|\mathbf{q}|, \tilde{\omega}) \frac{q^2}{|\mathbf{q}|^2} \left(\frac{q^2}{\tilde{q}^2} - 1 \right) + \frac{w_2^p(|\mathbf{q}|, \tilde{\omega})}{m^2} \left(\frac{q^4}{|\mathbf{q}|^4} \left(E_k - \tilde{\omega} \frac{E_k \tilde{\omega} - \mathbf{k} \cdot \mathbf{q}}{\tilde{q}^2} \right)^2 - \frac{1}{2} \frac{q^2}{|\mathbf{q}|^2} \frac{|\mathbf{k} \times \mathbf{q}|^2}{|\mathbf{q}|^2} \right) \right] + \dots \right\}$$

where the ellipses denote the neutron contributions.

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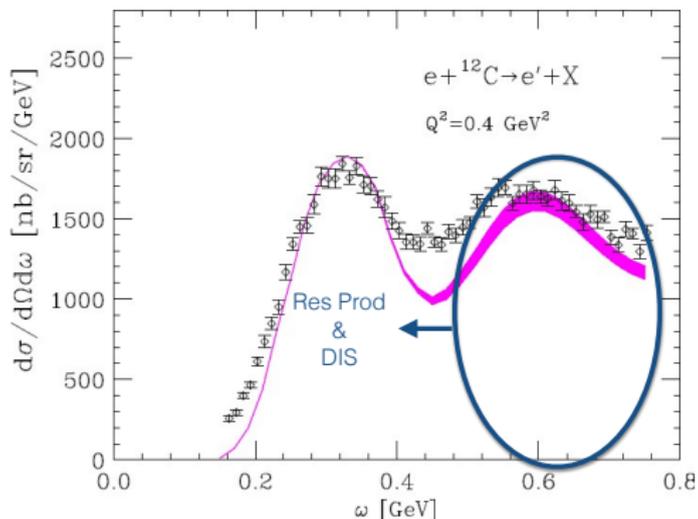
Nucleon structure functions

In the QE channel, the energy conserving δ -function enforces the condition that the scattering process be elastic

$$\begin{aligned}\tilde{w}_1^N &= \tau G_{MN}^2 \delta\left(\tilde{\omega} + \frac{\tilde{q}^2}{2m}\right), \\ \tilde{w}_2^N &= \frac{1}{(1+\tau)} \left(G_{EN}^2 + \tau G_{MN}^2\right) \delta\left(\tilde{\omega} + \frac{\tilde{q}^2}{2m}\right),\end{aligned}\tag{1}$$

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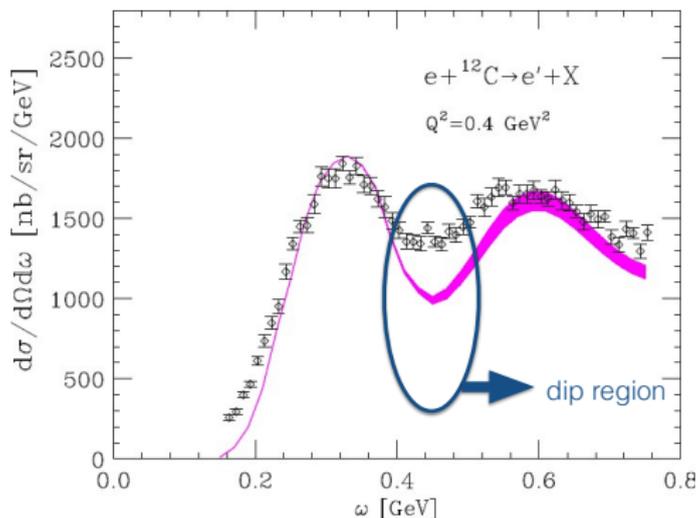
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In the Resonance production and DIS region

To take into account the possible production of hadrons other than protons and neutrons one has to introduce [the inelastic nucleon structure functions](#) extracted from the analysis of electron-proton and electron-deuteron scattering data (Bodek-Ritchie).

Range of applicability of the IA

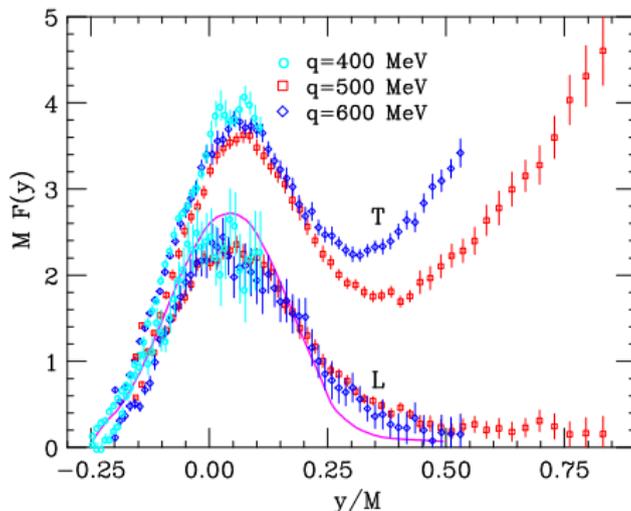
- Electron-Carbon cross section for $E_e = 1.3$ GeV, $\theta_e = 37.5$.



- The IA provides a unified framework, suitable to describe the measured cross section in different IA kinematical regimes, except in the *dip region*, where two-body currents are expected to contribute.

Role of reaction mechanism beyond IA

- Scaling functions associated with the longitudinal (L) and transverse (T) response of Carbon extracted from electron scattering data

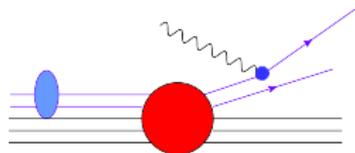


- ▶ the onset of scaling is clearly visible in the region of QE peak, corresponding to $y \sim 0$.
- ▶ large scaling violations appear in $F_T(y)$ at $y > 0$.

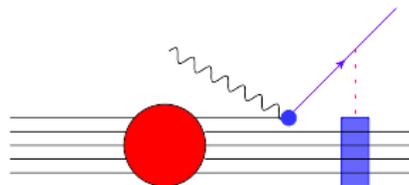
How can $2p2h$ final states be produced?

In a model accounting for NN correlations, $2p2h$ final states can be produced through 3 different reaction mechanisms.

- Initial State Correlations (ISC):

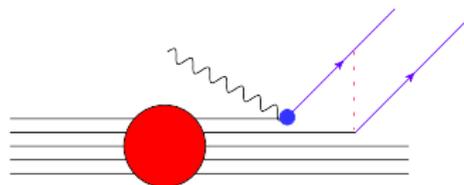
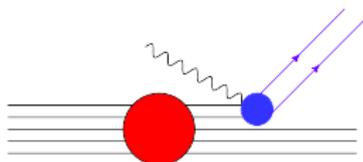


- Final State Interactions (FSI):



(a)

- Meson Exchange Currents (MEC):



(b)

Extending the factorization scheme

- Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the factorization scheme to two-nucleon emission amplitude

- ▶ Rewrite the hadronic final state $|X\rangle$ in the factorized form:

$$|X\rangle \longrightarrow |\mathbf{p} \mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)}; \mathbf{p} \mathbf{p}'\rangle ,$$

where $|n_{(A-2)}\rangle$ describes the spectator $(A - 2)$ -nucleon system, carrying momentum \mathbf{p}_n .

- ▶ The two nucleon current simplifies

$$\langle X | j_{ij}^\mu | 0 \rangle \rightarrow \int d^3k d^3k' M_n(\mathbf{k}, \mathbf{k}') \langle \mathbf{p} \mathbf{p}' | j_{ij}^\mu | \mathbf{k} \mathbf{k}' \rangle \delta(\mathbf{k} + \mathbf{k}' - \mathbf{p}_n) ,$$

- ▶ The nuclear amplitude: $M_n(\mathbf{k}, \mathbf{k}') = \langle n_{(A-2)}; \mathbf{k} \mathbf{k}' | 0 \rangle$ is independent of \mathbf{q} , and can therefore be obtained within NMBT.

Two nucleon spectral function

- Two-nucleon spectral function of uniform and isospin nuclear matter

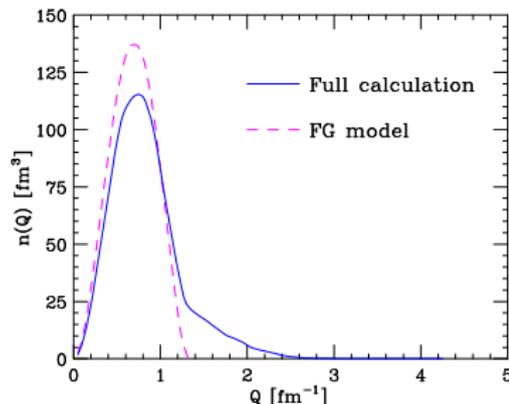
$$P(\mathbf{k}, \mathbf{k}', E) = \sum_n |M_n(\mathbf{k}, \mathbf{k}')|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}, \mathbf{k}') = \int dE P(\mathbf{k}, \mathbf{k}', E)$$

- Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi |\mathbf{Q}|^2 \int d^3K n\left(\mathbf{Q} + \frac{\mathbf{K}}{2}, \mathbf{Q} - \frac{\mathbf{K}}{2}\right)$$

$$\mathbf{K} = \mathbf{k} + \mathbf{k}' \quad , \quad \mathbf{Q} = \frac{\mathbf{k} - \mathbf{k}'}{2} .$$



- ▶ Correlation effects lead to **a quenching of the peak** of the distributions and **an enhancement of the high momentum tail**

1p1h and 2p2h contributions to the nuclear cross section

- ▶ The factorization scheme allows for a clear identification of the 1p1h and 2p2h contributions

$$d\sigma = d\sigma_{1p1h} + d\sigma_{2p2h} \propto L_{\mu\nu} (W_{1p1h}^{\mu\nu} + W_{2p2h}^{\mu\nu})$$

- ▶ 2p2h response tensor

$$W_{2p2h}^{\mu\nu} = \sum_{h,h' < k_F} \sum_{p,p' > k_F} \langle 0 | J^{\mu\dagger} | \mathbf{h}\mathbf{h}'\mathbf{p}\mathbf{p}' \rangle \langle \mathbf{h}\mathbf{h}'\mathbf{p}\mathbf{p}' | J^\nu | 0 \rangle \\ \times \delta(\omega + E_0 - E_{hh'pp'}) \delta(\mathbf{q} + \mathbf{h} + \mathbf{h}' - \mathbf{p} - \mathbf{p}') ,$$

- ▶ Current operator in momentum space:

$$J^\mu(\mathbf{k}_1, \mathbf{k}_2) = j_1^\mu(\mathbf{k}_1)\delta(\mathbf{k}_2) + j_2^\mu(\mathbf{k}_2)\delta(\mathbf{k}_1) + j_{12}^\mu(\mathbf{k}_1, \mathbf{k}_2) ,$$

$$W_{2p2h}^{\mu\nu} = W_{2p2h,11}^{\mu\nu} + W_{2p2h,22}^{\mu\nu} + W_{2p2h,12}^{\mu\nu}$$

- 1 Initial state correlations
- 2 MEC, two-body response
- 3 Interference

Initial state correlations

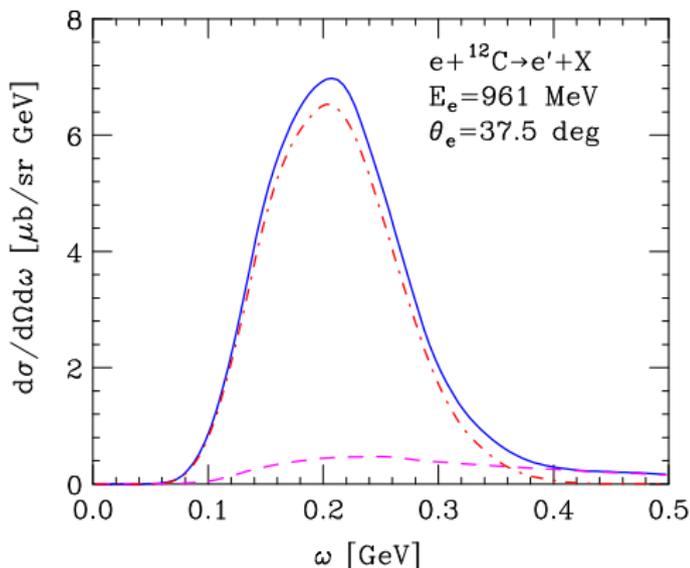
Within the IA...

$$W_{2p2h,11}^{\mu\nu} = \int d^3k \int dE P_{2h1p}(\mathbf{k}, E) w_{11}^{\mu\nu}$$

$$P_{2h1p}(\mathbf{k}, E) = \sum_{h,h' < k_F} \sum_{p' > k_F} |\Phi_k^{hh'p'}|^2$$

$\times \delta(E + e_h + e_{h'} - e_{p'})$,

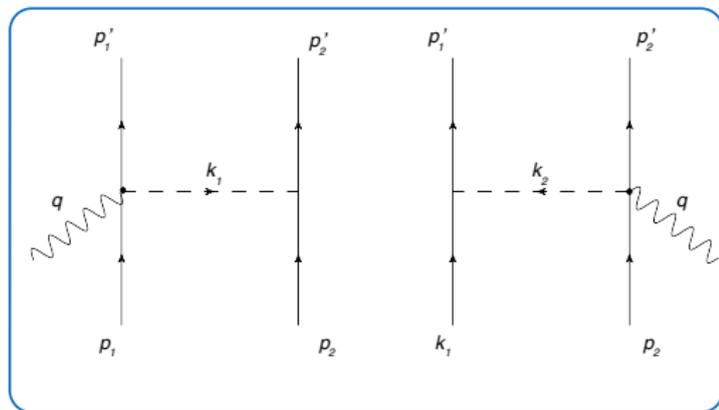
- appearance of the tail of the cross section, extending to large energy loss. This contribution amounts to $\sim 10\%$ of the integrated spectrum.



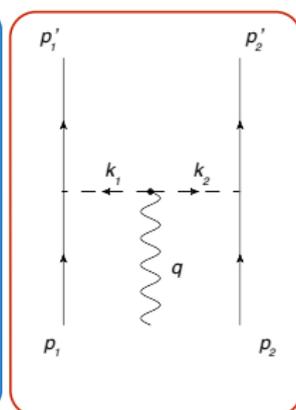
Production of 2p2h final states

- 1 Initial state correlations
- 2 **MEC, two-body response**
- 3 Interference

MEC: Pion exchange

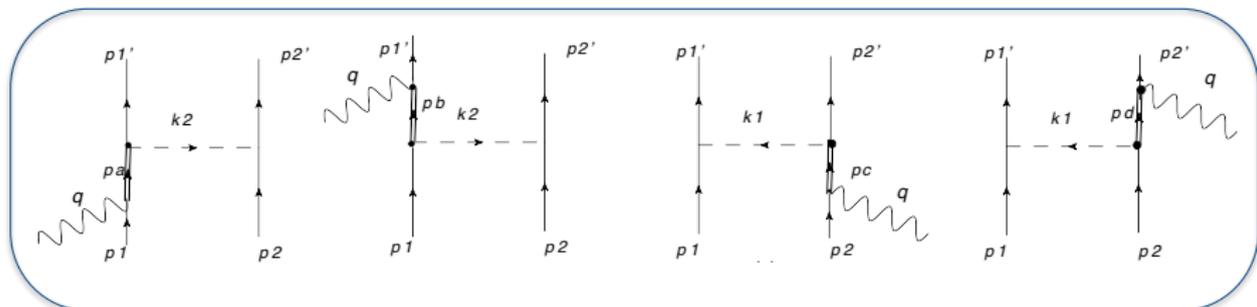


Seagull
or
contact
term



Pion
in
flight
term

MEC: Δ -isobar exchange



The Rarita-Schwinger (RS) expression for the Δ propagator reads

$$S^{\beta\gamma}(p, M_\Delta) = \frac{\not{p} + M_\Delta}{p^2 - M_\Delta^2} \left(g^{\beta\gamma} - \frac{\gamma^\beta \gamma^\gamma}{3} - \frac{2p^\beta p^\gamma}{3M_\Delta^2} - \frac{\gamma^\beta p^\gamma - \gamma^\gamma p^\beta}{3M_\Delta} \right)$$

WARNING

If the condition $p_\Delta^2 > (m_N + m_\pi)^2$ the real resonance mass has to be replaced by $M_\Delta \rightarrow M_\Delta - i\Gamma(s)/2$ where $\Gamma(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2} \frac{k^3}{\sqrt{s}} (m_N + E_k)$.

2p-2h Transverse Response

From the 2p-2h hadron tensor...

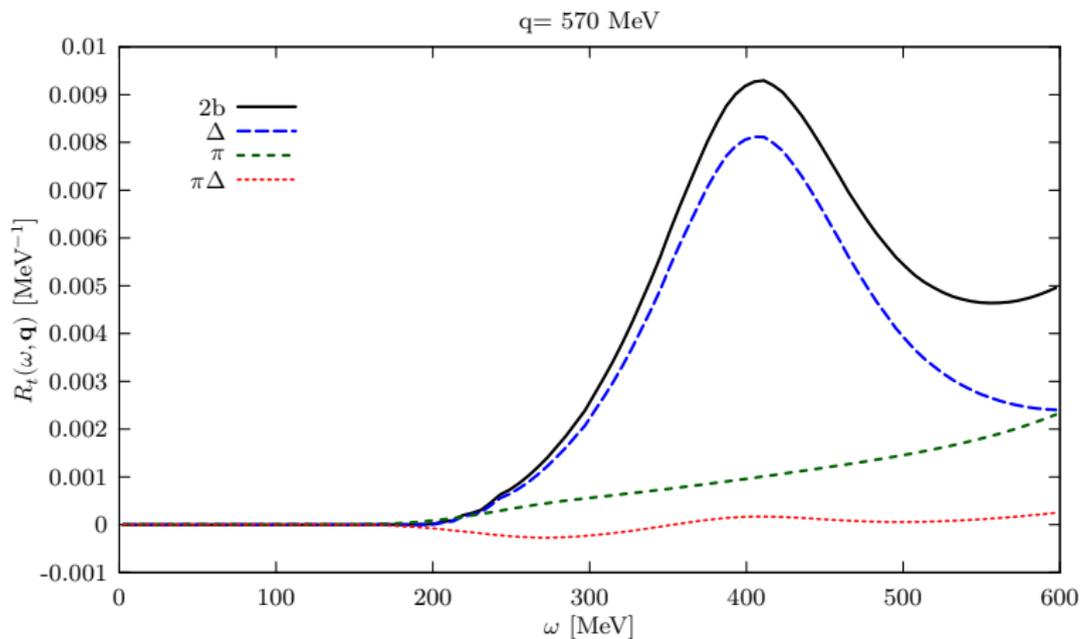
$$W_{2p2h,22}^{\mu\nu} = \int d^3k d^3k' d^3p d^3p' \int dE P_{2h}(\mathbf{k}, \mathbf{k}', E) \langle \mathbf{k}\mathbf{k}' | j_{12}^{\mu} | \mathbf{p}\mathbf{p}' \rangle \langle \mathbf{p}\mathbf{p}' | j_{12}^{\nu} | \mathbf{k}\mathbf{k}' \rangle \\ \times \delta(\mathbf{k} + \mathbf{k}' + \mathbf{q} - \mathbf{p} - \mathbf{p}') \delta(\omega - E - e_p - e_{p'}) \theta(|\mathbf{p}| - k_F) \theta(|\mathbf{p}'| - k_F) .$$

$$P_{2h}(\mathbf{k}, \mathbf{k}', E) = \sum_{h,h' < k_F} |\Phi_{kk'}^{hh'}|^2 \delta(E + e_h + e_{h'})$$

- ▶ 12D integral, can be analytically reduced to a 7D integral → [Monte Carlo integration technique](#)
- ▶ both the direct and Pauli exchange contribution have to be considered (more than 100,000 terms) → [Mathematica and Fortran code](#)

Contribution of the MEC to the transverse response

Separate contributions to the transverse response function $R_T(\omega, q)$ at $q = 570$ MeV: pionic, pionic- Δ interference, Δ and total.



Non Relativistic expression of the 2p2h contribution to $\mathcal{R}_T(\omega, q)$

PHYSICAL REVIEW C

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Relativistic meson exchange and isobar currents in electron scattering: Noninteracting Fermi gas analysis

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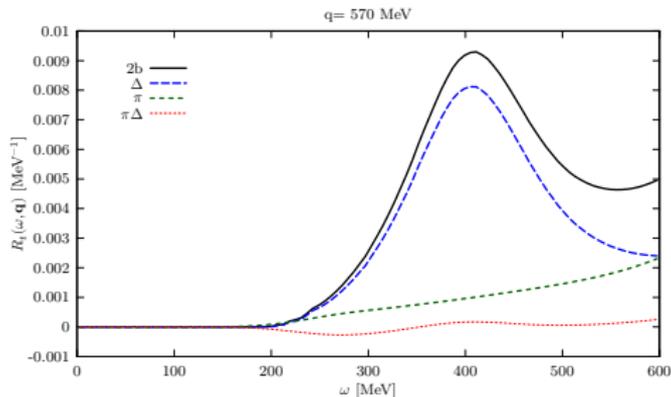
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(Received 22 November 1993)

$$\begin{aligned} \mathcal{R}_T = & 64c_N^2 \left[2 \frac{\mathbf{k}_1^2}{(\mathbf{k}_1^2 + m_\pi^2)^2} + \mathbf{k}_{1T}^2 \left(2 \frac{\mathbf{k}_1^2 \mathbf{k}_2^2}{(\mathbf{k}_1^2 + m_\pi^2)^2 (\mathbf{k}_2^2 + m_\pi^2)^2} - 4 \frac{\mathbf{k}_1^2}{(\mathbf{k}_1^2 + m_\pi^2)^2 (\mathbf{k}_2^2 + m_\pi^2)} + \frac{1}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)} \right) \right] \\ & + 64c_\Delta^2 \mathbf{k}_1^2 \left(\mathbf{k}_1^2 \mathbf{q}^2 (2\bar{b}^2 + \bar{a}^2) - (2\bar{b}^2 - \bar{a}^2)(\mathbf{k}_1 \cdot \mathbf{q})^2 \right) \frac{1}{(\mathbf{k}_1^2 + m_\pi^2)^2} + 64c_\Delta^2 \bar{a}^2 \frac{\mathbf{q}^4 \mathbf{k}_{1T}^2}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)} \\ & + 64c_\Delta c_N \bar{a} \left(\frac{4\mathbf{q}^2 \mathbf{k}_1^2 \mathbf{k}_{1T}^2}{(\mathbf{k}_1^2 + m_\pi^2)^2 (\mathbf{k}_2^2 + m_\pi^2)} - 2 \frac{\mathbf{q}^2 \mathbf{k}_{1T}^2}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)} - 4 \frac{\mathbf{k}_1^2 \mathbf{k}_1 \cdot \mathbf{q}}{(\mathbf{k}_1^2 + m_\pi^2)^2} \right) + (1 \leftrightarrow 2) \end{aligned} \quad (5.11)$$

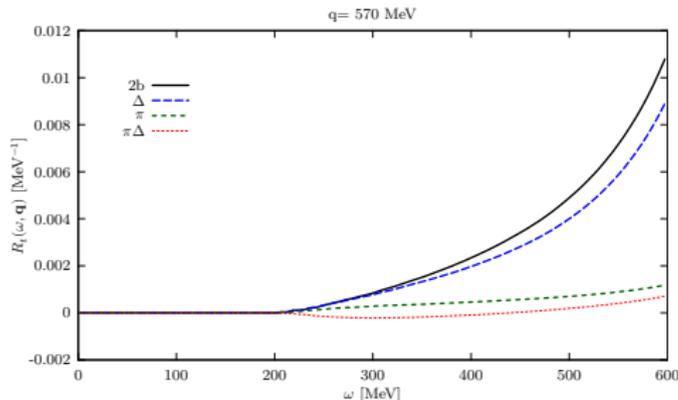
The impact of relativistic effects in the two-body response

Relativity dramatically affects the behaviour of the response.

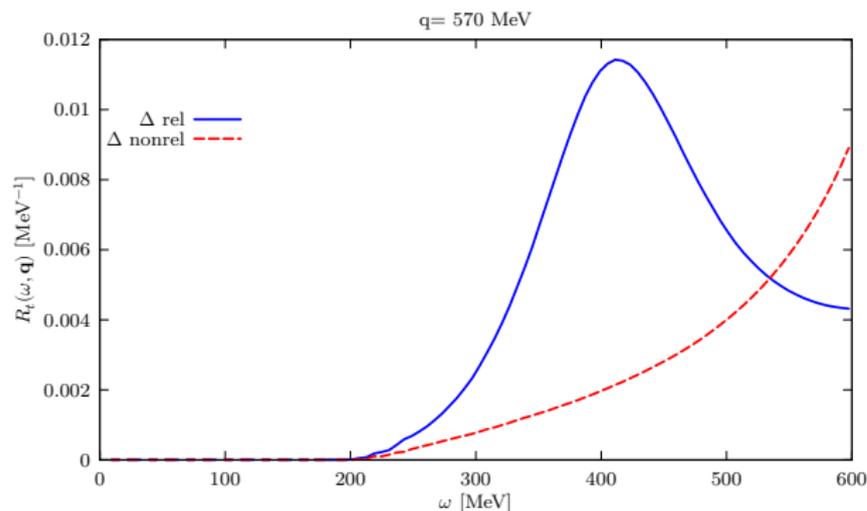


Relativistic

Nonrelativistic



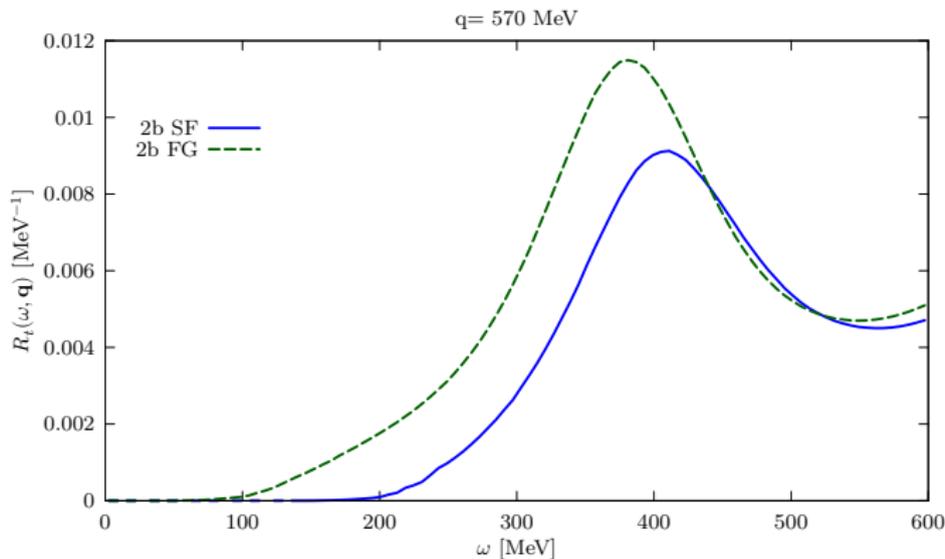
The impact of relativistic effects in the two-body response



The most important effect introduced by relativity is the peak produced by the dynamic Δ -propagation.

The overall effects are **small** in the domain of the QEP, **modest** in the dip region and **substantial** in the region of the Δ -peak. Beyond the Δ peak, relativity yields a **substantial reduction** of the response.

Beyond the non interacting FG ...



Sizable differences

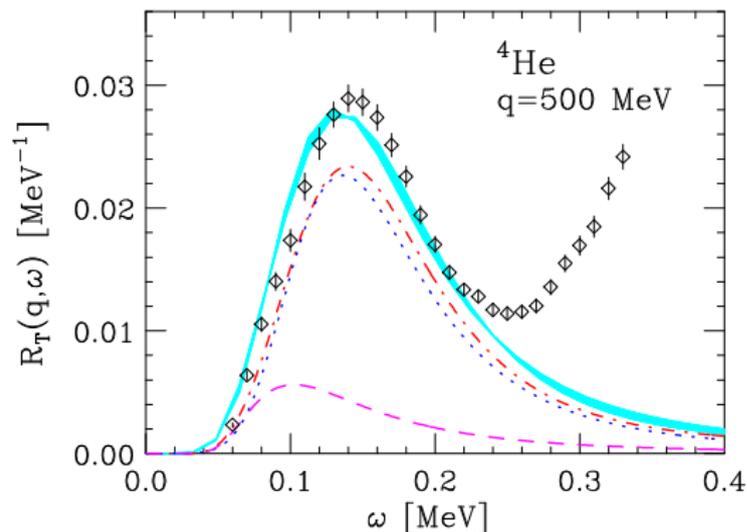
Different threshold \Rightarrow different treatment of the initial state energies of the knocked-out nucleons.

Significant quenching of the response ($\sim 20\%$) \Rightarrow short range correlations.

Production of 2p2h final states

- 1 Initial state correlations
- 2 MEC, two-body response
- 3 **Interference**

The relevance of the interference term. . . $R_T(q, \omega)$



- ▶ Green's Function Monte Carlo calculation of the transverse electromagnetic response function of ${}^4\text{He}$.
- ▶ MEC significantly enhance the transverse response function, not only in the dip region, but also in the quasielastic peak and threshold regions.

It cannot be written in terms of SF...

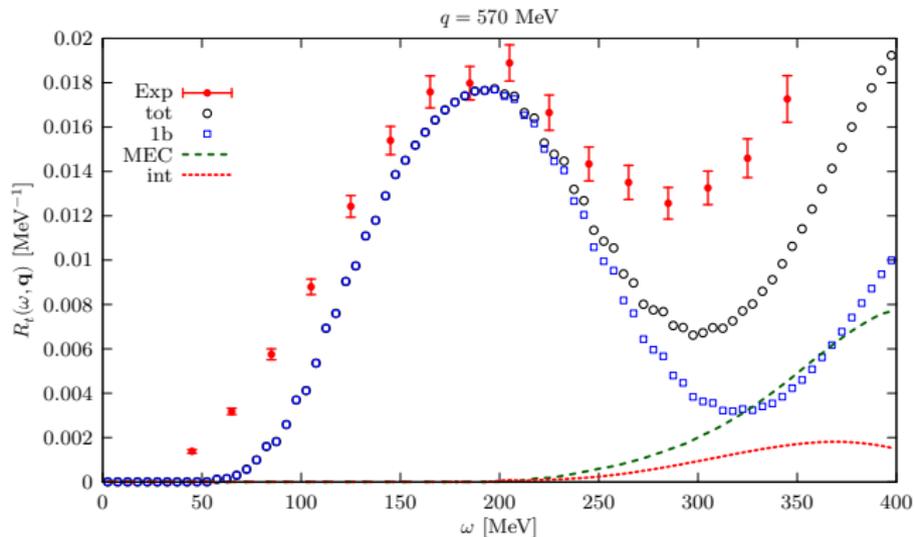
$$\begin{aligned} W^{\mu\nu}_{2p2h,12} = & \int d^3k d^3\xi d^3\xi' d^3h d^3h' d^3p d^3p' \phi_{\xi\xi'}^{hh'*} \left[\Phi_k^{hh'p'} \langle \mathbf{k} | j_1^\mu | \mathbf{p} \rangle \right. \\ & + \left. \Phi_k^{hh'p'} \langle \mathbf{k} | j_2^\mu | \mathbf{p}' \rangle \right] \langle \mathbf{p}, \mathbf{p}' | j_{12}^\nu | \xi, \xi' \rangle \delta(\mathbf{h} + \mathbf{h}' + \mathbf{q} - \mathbf{p} - \mathbf{p}') \\ & \times \delta(\omega + e_h + e_{h'} - e_p - e_{p'}) \theta(|\mathbf{p}| - k_F) \theta(|\mathbf{p}'| - k_F) + \text{h.c.} . \end{aligned}$$

Additional difficulty... This term involves the product of nuclear amplitudes entering in $P(k, E)$ and $P(k, k', E)$

WARNING

This interference contribution would be zero if correlations were not accounted for!

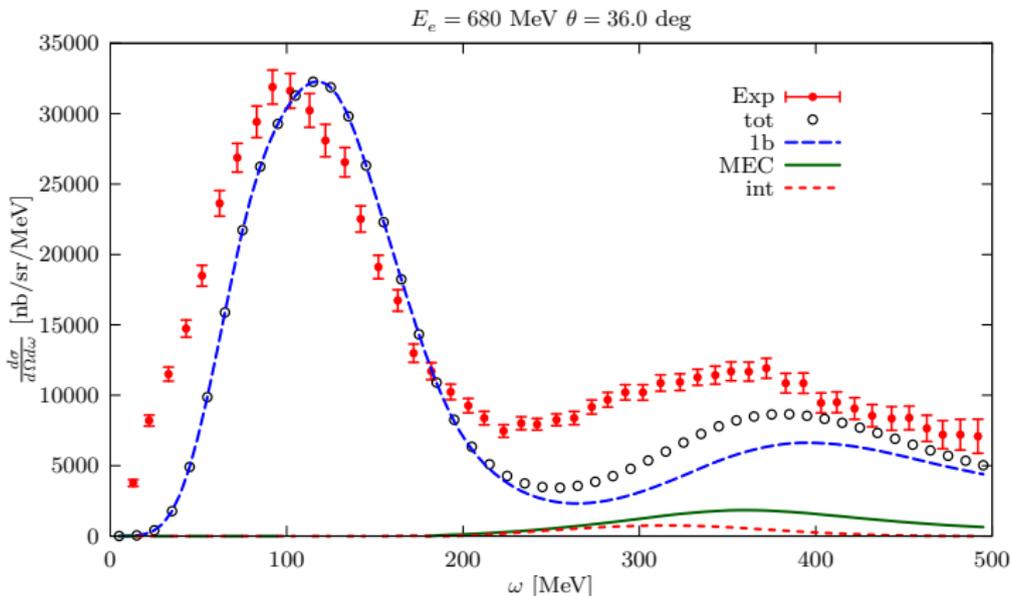
^{12}C electromagnetic response



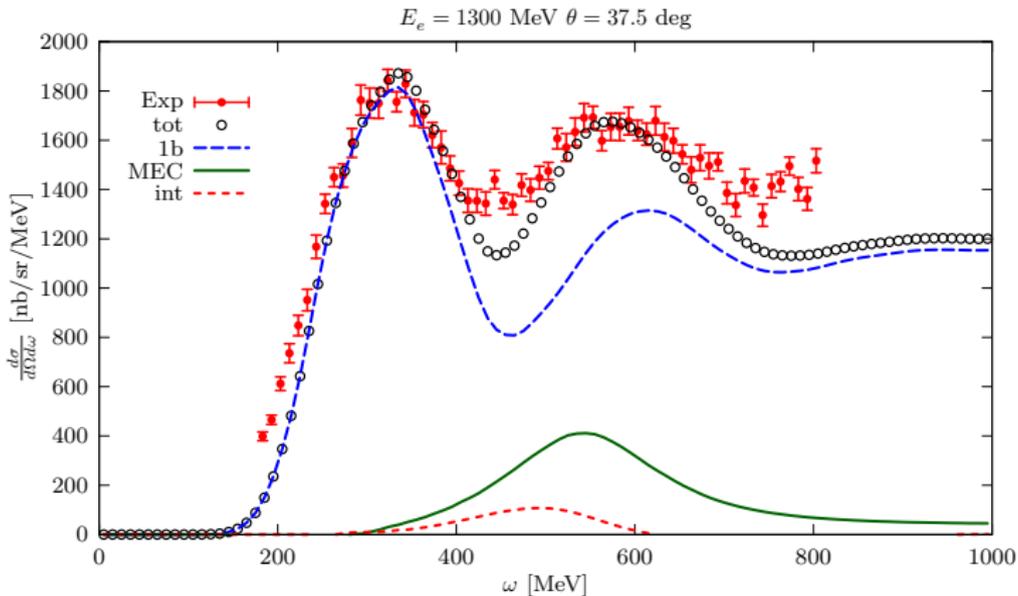
^{12}C calculations indicate a sizable enhancement of the electromagnetic transverse response.

The x -section can be rewritten in terms of R_T and R_L such as

$$\frac{d\sigma}{dE'_e d\Omega} = \sigma_{Mott} \left[\left(\frac{q^2}{\mathbf{q}^2} \right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T \right]$$

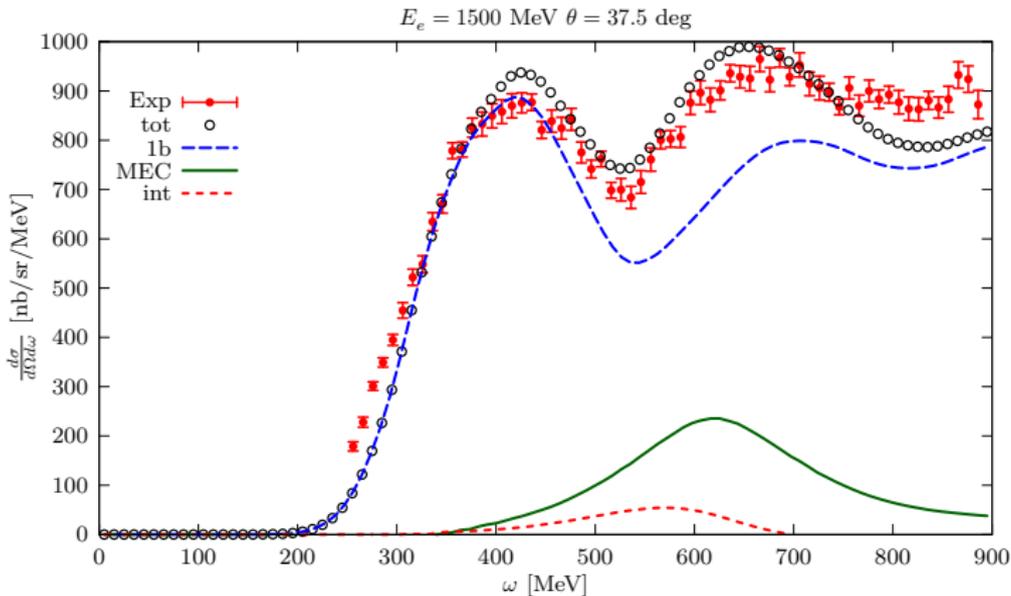


$e^- - {}^{12}\text{C}$ inclusive cross section



The contribution given by the interference term and MEC currents turns out to be sizable in the *dip* region.

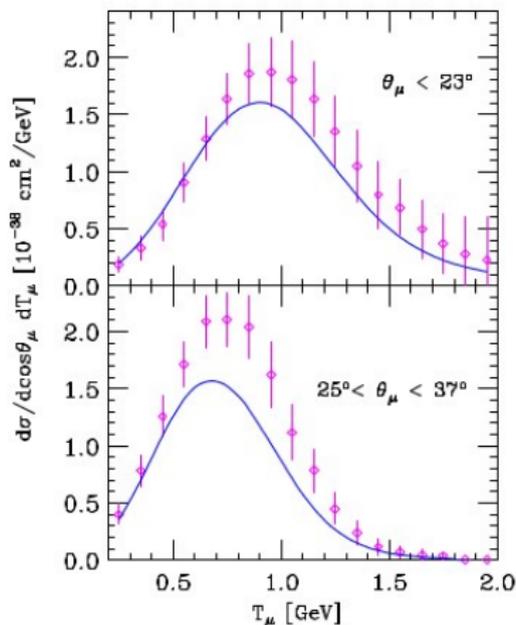
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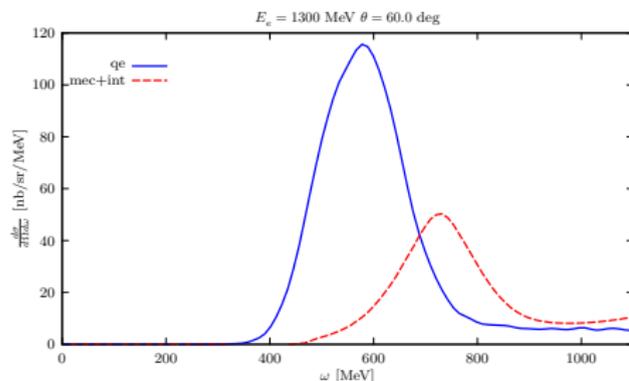
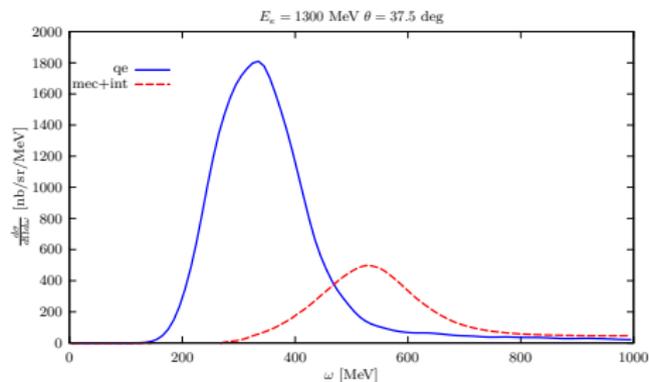
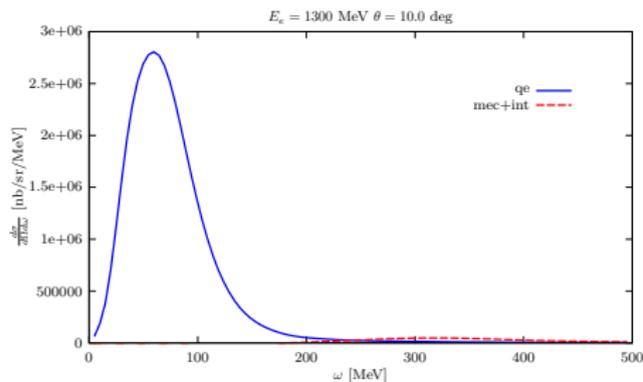
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May MEC explain the MiniBooNE data?

- It is apparent that the disagreement between theoretical calculations not including MEC and data is less pronounced at small θ_μ

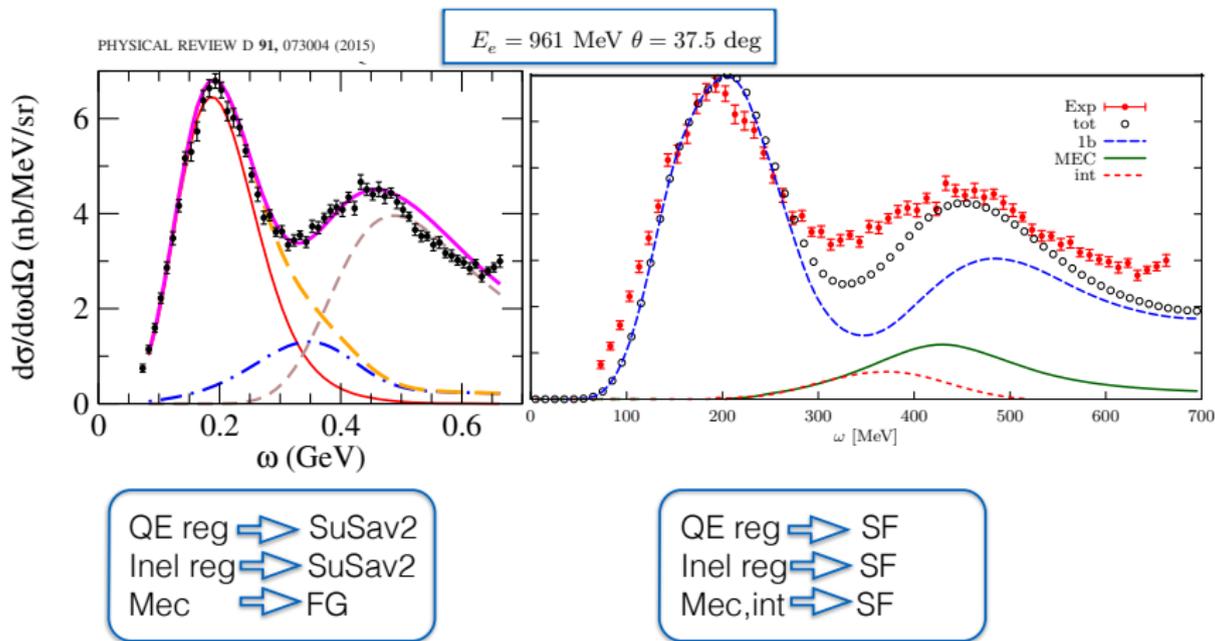


Angular dependence of the two-body contribution



The relative strength due to two-body processes increases for larger values of the scattering angle where the transverse response becomes dominant .

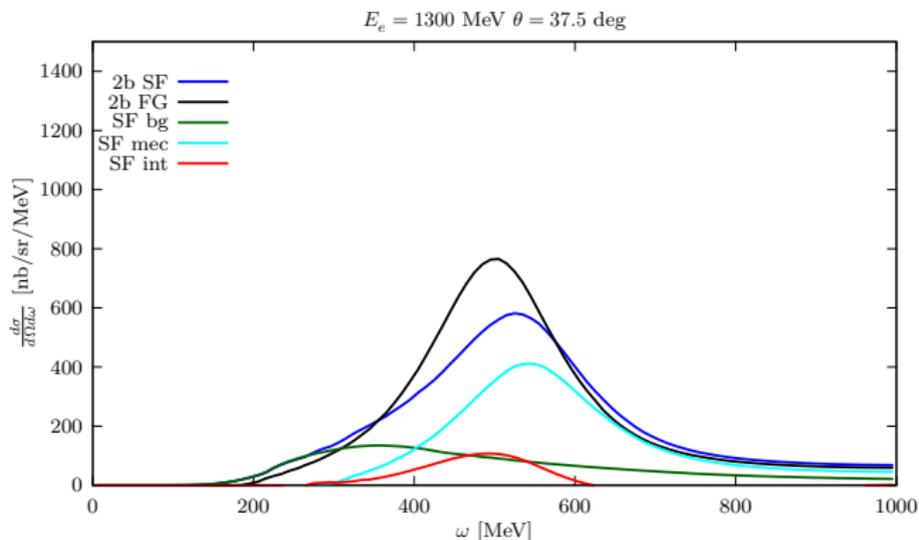
Two-body contribution within the SF and FG formalism



Comparison of inclusive $^{12}\text{C}(e, e')$ cross sections and predictions, using different theoretical approaches.

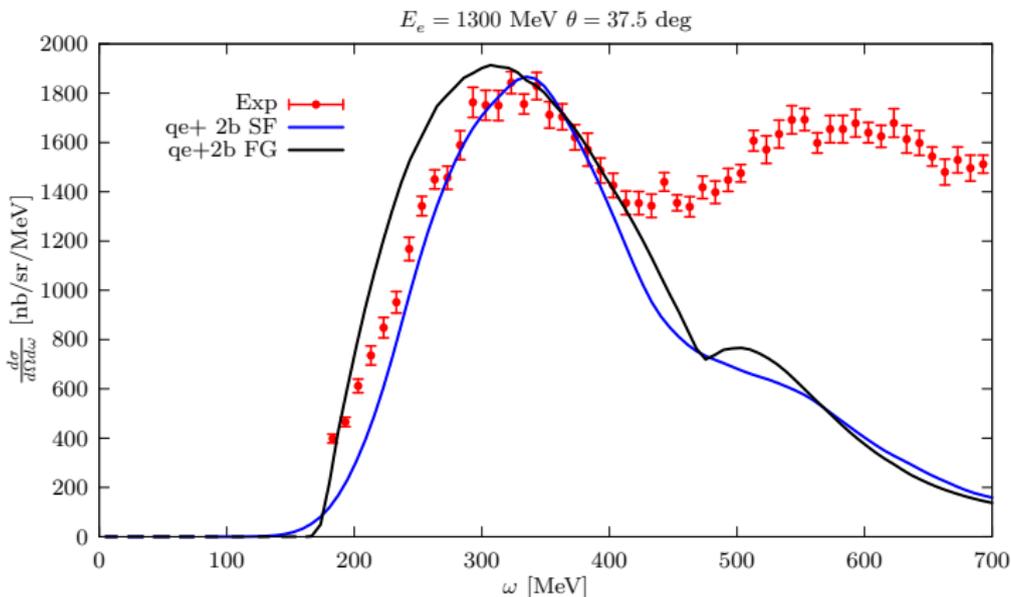
Two-body contribution within the SF and FG formalism

The introduction of the two-nucleon current contributions in theoretical approaches based on the independent particle model (IPM) of nuclear structure, provides a quantitative wealth of the experimental data.



- The total two-body contribution obtained within the SF formalism do not differs too much from the FG result.

e^- - ^{12}C cross section within the SF and FG formalism



- While there are sizable differences both in the position and width of the QE peak, in the “dip” region the results obtained for the e^- - ^{12}C cross section within the SF and FG approaches do not differ significantly.

Convolution scheme

$$\frac{d\sigma^{\text{FSI}}}{d\omega d\Omega} = \int d\omega' f_{\mathbf{q}}(\omega - \omega') \frac{d\sigma^{\text{IA}}}{d\omega d\Omega}$$

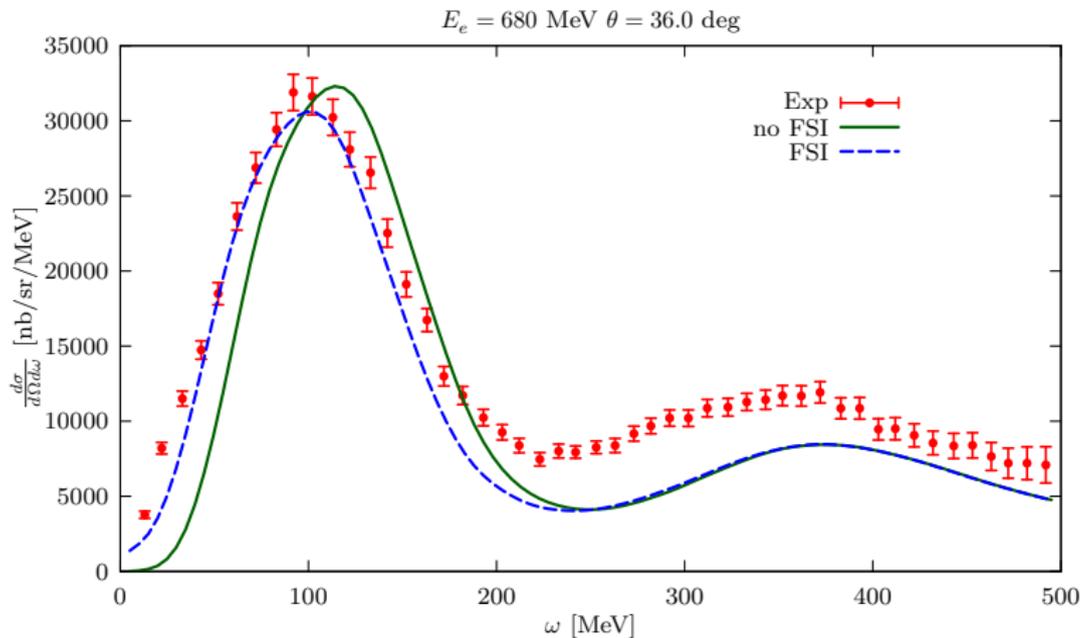
The folding function can be decomposed in the form

$$f_{\mathbf{q}}(\omega) = \delta(\omega) \sqrt{T_A} + (1 - \sqrt{T_A}) F_{\mathbf{q}}(\omega)$$

showing that the strength of FSI is driven by

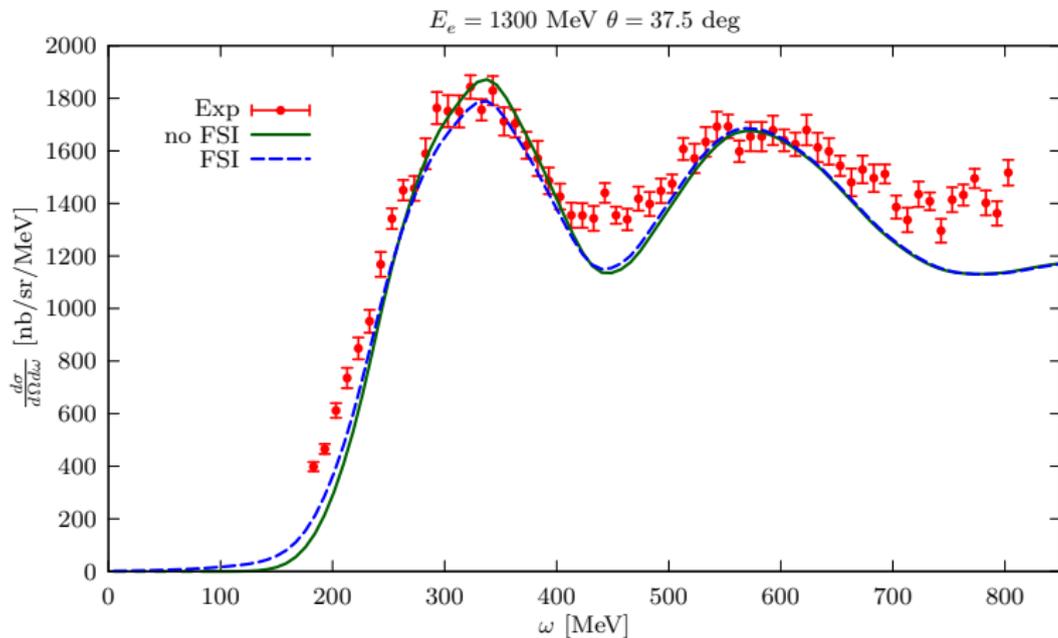
- ▶ the nuclear transparency T_A
- ▶ the finite-width function $F_{\mathbf{q}}(\omega)$
- A. Ankowski *et al.*, Phys. Rev. D 91, 033005 (2015)
- O. Benhar, Phys. Rev. C 87, 024606 (2013).

$e^- - {}^{12}\text{C}$ inclusive cross section including FSI effect



The contribution given by FSI is sizable \rightarrow shift of the QE peak and redistribution of the strength from the peak to the tail in the low ω domain.

$e^- - {}^{12}\text{C}$ inclusive cross section including FSI effect

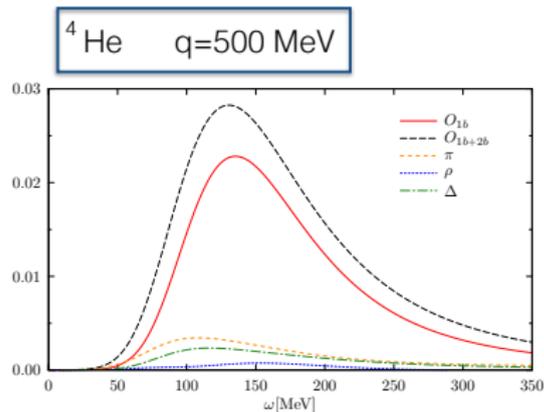
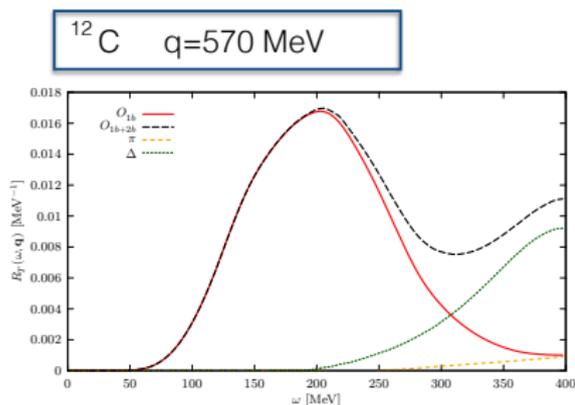


The contribution given by FSI is sizable \rightarrow shift of the QE peak and redistribution of the strength from the peak to the tail in the low ω domain.

- ▶ The treatment of nuclear effects is one of the main sources of systematic uncertainty in the analysis of neutrino-oscillation parameters
- ▶ A description of neutrino - nucleus interactions has to be validated through extensive comparison to the large body of electron-nucleus scattering data
- ▶ The extension of the factorization scheme underlying the IA is a viable option for the development of a unified treatment of processes involving one- and two- nucleon currents in the region of large momentum transfer

- ▶ We analysed the different mechanisms leading to the appearance of $2p2h$ final states in the response of interacting many-body systems. The fully relativistic expression of MEC has been implemented within the SF approach. Both the two-body response and the interference between one- and two- nucleon current have been fully accounted for.
- ▶ The factorized nuclear transition amplitudes involving the one-nucleon current can be corrected to include the effects of FSI in the quasi elastic sector using an extension of the spectral function formalism

Different results obtained within GFMC and SF approach



These differences should be ascribed to...

- Differences in the two-nucleon currents employed in the two cases
- The non relativistic nature of the GFMC calculations
- Interference between amplitudes involving the one- and two-body currents and 1p1h final states

- ▶ We are analysing the contribution of the interference between amplitudes involving the one- and two-body currents and $1p1h$ final states.
- ▶ The implementation of our results in the determination of the nuclear response to electroweak probes will require the introduction of the one- and two-nucleon axial currents, and the calculation of the associated axial-axial and vector-axial responses for both the two-body and interference terms.
- ▶ We will apply our approach in the data analysis of new generation neutrino experiments which use liquid Argon detectors. To do that, it will be necessary to extend the spectral function formalism in order to describe the non-isospin symmetric nuclei.

backup slides

Convolution scheme

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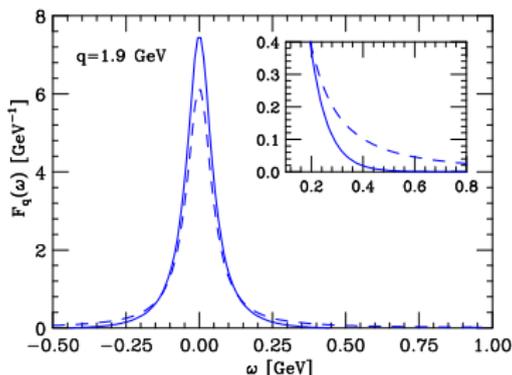
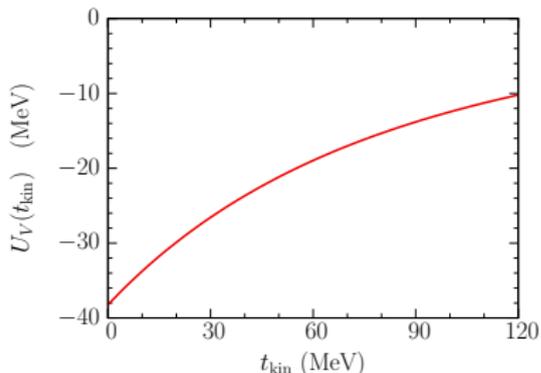
- ▶ the nuclear transparency T_A
- ▶ the finite-width function $F_{\mathbf{q}}(\omega)$

Inclusion of Final State Interaction contribution

- ▶ $f_{\mathbf{q}}(\omega - \omega' - U_V)$
- ▶ We consider $T_A = T_A(t_{kin})$ and $U_V = U_V(t_{kin})$ where

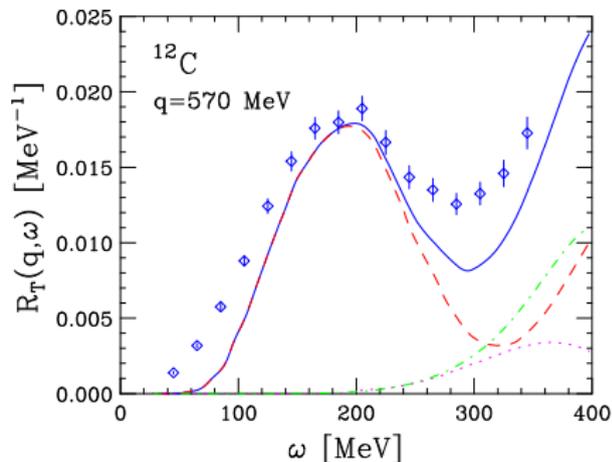
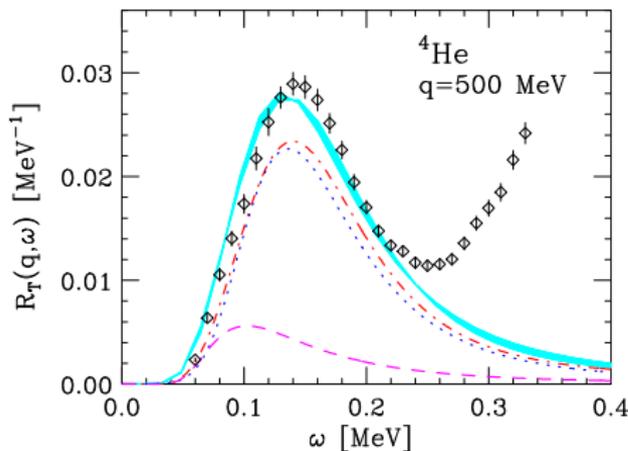
$$t_{kin} = \frac{E_k^2(1 - \cos\theta)}{M + E_K(1 - \cos\theta)}$$

- ▶ We neglect the \mathbf{q} dependence of $F_{\mathbf{q}}(\omega)$
- ▶ $F_{\mathbf{q}}(\omega)$ at $|\mathbf{q}| \sim 2$ GeV, including NN correlations



- A. Ankowski *et al.*, Phys. Rev. D 91, 033005 (2015)
- O. Benhar, Phys. Rev. C 87, 024606 (2013).

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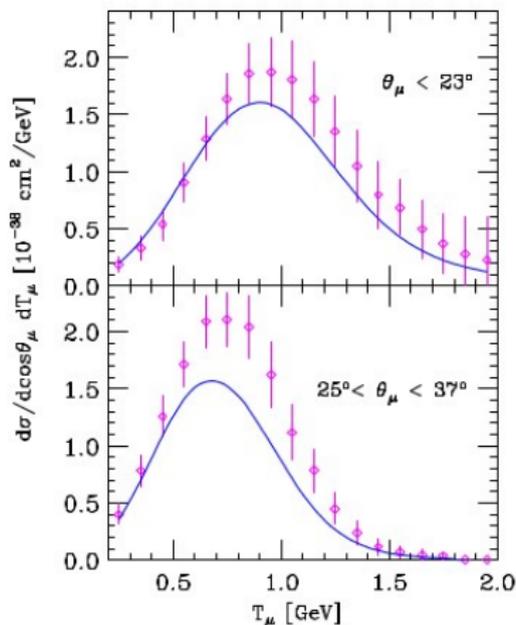


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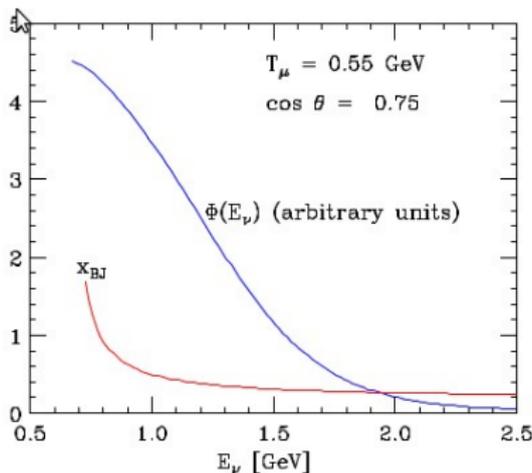
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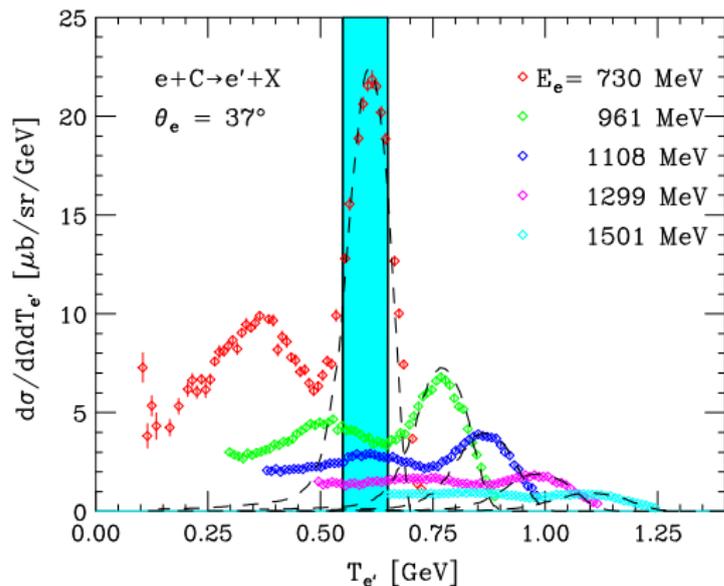
Contribution of different reaction mechanisms

- As neutrino beams are produced as secondary decay products their energy is broadly distributed
- The flux-averaged cross section at fixed T_μ and θ_μ picks up contributions at different beam energies



- ▶ $x=0.5 \rightarrow E_\nu = 0.788 \text{ GeV}$, $x=1 \rightarrow E_\nu = 0.975 \text{ GeV}$.
- ▶ $\Phi(0.975)/\Phi(0.788) = 0.83$

"Flux averaged" QE electron-Carbon cross section



- Electron-Carbon scattering cross sections at $\theta_e = 37^\circ$ plotted as a function of $T_{e'}$.
- Reaction mechanisms other than single-nucleon knockout contribute to the "flux-averaged" cross section.

- ▶ development of models based on a **new paradigm**, in which all relevant reaction mechanisms are *consistently* taken into account within a unified description of nuclear dynamics.

- The hadronic tensor can be written in the simple form

$$W_A^{\mu\nu} = \int d^3p dE P(\mathbf{p}, E) \frac{M}{E_p} [Z W_p^{\mu\nu} + (A - Z) W_n^{\mu\nu}] ,$$

- Elements entering the definition of the IA x-section

- ▶ the tensor describing the interactions of the i -th nucleon in free space

$$W_\alpha^{\mu\nu} = \sum_X \langle -\mathbf{p}_R, N | j_\alpha^{\mu\dagger} | X, \mathbf{p}_X \rangle \langle X, \mathbf{p}_X | j_\alpha^\nu | -\mathbf{p}_R, N \rangle \delta^{(4)}(\tilde{q} - p_R - p_X) .$$

$$\tilde{\omega} = E_X - \sqrt{\mathbf{p}^2 + M^2} = \omega + M - E - \sqrt{\mathbf{p}^2 + M^2}$$

- ▶ The nucleon energy and momentum distribution, described by the hole spectral functions

Violation of current conservation

The replacement of ω with $\tilde{\omega}$ leads to a violation of the current conservation:

$$q_\mu w_N^{\mu\nu} = 0$$

Prescription proposed by *de Forest*:

$$\tilde{w}_N^{\mu\nu} = w_N^{\mu\nu}(\tilde{q})$$

$$\tilde{w}_N^{3\nu} = \frac{\omega}{|\mathbf{q}|} w_N^{0\nu}(\tilde{q})$$

The violation of gauge invariance only affects the longitudinal response. As a consequence, it is expected to become less and less important as the momentum transfer increases, electron scattering at large $|\mathbf{q}|$ being largely dominated by transverse contributions.

Local Density Approximation (LDA) $P(\mathbf{k}, E)$ for oxygen

$$P_{LDA}(\mathbf{p}, E) = P_{MF}(\mathbf{p}, E) + P_{\text{corr}}(\mathbf{p}, E)$$

- $P_{MF}(\mathbf{p}, E) \rightarrow$ from $(e, e'p)$ data
- $P_{\text{corr}}(\mathbf{p}, E) \rightarrow$ from uniform nuclear matter calculations at different densities:

$$P_{MF}(\mathbf{p}, E) = \sum_{n \in \{F\}} Z_n |\phi_n(\mathbf{p})|^2 F_n(E - E_n)$$

$$P_{\text{corr}}(\mathbf{p}, E) = \int d^3r \varrho_A(\mathbf{r}) P_{\text{corr}}^{NM}(\mathbf{p}, E; \varrho = \varrho_A(\mathbf{r}))$$

Hadronic monopole form factors

$$\begin{aligned} F_{\pi NN}(k^2) &= \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - k^2} \\ F_{\pi N\Delta}(k^2) &= \frac{\Lambda_{\pi N\Delta}^2}{\Lambda_{\pi N\Delta}^2 - k^2} \end{aligned} \quad (3)$$

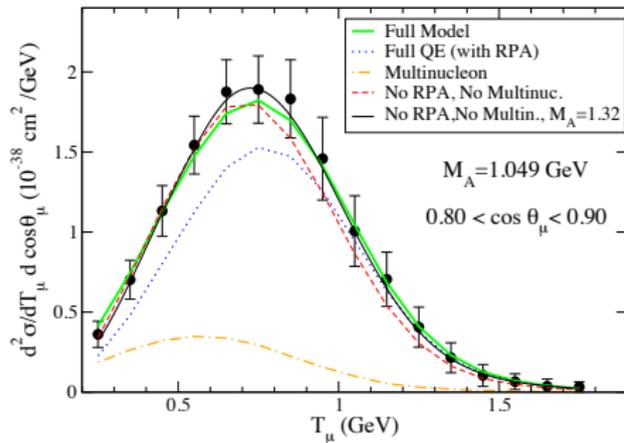
and the EM ones

$$\begin{aligned} F_{\gamma NN}(q^2) &= \frac{1}{(1 - q^2/\Lambda_D^2)^2}, \\ F_{\gamma N\Delta}(q^2) &= F_{\gamma NN}(q^2) \left(1 - \frac{q^2}{\Lambda_2^2}\right)^{-1/2} \left(1 - \frac{q^2}{\Lambda_3^2}\right)^{-1/2} \end{aligned} \quad (4)$$

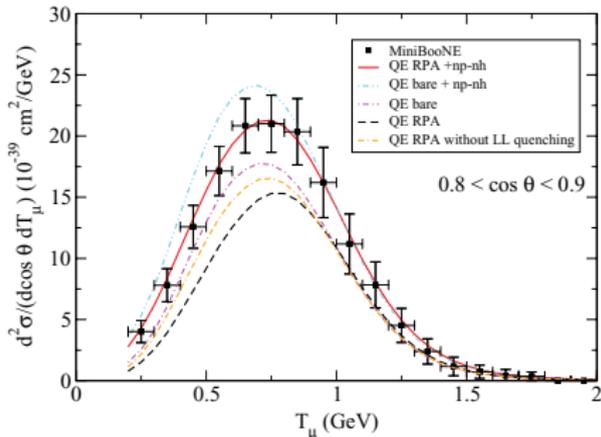
where $\Lambda_\pi = 1300$ MeV, $\Lambda_{\pi N\Delta} = 1150$ MeV, $\Lambda_D^2 = 0.71 \text{GeV}^2$,
 $\Lambda_2 = M + M_\Delta$ and $\Lambda_3^2 = 3.5 \text{GeV}^2$.

Including MEC within the IPM

- ▶ J. Nieves *et al*, Phys. Lett. B **707**, 72 (2012)



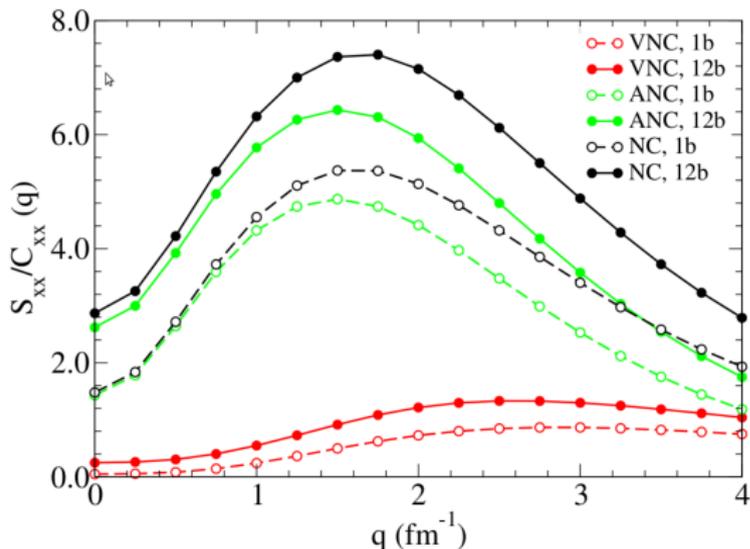
- ▶ M. Martini *et al*, Phys. Rev.C **80**, 065501 (2009);



- After the inclusion of MEC, both schemes turn out to provide a quantitative account of the data
- A fully consistent treatment of 2p2h processes requires a realistic model of nuclear structure, taking into account the effects of NN correlations.

Neutral weak current two-body contributions

The enhancement due to two- nucleon currents, at $q \simeq 1 \text{ fm}^{-1}$, is about 50% relative to the one-body values.



- ▶ Low momentum transfer the dominant contribution is given by: $\langle ij_{2b}^{\dagger} j_{2b} | i \rangle$
- ▶ At higher momentum transfer: $\langle ij_{2b}^{\dagger} j_{1b} | i \rangle + \langle ij_{1b}^{\dagger} j_{2b} | i \rangle$ plays a more important role.

- ▶ A.Lovato *et al.*, Phys. Rev. Lett. 112, 182502 (2014)